



# A common fixed point theorem for weakly compatible maps satisfying common property ( $E.A.$ ) and implicit relation in Intuitionistic fuzzy metric spaces

Saurav Manro

*School of Mathematics and Computer Applications, Thapar University, Patiala (Punjab) India*

*(Communicated by M. Eshaghi Gordji)*

---

## Abstract

In this paper, employing the common property ( $E.A.$ ), we prove a common fixed theorem for weakly compatible mappings via an implicit relation in Intuitionistic fuzzy metric space. Our results generalize the results of S. Kumar [S. Kumar, *Common fixed point theorems in Intuitionistic fuzzy metric spaces using property ( $E.A.$ )*, J. Indian Math. Soc., 76 (1-4) (2009), 94–103] and C. Alaca et al. [C. Alaca, D. Turkoglu and C. Yildiz, *Fixed points in Intuitionistic fuzzy metric spaces*, Chaos Solitons and Fractals, 29 (2006), 1073–1078].

*Keywords:* Intuitionistic fuzzy metric space; weakly compatible mappings; common property ( $E.A.$ ); implicit relation.

*2010 MSC:* Primary 54H25; Secondary 47H10.

---

## 1. Introduction

In 1986, Jungck [8] introduced the notion of compatible maps for a pair of self mappings. However, the study of common fixed points of non-compatible maps is also very interesting (see [16]). Aamri et al. [1] generalized the concept of non-compatibility by defining the notion of property ( $E.A.$ ) and in 2005, Liu et al. [13] defined common property ( $E.A.$ ) in metric spaces and proved common fixed point theorems under strict contractive conditions. Jungck et al. [9] initiated the study of weakly compatible maps in metric space and showed that every pair of compatible maps is weakly

---

*Email address:* sauravmanro@hotmail.com (Saurav Manro)

compatible but reverse is not true. In the literature, many results have been proved for contraction maps satisfying property  $(E.A.)$  in different settings such as probabilistic metric spaces [5, 7]; fuzzy metric spaces [12, 15]; Intuitionistic fuzzy metric spaces [11, 19]. In this paper, employing the common property  $(E.A.)$ , we prove a common fixed theorem for weakly compatible mappings via an implicit relation in Intuitionistic fuzzy metric space. Our results generalize the results of S. Kumar [11] and C. Alaca et al. [2].

## 2. Preliminaries and Definitions

The concepts of triangular norms ( $t$ -norms) and triangular conorms ( $t$ -conorms) are known as the axiomatic skelton that we use are characterization fuzzy intersections and union respectively. These concepts were originally introduced by Menger [14] in study of statistical metric spaces.

**Definition 2.1.** [18] A binary operation  $*$  :  $[0, 1][0, 1] \rightarrow [0, 1]$  is continuous  $t$ -norm if  $*$  satisfies the following conditions:

- (i)  $*$  is commutative and associative;
- (ii)  $*$  is continuous;
- (iii)  $a * 1 = a$  for all  $a \in [0, 1]$
- (iv)  $a * b \leq c * d$  whenever  $a \leq c$  and  $b \leq d$  for all  $a, b, c, d \in [0, 1]$ .

**Definition 2.2.** [18] A binary operation  $\diamond$  :  $[0, 1][0, 1][0, 1]$  is continuous  $t$ -conorm if  $\diamond$  satisfies the following conditions:

- (i)  $\diamond$  is commutative and associative;
- (ii)  $\diamond$  is continuous;
- (iii)  $a \diamond 0 = a$  for all  $a \in [0, 1]$
- (iv)  $a \diamond b \leq c \diamond d$  whenever  $a \leq c$  and  $b \leq d$  for all  $a, b, c, d \in [0, 1]$

Alaca et al. [2] using the idea of Intuitionistic fuzzy sets, defined the notion of Intuitionistic fuzzy metric space with the help of continuous  $t$ -norm and continuous  $t$ -conorms as a generalization of fuzzy metric space due to Kramosil et al. [10] as:

**Definition 2.3.** [2] A 5-tuple  $(X, M, N, *, \diamond)$  is said to be an Intuitionistic fuzzy metric space if  $X$  is an arbitrary set,  $*$  is a continuous  $t$ -norm,  $\diamond$  is a continuous  $t$ -conorm and  $M, N$  are fuzzy sets on  $X^2[0, \infty)$  satisfying the following conditions:

- (i)  $M(x, y, t) + N(x, y, t) \leq 1$  for all  $x, y \in X$  and  $t > 0$ ;
- (ii)  $M(x, y, 0) = 0$  for all  $x, y \in X$ ;
- (iii)  $M(x, y, t) = 1$  for all  $x, y \in X$  and if and only if  $x = y$ ;
- (iv)  $M(x, y, t) = M(y, x, t)$  for all  $x, y \in X$  and  $t > 0$ ;
- (v)  $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$  for all  $x, y \in X$  and  $s, t > 0$ ;
- (vi) for all  $x, y \in X$ ,  $M(x, y, \cdot) : [0, \infty) \rightarrow [0, 1]$  is left continuous;
- (vii)  $\lim_{t \rightarrow \infty} M(x, y, t) = 1$  for all  $x, y \in X$  and  $t > 0$ ;
- (viii)  $N(x, y, 0) = 1$  for all  $x, y \in X$  ;
- (ix)  $N(x, y, t) = 0$  for all  $x, y \in X$  and  $t > 0$  if and only if  $x = y$ ;
- (x)  $N(x, y, t) = N(y, x, t)$  for all  $x, y \in X$  and  $t > 0$ ;
- (xi)  $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$  for all  $x, y \in X$  and  $s, t > 0$ ;
- (xii) for all  $x, y \in X$ ,  $N(x, y, \cdot) : [0, \infty) \rightarrow [0, 1]$  is right continuous;

(xiii)  $\lim_{t \rightarrow \infty} N(x, y, t) = 0$  for all  $x, y \in X$  and  $t > 0$ .

Then  $(M, N)$  is called an Intuitionistic fuzzy metric space on  $X$ . The functions  $M(x, y, t)$  and  $N(x, y, t)$  denote the degree of nearness and the degree of non-nearness between  $x$  and  $y$  w.r.t.  $t$  respectively.

**Remark 2.4.** [2] Every fuzzy metric space  $(X, M, *)$  is an Intuitionistic fuzzy metric space of the form  $(X, M, 1 - M, *, \diamond)$  such that  $t$ -norm  $*$  and  $t$ -conorm  $\diamond$  are associated as  $x \diamond y = 1 - ((1 - x) * (1 - y))$  for all  $x, y \in X$ .

**Remark 2.5.** [2] In Intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$ ,  $M(x, y, \cdot)$  is non-decreasing and  $N(x, y, \cdot)$  is non-increasing for all  $x, y \in X$ .

Alaca et al.[2] introduced the following notions:

**Definition 2.6.** Let  $(X, M, N, *, \diamond)$  be an Intuitionistic fuzzy metric space. Then

(a) a sequence  $\{x_n\}$  in  $X$  is said to be Cauchy sequence if, for all  $t > 0$  and  $p > 0$ ,

$$\lim_{n \rightarrow \infty} M(x_{n+p}, x_n, t) = 1 \text{ and } \lim_{n \rightarrow \infty} N(x_{n+p}, x_n, t) = 0;$$

(b) a sequence  $\{x_n\}$  in  $X$  is said to be convergent to a point  $x \in X$  if, for all  $t > 0$ ,  $\lim_{n \rightarrow \infty} M(x_n, x, t) = 1$  and  $\lim_{n \rightarrow \infty} N(x_n, x, t) = 0$ .

**Definition 2.7.** [2] An Intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  is said to be complete if and only if every Cauchy sequence in  $X$  is convergent.

**Example 2.8.** [2] Let  $X = \{\frac{1}{n} : n \in \mathbb{N}\} \cup \{0\}$  and let  $*$  be the continuous  $t$ -norm and  $\diamond$  be the continuous  $t$ -conorm defined by and respectively, for all  $a, b \in [0, 1]$ . For each  $t \in (0, \infty)$  and  $x, y \in X$ , define  $(M, N)$  by

$$M(x, y, t) = \frac{t}{t + |x - y|} \text{ if } t > 0;$$

$$M(x, y, 0) = 0$$

and

$$N(x, y, t) = \frac{|x - y|}{t + |x - y|} \text{ if } t > 0;$$

$$N(x, y, 0) = 1$$

Clearly,  $(X, M, N, *, \diamond)$  is complete Intuitionistic fuzzy metric space.

**Definition 2.9.** [1] A pair of self mappings  $(T, S)$  of an Intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  is said to satisfy the property  $(E.A)$  if there exist a sequence  $\{x_n\}$  in  $X$  such that  $\lim_{n \rightarrow \infty} Tx_n = \lim_{n \rightarrow \infty} Sx_n = z$  for some  $z \in X$ .

**Example 2.10.** Let  $X = [0, \infty)$ . Consider  $(X, M, N, *, \diamond)$  be an Intuitionistic fuzzy metric space as in Example 2.8. Define  $T, S : X \rightarrow X$  by  $Tx = \frac{x}{5}$  and  $Sx = \frac{2x}{5}$  for all  $x \in X$ . Clearly, for sequence  $\{x_n\} = \{\frac{1}{n}\}$ ,  $T$  and  $S$  satisfies property  $(E.A)$ .

**Definition 2.11.** [9] Two pairs  $(A, S)$  and  $(B, T)$  of self mappings of an Intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  are said to satisfy the common property  $(E.A)$  if there exist two sequences  $\{x_n\}$  and  $\{y_n\}$  in  $X$  such that  $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} By_n = \lim_{n \rightarrow \infty} Ty_n = z$  for some  $z \in X$ .

**Example 2.12.** Let  $X = [-1,1]$ . Consider  $(X, M, N, *, \diamond)$  be an Intuitionistic fuzzy metric space as in Example 2.8. Define self mappings  $A, B, S$  and  $T$  on  $X$  as  $Ax = \frac{x}{3}, Bx = \frac{-x}{3}, Sx = x, Tx = -x$  for all  $x \in X$ . Then, with sequences  $\{x_n\} = \{\frac{1}{n}\}$  and  $\{y_n\} = \{\frac{-1}{n}\}$  in  $X$ , one can easily verify that  $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} By_n = \lim_{n \rightarrow \infty} Ty_n = 0$ . Therefore, pairs  $(A, S)$  and  $(B, T)$  satisfies the common property  $(E.A)$ .

**Definition 2.13.** [9] A pair of self mappings  $(T, S)$  of an Intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  is said to be weakly compatible if they commute at coincidence points i.e. if  $Tu = Su$  for some  $u \in X$ , then  $TSu = STu$ .

### 3. Main Results

Implicit relations play important role in establishing of common fixed point results. Let  $M_4$  be the set of all real continuous functions  $\psi, \phi : [0, 1]^4 \rightarrow R$ , non-decreasing in the first argument and satisfying the following conditions:

- (A)  $\phi(u, 1, u, 1) \geq 0 \Rightarrow u \geq 1$ ,
- (B)  $\phi(u, 1, 1, u) \geq 0 \Rightarrow u \geq 1$ ,
- (C)  $\phi(u, u, 1, 1) \geq 0 \Rightarrow u \geq 1$ ,
- (D)  $\psi(u, 0, u, 0) \leq 0 \Rightarrow u \leq 0$ ,
- (E)  $\psi(u, 0, 0, u) \leq 0 \Rightarrow u \leq 0$ ,
- (F)  $\psi(u, u, 0, 0) \leq 0 \Rightarrow u \leq 0$

for all  $u \geq 0$ .

**Example 3.1.** Define  $\psi, \phi : [0, 1]^4 \rightarrow R$  as  $\phi(t_1, t_2, t_3, t_4) = 14t_1 - 12t_2 + 6t_3 - 8t_4$  and  $\psi(t_1, t_2, t_3, t_4) = 12t_1 - 9t_2 + 8t_3 - 11t_4$ . Clearly, and satisfies all conditions (A), (B), (C), (D), (E) and (F). Therefore,  $\psi, \phi \in M_4$

We begin with following observation:

**Lemma 3.2.** Let  $\{A_i\}$ ,  $S$  and  $T$  be self mappings of an Intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  satisfying the following:

- (3.1) the pair  $(A_0, T)$  satisfies the property  $(E.A.)$ ;
- (3.2) for any  $x, y \in X$ , and  $\psi, \phi \in M_4$  and for all  $t > 0$ , there exists  $k \in (0, 1)$  such that,  
 $\phi(M(A_i x, A_0 y, kt), M(Sx, Ty, t), M(Sx, A_i x, t), M(Ty, A_0 y, t)) \geq 0$ ;  
 $\psi(N(A_i x, A_0 y, kt), N(Sx, Ty, t), N(Sx, A_i x, t), N(Ty, A_0 y, t)) \leq 0$ ;
- (3.3)  $A_i(X) \subseteq T(X)$  or  $A_0(X) \subseteq S(X)$ .

Then the pairs  $(A_i, S)$  and  $(A_0, T)$  share the common  $(E.A.)$  property.

**Proof .** As the pair  $(A_0, T)$  satisfies property  $(E.A.)$ , then there exist a sequence  $\{x_n\}$  in  $X$  such that  $\lim_{n \rightarrow \infty} A_0 x_n = \lim_{n \rightarrow \infty} T x_n = z$  for some  $z \in X$ . Since  $A_0(X) \subseteq S(X)$ , hence for each  $\{x_n\}$ , there exist  $\{y_n\}$  in  $X$  such that  $A_0 x_n = S y_n$ .

Therefore,  $\lim_{n \rightarrow \infty} A_0 x_n = \lim_{n \rightarrow \infty} S y_n = \lim_{n \rightarrow \infty} T x_n = z$ . Now, we claim that  $\lim_{n \rightarrow \infty} A_i y_n = z$ .

Suppose not, then applying inequality (3.2), we obtain

$$\phi(M(A_i y_n, A_0 x_n, kt), M(S y_n, T x_n, t), M(S y_n, A_i y_n, t), M(T x_n, A_0 x_n, t)) \geq 0;$$

$$\psi(N(A_i y_n, A_0 x_n, kt), N(S y_n, T x_n, t), N(S y_n, A_i y_n, t), N(T x_n, A_0 x_n, t)) \leq 0;$$

which on making  $n \rightarrow \infty$  reduces to

$$\phi(M(\lim_{n \rightarrow \infty} A_i y_n, z, kt), M(z, z, t), M(z, \lim_{n \rightarrow \infty} A_i y_n, t), M(z, z, t)) \geq 0;$$

$$\psi(N(\lim_{n \rightarrow \infty} A_i y_n, z, kt), N(z, z, t), N(z, \lim_{n \rightarrow \infty} A_i y_n, t), N(z, z, t)) \leq 0;$$

As  $\phi$  and  $\psi$  is non-decreasing in the first argument, we have

$$\phi(M(\lim_{n \rightarrow \infty} A_i y_n, z, t), M(z, z, t), M(z, \lim_{n \rightarrow \infty} A_i y_n, t), M(z, z, t)) \geq 0;$$

$$\psi(N(\lim_{n \rightarrow \infty} A_i y_n, z, t), N(z, z, t), N(z, \lim_{n \rightarrow \infty} A_i y_n, t), N(z, z, t)) \leq 0;$$

Using (B) and (E), we get

$$M(\lim_{n \rightarrow \infty} A_i y_n, z, t) \geq 1 \text{ and } N(\lim_{n \rightarrow \infty} A_i y_n, z, t) \leq 0.$$

$$\text{Hence } M(\lim_{n \rightarrow \infty} A_i y_n, z, t) = 1 \text{ and } N(\lim_{n \rightarrow \infty} A_i y_n, z, t) = 0.$$

Therefore,  $\lim_{n \rightarrow \infty} A_i y_n = z$ . Hence, the pairs  $(A_i, S)$  and  $(A_0, T)$  share the common (E.A.) property.

□

**Theorem 3.3.** *Let  $\{A_i\}$ ,  $S$  and  $T$  be self mappings of a Intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  satisfying the conditions (3.2) and*

(3.4) *the pair  $(A_i, S)$  and  $(A_0, T)$  share the common property (E.A);*

(3.5)  *$S(X)$  and  $T(X)$  are closed subsets of  $X$ .*

*Then the pairs and have a point of coincidence each. Moreover,  $\{A_i\}$ ,  $S$  and  $T$  have a unique common fixed point provided both the pairs  $(A_i, S)$  and  $(A_0, T)$  are weakly compatible.*

**Proof .** In view of (3.4), there exist two sequences  $\{x_n\}$  and  $\{y_n\}$  in  $X$  such that  $\lim_{n \rightarrow \infty} A_0 y_n = \lim_{n \rightarrow \infty} A_i x_n = \lim_{n \rightarrow \infty} S x_n = \lim_{n \rightarrow \infty} T y_n = z$  for some  $z \in X$ . Since  $S(X)$  is a closed subset of  $X$ , therefore, there exists a point  $u \in X$  such that  $z = Su$ . We claim that  $A_i u = z$ . If  $A_i u \neq z$ , then by

(3.2), take  $x = u, y = y_n$ ,

$$\phi(M(A_i u, A_0 y_n, kt), M(Su, T y_n, t), M(Su, A_i u, t), M(T y_n, A_0 y_n, t)) \geq 0;$$

on making  $n \rightarrow \infty$ , we get

$$\phi(M(A_i u, z, kt), M(z, z, t), M(z, A_i u, t), M(z, z, t)) \geq 0;$$

$$\phi(M(A_i u, z, kt), 1, M(z, A_i u, t), 1) \geq 0;$$

and

$$\psi(N(A_i u, A_0 y_n, kt), N(Su, T y_n, t), N(Su, A_i u, t), N(T y_n, A_0 y_n, t)) \leq 0;$$

on making  $n \rightarrow \infty$ , we get

$$\psi(N(A_i u, z, kt), N(z, z, t), N(z, A_i u, t), N(z, z, t)) \leq 0;$$

$$\psi(N(A_i u, z, kt), 0, N(z, A_i u, t), 0) \leq 0;$$

As  $\phi$  and  $\psi$  is non-decreasing in the first argument, we have

$$\phi(M(A_i u, z, t), 1, M(z, A_i u, t), 1) \geq 0;$$

and

$$\psi(N(A_i u, z, t), 0, N(z, A_i u, t), 0) \leq 0;$$

Using (A) and (D), we get  $M(A_i u, z, t) \geq 1$  and  $N(A_i u, z, t) \leq 0$ .

Hence  $M(A_i u, z, t) = 1$  and  $N(A_i u, z, t) = 0$ .

Therefore,  $A_i u = z = Su$  which shows that  $u$  is a coincidence point of the pair  $(A_i, S)$ .

Since  $T(X)$  is also a closed subset of  $X$ , therefore  $\lim_{n \rightarrow \infty} T y_n = z$  in  $T(X)$  and hence there exists  $v \in X$  such that  $Tv = z = A_i u = Su$ .

Now, we show that  $A_0 v = z$ . If not, then by using inequality (3.2), take  $x = u, y = v$ , we have

$$\phi(M(A_i u, A_0 v, kt), M(Su, T v, t), M(Su, A_i u, t), M(T v, A_0 v, t)) \geq 0;$$

$$\phi(M(z, A_0 v, kt), 1, 1, M(z, A_0 v, t)) \geq 0;$$

and

$$\psi(N(A_i u, A_0 v, kt), N(Su, T v, t), N(Su, A_i u, t), N(T v, A_0 v, t)) \leq 0;$$

$$\psi(N(z, A_0 v, kt), 0, 0, N(z, A_0 v, t)) \leq 0;$$

As  $\phi$  and  $\psi$  is non-decreasing in the first argument, we have

$$\phi(M(z, A_0v, kt), 1, 1, M(z, A_0v, t)) \geq 0;$$

and

$$\psi(N(z, A_0v, kt), 0, 0, N(z, A_0v, t)) \leq 0;$$

Using (B) and (E), we get  $M(z, A_0v, kt) \geq 1$  and  $N(z, A_0v, kt) \leq 0$ .

Hence  $M(z, A_0v, kt) = 1$  and  $N(z, A_0v, kt) = 0$ . Therefore,  $A_0v = z = Tv$  which shows that  $v$  is a coincidence point of the pair  $(A_0, T)$ .

Since the pairs  $(A_i, S)$  and  $(A_0, T)$  are weakly compatible and  $A_iu = Su, A_0v = Tv$ , therefore,  $A_iz = A_iSu = SA_iu = Sz, A_0z = A_0Tv = TA_0v = Tz$ . If  $A_iz \neq z$ , then by using inequality (3.2), we have

$$\phi(M(A_iz, A_0v, kt), M(Sz, Tv, t), M(Sz, A_iz, t), M(Tv, A_0v, t)) \geq 0;$$

$$\phi(M(A_iz, z, kt), M(A_iz, z, t), M(A_iz, A_iz, t), M(z, z, t)) \geq 0;$$

$$\phi(M(A_iz, z, kt), M(A_iz, z, t), 1, 1) \geq 0;$$

and

$$\psi(N(A_iz, A_0v, kt), N(Sz, Tv, t), N(Sz, A_iz, t), N(Tv, A_0v, t)) \leq 0;$$

$$\psi(N(A_iz, z, kt), N(A_iz, z, t), N(A_iz, A_iz, t), N(z, z, t)) \leq 0;$$

$$\psi(N(A_iz, z, kt), N(A_iz, z, t), 0, 0) \leq 0;$$

As  $\phi$  and  $\psi$  is non-decreasing in the first argument, we have

$$\phi(M(A_iz, z, t), M(A_iz, z, t), 1, 1) \geq 0;$$

and

$$\psi(N(A_iz, z, kt), N(A_iz, z, t), 0, 0) \leq 0;$$

Using (C) and (F), we get

$$M(A_iz, z, t) \geq 1 \text{ and } N(A_iz, z, t) \leq 0.$$

Hence  $M(A_iz, z, t) = 1$  and  $N(A_iz, z, t) = 0$ .

Therefore,  $A_iz = z = Sz$ .

Similarly, one can prove that  $A_0z = Tz = z$ . Hence  $A_0z = A_iz = Sz = Tz$  and  $z$  is common fixed point of  $A_i, A_0, S$  and  $T$ .

Uniqueness: Let  $z$  and  $w$  be two common fixed points of  $A_i, A_0, S$  and  $T$ . If  $z \neq w$ , then by using inequality (3.2), we have

$$\phi(M(A_iz, A_0w, kt), M(Sz, Tw, t), M(Sz, A_iz, t), M(Tw, A_0w, t)) \geq 0;$$

$$\phi(M(z, w, kt), M(z, w, t), M(z, z, t), M(w, w, t)) \geq 0;$$

$$\phi(M(z, w, t), M(z, w, t), M(z, z, t), M(w, w, t)) \geq 0;$$

$$\phi(M(z, w, t), M(z, w, t), 1, 1) \geq 0;$$

and

$$\psi(N(A_iz, A_0w, kt), N(Sz, Tw, t), N(Sz, A_iz, t), N(Tw, A_0w, t)) \leq 0;$$

$$\psi(N(z, w, kt), N(z, w, t), N(z, z, t), N(w, w, t)) \leq 0;$$

$$\psi(N(z, w, t), N(z, w, t), N(z, z, t), N(w, w, t)) \leq 0;$$

$$\psi(N(z, w, t), N(z, w, t), 0, 0) \leq 0;$$

Using (C) and (F), we have

$$M(z, w, t) \geq 1 \text{ and } N(z, w, t) \leq 0.$$

Hence,  $M(z, w, t) = 1$  and  $N(z, w, t) = 0$ .

Therefore,  $z = w$ .  $\square$

By choosing  $A_i, A_0, S$  and  $T$  suitably, one can derive corollaries involving two or more mappings. As a sample, we deduce the following natural result for a pair of self mappings by setting  $A_0 = A_i$  and  $T = S$  in above theorem:

**Corollary 3.4.** Let  $A_i$  and  $S$  be self mappings of an Intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  satisfying the following:



(3.6) the pair  $(A_i, S)$  satisfies the property  $(E.A.)$ ;

(3.7) for any  $x, y \in X$ ,  $\phi$  and  $\psi$  in  $M_4$  and for all  $t > 0$ ,

$$\phi(M(A_i x, A_i y, kt), M(Sx, Sy, t), M(Sx, A_i x, t), M(Sy, A_i y, t)) \geq 0;$$

$$\psi(N(A_i x, A_i y, kt), N(Sx, Sy, t), N(Sx, A_i x, t), N(Sy, A_i y, t)) \leq 0;$$

(3.8)  $S(X)$  is a closed subset of  $X$ .

Then,  $A_i$  and  $S$  have a point of coincidence each. Moreover, if the pairs  $(A_i, S)$  is weakly compatible, then  $A_i$  and  $S$  have a unique common fixed point.

The following example illustrates Theorem 3.3.

**Example 3.5.** Let  $(X, M, N, *, \diamond)$  be an Intuitionistic fuzzy metric space as in Example 2.8 where  $X = [0, 2)$  and define  $\psi, \phi : [0, 1]^4 \rightarrow R$  as  $\phi(t_1, t_2, t_3, t_4) = 14t_1 - 12t_2 + 6t_3 - 8t_4$  and  $\psi(t_1, t_2, t_3, t_4) = 12t_1 - 9t_2 + 8t_3 - 11t_4$ . Clearly, all conditions  $(A), (B), (C), (D), (E)$  and  $(F)$  hold. Therefore  $\psi, \phi \in M_4$

Define  $A_i, A_0, S$  and  $T$  by

$$A_i x = A_0 x = 1,$$

$$Sx = 1 \text{ if } x \in Q, Sx = \frac{2}{3} \text{ otherwise}$$

and

$$Tx = 1 \text{ if } x \in Q, Tx = \frac{1}{3} \text{ otherwise}$$

for all  $x, y \in X = [0, 2)$  and  $t > 0$ . Then with sequences  $\{x_n = \frac{1}{n}\}$  and  $\{y_n = \frac{-1}{n}\}$  in  $X$ , we have  $\lim_{n \rightarrow \infty} A_0 x_n = \lim_{n \rightarrow \infty} A_i y_n = \lim_{n \rightarrow \infty} S y_n = \lim_{n \rightarrow \infty} T x_n = 1 \in X$  which shows that pairs  $(A_i, S)$  and  $(A_0, T)$  share the common property  $(E.A.)$ . By a routine calculation, one can verify the condition (3.2). Thus, all the conditions of Theorem 3.3 are satisfied and  $x = 1$  is the unique common fixed point of  $A_i, A_0, S$  and  $T$ .

## References

- [1] M. Aamri and D. El Moutawakil, *Some new common fixed point theorems under strict contractive conditions*, J. Math. Anal. Appl. 270 (2002) 181–188.
- [2] C. Alaca, D. Turkoglu and C. Yildiz, *Fixed points in Intuitionistic fuzzy metric spaces*, Chaos Solitons and Fractals 29 (2006) 1073–1078.
- [3] K. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets Syst. 20 (1986) 87–96.
- [4] D. Coker, *An introduction to Intuitionistic Fuzzy topological spaces*, Fuzzy Sets Syst. 88 (1997) 81–89.
- [5] J. X. Fang and Y. Gao, *Common fixed point theorems under strict contractive conditions in Menger spaces*, Nonlinear Anal. 70 (2009) 184–193.
- [6] A. George and P. Veeramani, *On some results in fuzzy metric spaces*, Fuzzy Sets Syst. 64 (1994) 395–399.
- [7] M. Imdad, M. Tanveer and M. Hasan, *Some common fixed point theorems in Menger PM-spaces*, Fixed Point Theory and Applications Volume 2010, Article ID 819269, 14 pages.
- [8] G. Jungck, *Compatible mappings and common fixed points*, Internat. J. Math. Math. Sci. 9 (1986) 771–779.
- [9] G. Jungck and B.E. Rhoades, *Fixed points for set valued functions without continuity*, Indian J. Pure Appl. Math. 29 (1998) 227–238.
- [10] I. Kramosil and J. Michalek, *Fuzzy metric and statistical spaces*, Kybernetika 11 (1975) 336–344.
- [11] S. Kumar, *Common fixed point theorems in Intuitionistic fuzzy metric spaces using property  $(E.A.)$* , J. Indian Math. Soc. 76 (2009) 94–103.
- [12] S. Kumar and B. Fisher, *A common fixed point theorem in fuzzy metric space using property  $(E.A.)$  and implicit relation*, Thai J. Math. 8 (2010) 439–446.
- [13] W. Lui, J. Wu and Z. Li, *Common fixed points of single-valued and multivalued maps*, Int. J. Math. Math. Sci. 19 (2005) 3045–3055.
- [14] K. Menger, *Statistical metrics*, Proc. Nat. Acad. Sci. (USA) 28 (1942) 535–537.
- [15] D. Mihet, *Fixed point theorems in fuzzy metric spaces using property  $E.A.$* , Nonlinear Anal. 73 (2010) 2184–2188.
- [16] R.P. Pant, *Common fixed points of contractive maps*, J. Math. Anal. Appl. 226 (1998) 251–258.
- [17] J.H. Park, *Intuitionistic fuzzy metric spaces*, Chaos Solitons and Fractals 22 (2004) 1039–1046.

- [18] B. Schweizer and A. Sklar, *Probabilistic Metric Spaces*, North Holland Amsterdam, 1983.
- [19] S. Sharma and B. Deshpande, *Compatible mappings of type (I) and (II) on Intuitionistic fuzzy metric spaces in consideration of common fixed point*, Commun. Korean Math. Soc. 24 (2009) 197–214.
- [20] L.A. Zadeh, *Fuzzy sets*, Infor. and Control. 8 (1965) 338–353.