COMMON FIXED POINTS OF FOUR MAPS USING GENERALIZED WEAK CONTRACTIVITY AND WELL-POSEDNESS

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Abstract. In this paper, we introduce the concept of generalized φ-contractivity of a pair of maps w.r.t. another pair. We establish a common fixed point result for two pairs of self-mappings, when one of these pairs is generalized φ-contraction w.r.t. the other and study the well-posedness of their fixed point problem. In particular, our fixed point result extends the main result of a recent paper of Qingnian Zhang and Yisheng Song.

1. Introduction

The concept of the weak contraction was defined by Alber and Guerre-Delabriere [1] in 1997. Actually in [1], the authors defined such mappings for single-valued maps on Hilbert spaces and proved the existence of fixed points.

Definition 1.1. Let (X, d) be a metric space and S be self-mapping of X. Let φ : [0, ∞) → [0, ∞) be a function such that φ(0) = 0 and φ is positive on (0, ∞). We say that T is a φ-weak contraction if we have

\[ d(Tx, Ty) \leq d(fx, fy) - \phi(d(fx, fy)) \]  \hspace{1cm} (1.1)

for all x, y in X

Rhoades [9] showed that most results of [1] are still true for any Banach space. Also Rhoades [9] proved the following important fixed point theorem which is one of generalizations of the Banach contraction principle [3], because it contains contractions as special case (φ(t) = (1 − k)t).

Theorem 1.2. (Rhoades [9], Theorem 2). Let (X, d) be a complete metric space, and let T be a φ-weak contraction on X. If φ : [0, ∞) → [0, ∞) is a continuous and nondecreasing function such that φ(0) = 0 and φ is positive on (0, ∞), then T has a unique fixed point.

Two generalizations of this result were given by I. Beg and M. Abbas in [4] and by A. Azam and M. Shakeel in [2].

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Recently, this theorem was recently extended by Qingnian Zhang and Yisheng Song (see [12]) to the context of generalized weak contractions. More precisely, the following result was established in [12].

**Theorem 1.3.** ([12]) Let \((X, d)\) be a complete metric space and \(S, T : X \to X\) be self-mappings of \(X\) such that
\[
d(Tx, Sy) \leq N(x, y) - \phi(N(x, y)), \quad \forall \, x, y \in X,
\]
where \(\phi : [0, \infty) \to [0, \infty)\) is a lower semi-continuous function with \(\phi(t) > 0\) for all \(t \in (0, \infty)\) and \(\phi(0) = 0\) and
\[
N(x, y) = \max\{d(x, y), d(Tx, x), d(Sy, y), \frac{1}{2}[d(y, Tx) + d(x, Sy)]\}.
\]
Then there exists a unique point \(u \in X\) such that \(u = Tu = Su\).

In this paper, we introduce the concept of a pair of mappings which is generalized weakly contractive w.r.t. another pair of mappings by means of a function \(\phi\) in the class \(\Phi\) of functions considered in Theorem 1.3. We establish a common fixed point result for two pairs of self-mappings, when one of these pairs is generalized \(\phi\)-contraction w.r.t. the other and study the well-posedness of their fixed point problem. In particular, our fixed point result (see Theorem 2.4 below) extends Theorem 1.3 of Qingnian Zhang and Yisheng Song (see [12]).

The main result of the second section is Theorem 2.4.

In the third section, we study the well-posedness of the common fixed point problem for two pairs of self-mappings of a metric space such that one of them is \(\phi\)-weakly contractive w.r.t. the other. The main result of this section is Theorem 3.3.

2. Coincidence and common fixed points

We start with some definitions.

**Definition 2.1.** Let \(X\) be a nonempty set and \(S, T\) self-mappings on \(X\).

A point \(x \in X\) is called a coincidence point of \(S\) and \(T\) if \(Sx = Tx\).

A point \(w \in X\) is called a point of coincidence of \(S\) and \(T\) if there exists a coincidence point \(x \in X\) of \(S\) and \(T\) such that \(w = Sx = Tx\).

S and \(T\) are weakly compatible if they commute at their coincidence points, that is if \(STx = TSx\), whenever \(Sx = Tx\).

We recall that the concept of weak compatibility was introduced by Jungck and Rhoades [6].

**Definition 2.2.** Let \(\Phi\) be the set of functions \(\phi : [0, \infty) \to [0, \infty)\) satisfying the following properties:

\((\phi_1)\): \(\phi\) is lower semi-continuous.

\((\phi_2)\): \(\phi(0)\) and \(\phi(t) > 0\) for all \(t > 0\).

**Definition 2.3.** Let \((X, d)\) be a metric space. Let \(S, T, I, J : X \to X\) be four self-mappings of \(X\).

Let \(\phi \in \Phi\). The pair \((S, T)\) is called generalized \(\phi\)-weakly contractive with respect to the pair \((I, J)\) if we have
\[
d(Sx, Ty) \leq M(x, y) - \phi(M(x, y)),
\]
where \(\phi : [0, \infty) \to [0, \infty)\) is a lower semi-continuous function with \(\phi(t) > 0\) for all \(t > 0\) and \(\phi(0) = 0\) and
\[
M(x, y) = \max\{d(x, y), d(Sx, x), d(Ty, y), \frac{1}{2}[d(y, Tx) + d(x, Ty)]\}.
\]
for all \(x, y \in X\), where
\[
M(x, y) := \max\{d(Ix, Jy), d(Ix, Sx), d(Jy, Ty), \frac{1}{2}[d(Ix, Ty) + d(Jy, Sx)]\}.
\]

The pair \((S, T)\) is called generalized weakly contractive with respect to the pair \((I, J)\) if it is generalized \(\phi\)-weakly contractive with respect to \((I, J)\) with some \(\phi \in \Phi\).

We observe that if \(I = J = Id_X\) is the identity mapping, then \(N(x, y) = M(x, y)\) for all \(x, y \in X\).

The main result of this section reads as follows.

**Theorem 2.4.** Let \((X, d)\) be a metric space and let \(S, T, I, J\) be four self-mappings of \(X\). Let \(\phi \in \Phi\).

We suppose that:

1. \((H1): \text{The pair } (S, T) \text{ is generalized } \phi\text{-weakly contractive with respect to the pair } (I, J), \text{ that is}
   \[
   d(Sx, Ty) \leq M(x, y) - \phi(M(x, y)),
   \]
2. \((H2): S(X) \subset J(X) \text{ and } T(X) \subset I(X).
3. \((H3): \text{One of the subsets } S(X), T(X), I(X) \text{ or } J(X) \text{ is a complete subspace of } X.

Then,

a) the pair \(\{S, I\}\) has a point of coincidence,

b) the pair \(\{T, J\}\) has a point of coincidence.

Moreover, if the pairs \(\{S, I\}\) and \(\{T, J\}\) are weakly compatible, then the mappings \(S, T, I\) and \(J\) have a unique common fixed point in \(X\).

**Proof.** Let \(x_0\) be an arbitrary point in \(X\). Set \(y_0 = Sx_0\). Since \(S(X) \subset J(X)\), then we can find a point \(x_1 \in X\) such that \(y_0 = Sx_0 = Jx_1\). Set \(y_1 = Tx_1\). Since \(T(X) \subset I(X)\), then there exists a point \(x_2 \in X\) such that \(y_1 = Tx_1 = Ix_2\). By induction, we construct two sequences \((x_n)\) and \((y_n)\) in \(X\) satisfying for each nonnegative integer \(n\),
\[
y_{2n} = Sx_{2n} = Jx_{2n+1} \quad \text{and} \quad y_{2n+1} = Tx_{2n+1} = Ix_{2n+2}
\]
(3.3)

To simplify notation, for each nonnegative integer \(n\), we set \(t_n := d(y_n, y_{n+1})\).

For all nonnegative integer \(n\) we have
\[
t_{2n+1} = d(y_{2n+2}, y_{2n+1}) = d(Sx_{2n+2}, Tx_{2n+1})
\leq M(x_{2n+2}, x_{2n+1}) - \phi(M(x_{2n+2}, x_{2n+1}))
= \max\{t_{2n}, t_{2n+1}, \frac{1}{2}d(y_{2n}, y_{2n+2})\} - \phi(\max\{t_{2n}, t_{2n+1}, \frac{1}{2}d(y_{2n}, y_{2n+2})\}).
\]
(4.4)

Since \(\frac{1}{2}d(y_{2n}, y_{2n+2}) \leq \frac{1}{2}(t_{2n} + t_{2n+1})\), then
\[
\max\{t_{2n}, t_{2n+1}, \frac{1}{2}d(y_{2n}, y_{2n+2})\} = \max\{t_{2n}, t_{2n+1}\}.
\]
Suppose that \(t_{2n} < t_{2n+1}\). Then by (4.4) we obtain
\[
0 < t_{2n+1} \leq t_{2n+1} - \phi(t_{2n+1}) < t_{2n+1},
\]
a contradiction. Thus \(t_{2n} \geq t_{2n+1}\), and
\[
0 < t_{2n+1} \leq t_{2n} - \phi(t_{2n}).
\]
We conclude that for all nonnegative integer \( n \), we have
\[
\phi(t) \leq \lim \inf \phi(t_n) \leq \lim (t_n - t_{n+1}) = 0.
\]
Thus \( 0 \leq \phi(t) \leq 0 \), which implies that \( \phi(t) = 0 \). By property \((\phi_2)\), we obtain \( t = 0 \).

Let us show that \( \{y_n\} \) is a Cauchy sequence. Since \( \lim_{n \to \infty} d(y_n, y_{n+1}) = 0 \), then we need only to show that \( \{y_{2n}\} \) is a Cauchy sequence. To get a contradiction, let us suppose that there is a number \( \epsilon > 0 \) and two sequences \( \{2n(k)\}, \{2m(k)\} \) with \( 2k \leq 2m(k) < 2n(k), (k \in \mathbb{N}) \) verifying
\[
d(y_{2n(k)}, y_{2m(k)}) > \epsilon.
\]
For each integer \( k \), we shall denote \( 2n(k) \) the least even integer exceeding \( 2m(k) \) for which \((2.6)\) holds. Then we have
\[
d(y_{2m(k)}, y_{2n(k)-2}) \leq \epsilon \quad \text{and} \quad d(y_{2m(k)}, y_{2n(k)}) > \epsilon.
\]
For each integer \( k \), we set \( p_k := d(y_{2m(k)}, y_{2n(k)}) \), then we have
\[
\epsilon < p_k = d(y_{2m(k)}, y_{2n(k)}) \leq d(y_{2m(k)}, y_{2n(k)-2}) + d(y_{2n(k)-2}, y_{2n(k)-1}) + d(x_{2n(k)-1}, y_{2n(k)}) \leq \epsilon + t_{2n(k)-2} + t_{2n(k)-1}.
\]

Since the sequence \( \{t_n\} \) converges to 0, we deduce from \((2.8)\) that \( \{p_k\} \) converges to \( \epsilon \). For every integer \( k \in \mathbb{N} \) we set
\[
q_k := d(y_{2m(k)+1}, y_{2n(k)+2}), \quad r_k := d(y_{2m(k)}, y_{2n(k)+1}),
\]
\[
s_k := d(y_{2m(k)+1}, y_{2n(k)+1}), \quad v_k := d(y_{2m(k)}, y_{2n(k)+2}).
\]
By using the triangle inequality, for all integer \( k \), we obtain the following estimates:
\[
|r_k - p_k| \leq t_{2n(k)} \leq t_k,
\]
\[
|r_k - s_k| \leq t_{2m(k)} \leq t_k,
\]
\[
|s_k - q_k| \leq t_{2n(k)+1} \leq t_k,
\]
\[
|v_k - q_k| \leq t_{2m(k)} \leq t_k.
\]
Since the sequence \( \{t_n\} \) converges to 0, we deduce that the sequences: \( \{q_k\}, \{r_k\}, \{s_k\} \) and \( \{v_k\} \) converge to \( \epsilon \).

For all nonnegative integer \( k \), we have
\[
M(x_{2n(k)+2}, x_{2m(k)+1}) = \max\{d(y_{2n(k)+1}, y_{2m(k)}), d(y_{2n(k)+1}, y_{2n(k)+2}),
\]
\[
d(y_{2m(k)}, y_{2m(k)+2}), d(y_{2n(k)+1}, y_{2m(k)+1}), d(y_{2m(k)}, y_{2n(k)+2})\}
\]
\[
= \max\{r_k, t_{2n(k)+1}, t_{2m(k)}, s_k, v_k\}.
\]
Then, by using the condition (2.1), for every non-negative integer $k$, we have the following estimates:

$$q_k = d(y_{2n(k)+2}, y_{2m(k)+1}) = d(Sx_{2n(k)+2}, Tx_{2m(k)+1})$$

$$\leq M(x_{2n(k)+2}, x_{2m(k)+1}) - \phi(M(x_{2n(k)+2}, x_{2m(k)+1}))$$

$$\leq \max\{r_k, t_{2n(k)+1}, t_{2m(k)}, s_k, v_k\} - \phi(\max\{r_k, t_{2n(k)+1}, t_{2m(k)}, s_k, v_k\}).$$

Then, we obtain

$$\phi(\max\{r_k, t_{2n(k)+1}, t_{2m(k)}, s_k, v_k\}) \leq \max\{r_k, t_{2n(k)+1}, t_{2m(k)}, s_k, v_k\} - q_k.$$

Letting $k$ tend to $\infty$ and using the lower semicontinuity of $\phi$, we get

$$\phi(\epsilon) \leq \liminf_{k \to \infty} \phi(\max\{r_k, t_{2n(k)+1}, t_{2m(k)}, s_k, v_k\})$$

$$\leq \lim_{k \to \infty} (\max\{r_k, t_{2n(k)+1}, t_{2m(k)}, s_k, v_k\} - q_k) = 0,$$

which implies $\phi(\epsilon) = 0$ a contradiction to property $(\phi_2)$. Thus $\{y_n\}$ is a Cauchy sequence.

Suppose that $J(X)$ is a complete subspace of $X$, Since $M$ is complete, then the sequence $\{y_n\}$ converges to a point (say) $z \in J(X)$. Thus we have

$$z = \lim_{n \to \infty} Sx_{2n} = \lim_{n \to \infty} Jx_{2n+1} = \lim_{n \to \infty} Tx_{2n+1} = \lim_{n \to \infty}Ix_{2n}.$$  \hfill (2.9)

Let $u \in X$ such that $z = Ju$. By inequality (2.1), we obtain

$$d(y_{2n}, Tu) = d(Sx_{2n}, Tu)$$

$$\leq M(x_{2n}, u) - \phi(d(x_{2n}, u))$$

$$= \max\{d(Ix_{2n}, z), d(Ix_{2n}, Sx_{2n}), d(z, Tu), \frac{1}{2}[d(Ix_{2n}, Tu) + d(z, Sx_{2n})]\}$$

$$- \phi(\max\{d(Ix_{2n}, z), d(Ix_{2n}, Sx_{2n}), d(z, Tu), \frac{1}{2}[d(Ix_{2n}, Tu) + d(z, Sx_{2n})]\}),$$

from which, we get

$$\phi(\max\{d(Ix_{2n}, z), d(Ix_{2n}, Sx_{2n}), d(z, Tu), \frac{1}{2}[d(Ix_{2n}, Tu) + d(z, Sx_{2n})]\})$$

$$\leq \max\{d(Ix_{2n}, z), d(Ix_{2n}, Sx_{2n}), d(z, Tu), \frac{1}{2}[d(Ix_{2n}, Tu) + d(Sx_{2n}, z)]\} - d(y_{2n}, Tu).$$

By letting $n$ tend to infinity and using lower semi-continuity, we obtain

$$\phi(d(z, Tu))$$

$$\leq \liminf_{n \to \infty} \phi(\max\{d(Ix_{2n}, z), d(Ix_{2n}, Sx_{2n}), d(z, Tu), \frac{1}{2}[d(Ix_{2n}, Tu) + d(z, Sx_{2n})]\})$$

$$\leq \phi(d(z, Tu)) - d(z, Tu),$$

which implies that $d(z, Tu)$. Hence we have $z = Ju = Tu$. Since $T(X) \subset I(X)$, then there exists $w \in X$ such that $z = Tu = Tw$. By using inequality (2.1), we have

$$d(Sw, z) = d(Sw, Tu) \leq M(w, u) - \phi(M(w, u)).$$
Since
\[ M(w, u) = \max\{d(Iw, Ju), d(Iw, Sw), d(Ju, Tu), \frac{1}{2}[d(Iw, Tu) + d(Ju, Sw)]\} \]
\[ = \max\{0, d(z, Sw), 0, \frac{1}{2}[d(z, Sw)]\} \]
\[ = d(z, Sw). \]
We deduce that
\[ d(Sw, z) \leq d(z, Sw) - \phi(d(z, Sw)), \]
from which, we get \( \phi(d(z, Sw)) = 0 \), which implies that \( d(Sw, z) = 0 \), thus \( z = Sw = Iw \). We conclude that
\[ Sw = Iw = z = Ju = Tu. \] (2.10)
So the conclusions a) and b) are obtained. By similar arguments, the same conclusions will be obtained if we suppose that one of \( S(X) \), \( T(X) \) or \( I(X) \) is a complete subspace of \( X \).

Suppose that the pairs \( \{S, I\} \) and \( \{T, J\} \) are weakly compatible, then by (2.10), we have
\[ Sz = Iz \quad \text{and} \quad Tz = Jz. \]
Since
\[ M(w, z) = \max\{d(Iw, Jz), d(Iw, Sw), d(Jz, Tz), \frac{1}{2}[d(Iw, Tz) + d(Jz, Sw)]\} \]
\[ = \max\{d(z, Jz), 0, 0, \frac{1}{2}[d(z, Tz) + d(Jz, z)]\} \]
\[ = d(z, Tz), \]
then by inequality (2.1), we obtain
\[ d(z, Tz) = d(Sw, Tz) \leq M(w, z) - \phi(M(w, z)) = d(z, Tz) - \phi(d(z, Tz)), \]
which implies that \( \phi(d(z, Tz)) = 0 \). Thus, by property \( (\phi_2) \), we obtain \( d(z, Tz) = 0 \). So we have \( z = Tz = Jz \).

Again, by inequality (2.1), we obtain
\[ d(Sz, z) = d(Sz, Tz) \leq M(z, z) - \phi(M(z, z)) = d(Sz, z) - \phi(d(Sz, z)). \]
Hence \( \phi(d(Sz, z)) = 0 \), which by property \( (\phi_2) \), implies that \( d(Sz, z) = 0 \). So we have \( z = Sz = Iz \). Thus \( z \) is a common fixed point of the mappings \( S, T, I \) and \( J \).

Let \( q \) be another common fixed point of the mappings \( S, T, I \) and \( J \). Then, by using the inequality (2.1), we obtain
\[ d(z, q) = d(Sz, Tq) \leq M(z, q) - \phi(d(z, q)) = d(z, q) - \phi(d(z, q)), \]
which gives \( \phi(d(z, q)) = 0 \). By property \( (\phi_2) \), we conclude that \( z = q \). This completes the proof. \( \square \)
3. Well-posedness

The notion of well-posedness of a fixed point problem has evoked much interest to a several mathematicians, for examples, F.S. De Blasi and J. Myjak (see [5]), S. Reich and A. J. Zaslavski (see [8]), B.K. Lahiri and P. Das (see [7]) and V. Popa (see [10] and [11]).

**Definition 3.1.** Let \((X, d)\) be a metric space and \(T : (X, d) \to (X, d)\) a mapping. The fixed point problem of \(T\) is said to be well posed if:

(a) \(T\) has a unique fixed point \(z\) in \(X\);

(b) for any sequence \(\{x_n\}\) of points in \(X\) such that \(\lim_{n \to \infty} d(Tx_n, x_n) = 0\), we have \(\lim_{n \to \infty} d(x_n, z) = 0\).

For a set of mappings, it is natural to introduce the following definition.

**Definition 3.2.** Let \((X, d)\) be a metric space and let \(T\) be a set of self-mappings of \(X\). The fixed point problem of \(T\) is said to be well-posed if:

(a) \(T\) has a unique fixed point \(z\) in \(X\);

(b) for any sequence \(\{x_n\}\) of points in \(X\) such that \(\lim_{n \to \infty} d(Tx_n, x_n) = 0\), \(\forall T \in T\), we have \(\lim_{n \to \infty} d(x_n, z) = 0\).

Concerning the well-posedness of the common fixed point problem for four mappings satisfying the conditions of Theorem 2.4, we have the following result.

**Theorem 3.3.** Let \((X, d)\) be a metric space and let \(S, T, I, J\) be four self-mappings of \(X\). Let \(\phi \in \Phi\).

We suppose that:

\((H1)\) : The pair \((S, T)\) is \(\phi\)-weakly contractive with respect to the pair \((I, J)\), that is

\[
d(Sx, Ty) \leq M(x, y) - \phi(M(x, y)),
\]

for all \(x, y\) in \(X\).

\((H2)\) : \(S(X) \subset J(X)\) and \(T(X) \subset I(X)\).

\((H3)\) : The pairs \(\{S, I\}\) and \(\{T, J\}\) are weakly compatible.

\((H4)\) : One of the subsets \(S(X), T(X), I(X)\) or \(J(X)\) is a complete subspace of \(X\).

\((H5)\) : The function \(\phi\) is nondecreasing on \([0, \infty)\).

Then, the common fixed point problem for the set of mappings \(\{S, T, I, J\}\) is well-posed.

**Proof.** We know, by Theorem 2.4, that the mappings \(S, T, I\) and \(J\) have a unique common fixed point (say) \(z \in X\). Let \(\{x_n\}\) of points in \(X\) such that

\[
\lim_{n \to \infty} d(Sx_n, x_n) = \lim_{n \to \infty} d(Tx_n, x_n) = \lim_{n \to \infty} d(Ix_n, x_n) = \lim_{n \to \infty} d(Tx_n, x_n) = 0. \tag{3.2}
\]

We observe that for all nonnegative integer \(n\), we have

\[
M(z, x_n) = \max\{d(z, Jx_n), d(Jx_n, Tx_n), \frac{1}{2}[d(z, Tx_n) + d(Jx_n, z)]\} \\
\leq d(z, x_n) + d(x_n, Jx_n) + d(x_n, Tx_n).
\]
By the triangle inequality and inequality (3.1), we have
\[ d(z, x_n) \leq d(Sz, Tx_n) + d(Tx_n, x_n) \]
\[ \leq M(z, x_n) - \phi(M(z, x_n)) + d(Tx_n, x_n) \]
\[ \leq d(z, x_n) + d(x_n, Jx_n) + 2d(Tx_n, x_n) - \phi(M(z, x_n)). \]
We deduce that
\[ \phi(M(z, x_n)) \leq d(x_n, Jx_n) + 2d(Tx_n, x_n). \]
Thus we have
\[ \lim_{n \to \infty} \phi(M(z, x_n)) = 0. \] (3.4)
To get a contradiction, let us suppose that the sequence \( \{x_n\} \) does not converge to \( z \). Then the sequence \( \{Jx_n\} \) does not converge to \( z \). Then, there exists a positive number \( \epsilon > 0 \) and a subsequence \( \{x_{n_k}\} \) such that
\[ d(z, Jx_{n_k}) \geq \epsilon, \quad \text{for all integer } k. \] (3.5)
Since \( \phi \) is nondecreasing, from (3.3) and (3.5), we obtain
\[ \phi(\epsilon) \leq \phi(d(z, Jx_{n_k})) \leq \phi(M(z, Jx_{n_k})) \leq d(x_{n_k}, Jx_{n_k}) + 2d(Tx_{n_k}, x_{n_k}). \]
By letting \( k \) to infinity, we get
\[ \phi(\epsilon) = 0, \]
a contradiction to the property \( (\phi_2) \). This completes the proof. \( \Box \)

As a consequence, we have the following improvement to Theorem 1.3 of [12].

**Corollary 3.4.** Let \((X, d)\) be a complete metric space and \(S, T : X \to X\) be self mappings of \(X\) such that
\[ d(Tx, Sy) \leq N(x, y) - \phi(N(x, y)), \quad \forall \ x, y \in X. \] (1.2)
where \( \phi : [0, \infty) \to [0, \infty) \) is a lower semi-continuous function with \( \phi(t) > 0 \) for all \( t \in (0, \infty) \) and \( \phi(0) = 0 \) and
\[ N(x, y) = \max\{d(x, y), d(Tx, x), d(Sy, y), \frac{1}{2}[d(y, Tx) + d(x, Sy)]\}. \]
Then, there exists a unique point \( u \in X \) such that \( u = Tu = Su \).
Moreover, if \( \phi \) is nondecreasing then the common fixed point problem for the pair \( \{S, T\} \) is well-posed.

**References**


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