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On positive solutions for a class of infinite semipositone problems

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Abstract

We discuss the existence of a positive solution to the infinite semipositone problem

$$-\Delta u = au - bu^{\gamma} - f(u) - \frac{c}{u^{\alpha}},\ x \in \Omega, \quad u = 0,\ x \in \partial\Omega,$$

where Δ is the Laplacian operator, $\gamma > 1$, $\alpha \in (0,1)$, a, b and c are positive constants, Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$, and $f : [0,\infty) \to \mathbb{R}$ is a continuous function such that $f(u) \to \infty$ as $u \to \infty$. Also we assume that there exist $A > 0$ and $\beta > 1$ such that $f(s) \le As^{\beta}$, for all $s \ge 0$. . We obtain our result via the method of sub- and supersolutions.

Keywords: Positive solution, Infinite semipositone, Sub- and supersolutions.
2010 MSC: Primary 35J25; Secondary 35J55.

1. Introduction

We consider the positive solution to the boundary value problem

$$\begin{cases} -\Delta u = au - bu^{\gamma} - f(u) - \dfrac{c}{u^{\alpha}}, & x \in \Omega, \\ u = 0, & x \in \partial\Omega, \end{cases} \tag{1.1}$$

where Δ denotes the Laplacian operator, $\gamma > 1$, $\alpha \in (0,1)$, a, b and c are positive constants, Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$, and $f : [0,\infty) \to \mathbb{R}$ is a continuous function. We make the following assumptions:

(H1) $f : [0,+\infty) \to \mathbb{R}$ is continuous function such that $\lim_{s\to+\infty} f(s) = \infty$.

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(H2) There exist $A > 0$ and $\beta > 1$ such that $f(s) \leq As^{\beta}$, for all $s \geq 0$.

In [9], the authors have studied the equation $-\Delta u = g(u) - (c/u^{\alpha})$ with Dirichlet boundary conditions,where g is nonnegative and nondecreasing and $\lim_{u\to\infty} g(u) = \infty$. The case $g(u) := au - f(u)$ has been study in [8], where $f(u) \geq au - M$ and $f(u) \leq Au^{\beta}$ on $[0, \infty)$ for some $M, A > 0, \beta > 1$ and this g may have a falling zero. In this paper, we study the equation $-\Delta u = au - bu^{\gamma} - f(u) - (c/u^{\alpha})$ with Dirichlet boundary conditions. Our result in this paper include the result of [8], where say in Remark 2.2. Let $F(u) := au - bu^{\gamma} - f(u) - (c/u^{\alpha})$, then $\lim_{u\to 0^+} F(u) = -\infty$ and hence we refer to (1.1) as an infinite semipositone problem.

In recent years, there has been considerable progress on the study of semipositione problems ($F(0) < 0$ but finite)(see [2],[3],[6]). Many results have been obtained on kind of infinite semipositone problems; see for example [7], [8], [9] and [10]. One of the main tools used in these studies is the method of sub-super solutions. By a subsolution of (1.1) we mean a function $\psi \in C^2(\Omega) \cap C(\overline{\Omega})$ that satisfies

$$-\Delta\psi \leq a\psi - b\psi^{\gamma} - f(\psi) - \frac{c}{\psi^{\alpha}} \quad \text{in } \Omega$$
$$\psi \leq 0 \quad \text{on } \partial\Omega,$$

and by a supersolution of (1.1) we mean a function $Z \in C^2(\Omega) \cap C(\overline{\Omega})$ that satisfies

$$-\Delta Z \geq aZ - bZ^{\gamma} - f(Z) - \frac{c}{Z^{\alpha}} \quad \text{in } \Omega$$
$$Z \geq 0 \quad \text{on } \partial\Omega.$$

Then we have the following Lemma.

Lemma 1.1 ([1, 4]). *If there exist a subsolution ψ and a supersolution Z of (1.1) such that $\psi \leq Z$ on $\overline{\Omega}$, then (1.1) has at least one solution $u \in C^2(\Omega) \cap C(\overline{\Omega})$ satisfying $\psi \leq u \leq Z$ on $\overline{\Omega}$.*

2. The main result

We shall establish the following result.

Theorem 2.1. *Let (H1) and (H2) hold. If $a > (\frac{2}{1+\alpha})\lambda_1$, Then there exists positive constant $c^* := c^*(a, A, \alpha, \beta, \gamma, \Omega)$ such that for $c \leq c^*$, problem (1.1) has a positive solution, where λ_1 be the first eigenvalue of the Laplacian operator with Dirichlet boundary conditions.*

Remark 2.2. Theorem 2.1 was established in [8] for the case $f(u) := g(u) - bu^{\gamma}$, where the function g satisfy the following assumptions:
- $g(u) \approx bu^{\theta}$ for some $\theta > \gamma$.
- There exist $A > 0$ and $\beta > 1$ such that $g(u) \leq Au^{\beta}$, for all $u \geq 0$.
- There exist $M > 0$ such that $g(u) \geq au - M$, for all $u \geq 0$.

In fact, the function f satisfy the hypotheses of Theorem 2.1 in this paper (Since $\lim_{u\to\infty} (g(u)/bu^{\theta}) = 1$, hence $\lim_{u\to\infty} f(u) = \infty$) and g satisfy the hypotheses of Theorem 2.1 in [8], where (1.1) changes to equation $-\Delta u = au - g(u) - (c/u^{\alpha})$ with Dirichlet boundary conditions.

Proof . We shall establish Theorem 2.1 by constructing positive sub-supersolutions to equation (1.1). From an anti-maximum principle (see [5, pages 155-156]), there exists $\sigma(\Omega) > 0$ such that the solution z_λ of

$$\begin{cases} -\Delta z - \lambda z = -1, & x \in \Omega, \\ z = 0, & x \in \partial\Omega, \end{cases}$$

for $\lambda \in (\lambda_1, \lambda_1 + \sigma)$ is positive in Ω and is such that $\frac{\partial z}{\partial \nu} < 0$ on $\partial\Omega$, where ν is outward normal vector on $\partial\Omega$. Fix $\lambda^* \in (\lambda_1, \min\{\lambda_1 + \sigma, (\frac{1+\alpha}{2})a\})$ and let

$$K := \min\Big\{ \Big(\frac{(2/1+\alpha)}{2b \left\| z_{\lambda^*} \right\|_\infty^{\frac{2\gamma-\alpha+1}{1+\alpha}}} \Big)^{\frac{1}{\gamma-1}}, \Big(\frac{a - (\frac{2}{1+\alpha})\lambda^*}{3b \left\| z_{\lambda^*} \right\|_\infty^{\frac{2(\gamma-1)}{1+\alpha}}} \Big)^{\frac{1}{\gamma-1}}, \Big(\frac{(2/1+\alpha)}{2A \left\| z_{\lambda^*} \right\|_\infty^{\frac{2\beta-\alpha+1}{1+\alpha}}} \Big)^{\frac{1}{\beta-1}}, \Big(\frac{a - (\frac{2}{1+\alpha})\lambda^*}{3A \left\| z_{\lambda^*} \right\|_\infty^{\frac{2(\beta-1)}{1+\alpha}}} \Big)^{\frac{1}{\beta-1}} \Big\}$$

Define $\psi = K z_{\lambda^*}^{\frac{2}{1+\alpha}}$. Then

$$\nabla\psi = K(\frac{2}{1+\alpha}) z_{\lambda^*}^{\frac{1-\alpha}{1+\alpha}} \nabla z_{\lambda^*}$$

and

$$\begin{aligned} -\Delta\psi &= -\operatorname{div}(\nabla\psi) \\ &= -K(\frac{2}{1+\alpha}) \left\{ (\frac{1-\alpha}{1+\alpha}) z_{\lambda^*}^{\frac{-2\alpha}{1+\alpha}} |\nabla z_{\lambda^*}|^2 + z_{\lambda^*}^{\frac{1-\alpha}{1+\alpha}} \Delta z_{\lambda^*} \right\} \\ &= -K(\frac{2}{1+\alpha}) \left\{ (\frac{1-\alpha}{1+\alpha}) z_{\lambda^*}^{\frac{-2\alpha}{1+\alpha}} |\nabla z_{\lambda^*}|^2 + z_{\lambda^*}^{\frac{1-\alpha}{1+\alpha}} (1 - \lambda^* z_{\lambda^*}) \right\} \\ &= K(\frac{2}{1+\alpha}) \left\{ \lambda^* z_{\lambda^*}^{\frac{2}{1+\alpha}} - z_{\lambda^*}^{\frac{1-\alpha}{1+\alpha}} - \left(\frac{1-\alpha}{1+\alpha} \right) \frac{|\nabla z_{\lambda^*}|^2}{z_{\lambda^*}^{\frac{2\alpha}{1+\alpha}}} \right\} \end{aligned}$$

Let $\delta > 0$, $\mu > 0$, $m > 0$ be such that $|\nabla z_{\lambda^*}|^2 \geq m$ in $\overline{\Omega}_\delta$ and $z_{\lambda^*} \geq \mu$ in $\Omega \setminus \overline{\Omega}_\delta$, where $\overline{\Omega}_\delta := \{x \in \Omega : d(x, \partial\Omega) \leq \delta\}$. Let

$$c^* := K^{1+\alpha} \min\left\{ (\frac{2}{1+\alpha}) \Big(\frac{1-\alpha}{1+\alpha} \Big) m^2, \frac{1}{3}\mu^2 \Big(a - (\frac{2}{1+\alpha})\lambda^* \Big) \right\}.$$

Let $x \in \overline{\Omega}_\delta$ and $c \leq c^*$. Since $(\frac{2}{1+\alpha})\lambda^* < a$, we have

$$K(\frac{2}{1+\alpha}) \lambda^* z_{\lambda^*}^{\frac{2}{1+\alpha}} < a\Big(K z_{\lambda^*}^{\frac{2}{1+\alpha}} \Big). \tag{2.1}$$

From the choice of K , we have

$$\frac{1}{2}(\frac{2}{1+\alpha}) \geq bK^{\gamma-1} \left\| z_{\lambda^*} \right\|_\infty^{\frac{2\gamma-\alpha+1}{1+\alpha}} \tag{2.2}$$

$$\frac{1}{2}(\frac{2}{1+\alpha}) \geq AK^{\beta-1} \left\| z_{\lambda^*} \right\|_\infty^{\frac{2\beta-\alpha+1}{1+\alpha}} \tag{2.3}$$

and by (2.2),(2.3) and (H2), we know that

$$-\frac{1}{2}K(\frac{2}{1+\alpha})z_{\lambda^*}^{\frac{1-\alpha}{1+\alpha}} \leq -b\Big(Kz_{\lambda^*}^{\frac{2}{1+\alpha}}\Big)^{\gamma} \quad (2.4)$$

$$-\frac{1}{2}K(\frac{2}{1+\alpha})z_{\lambda^*}^{\frac{1-\alpha}{1+\alpha}} \leq -A\Big(Kz_{\lambda^*}^{\frac{2}{1+\alpha}}\Big)^{\beta} \leq -f\Big(Kz_{\lambda^*}^{\frac{2}{1+\alpha}}\Big) \quad (2.5)$$

Since $|\nabla z_{\lambda^*}|^2 \geq m$ in $\overline{\Omega}_\delta$, from the choice of c^* we have

$$\begin{aligned} & -K(\frac{2}{1+\alpha})\Big(\frac{1-\alpha}{1+\alpha}\Big)\frac{|\nabla z_{\lambda^*}|^2}{z_{\lambda^*}^{\frac{2\alpha}{1+\alpha}}} \\ & \leq -K(\frac{2}{1+\alpha})\Big(\frac{1-\alpha}{1+\alpha}\Big)\frac{m^2}{z_{\lambda^*}^{\frac{2\alpha}{1+\alpha}}} \\ & \leq -\frac{c}{\Big(Kz_{\lambda^*}^{\frac{2}{1+\alpha}}\Big)^{\alpha}}. \end{aligned} \quad (2.6)$$

Hence for $c \leq c^*$, combining (2.1),(2.4),(2.5) and (2.6) we have

$$\begin{aligned} -\Delta\psi &= K(\frac{2}{1+\alpha})\left\{\lambda^* z_{\lambda^*}^{\frac{2}{1+\alpha}} - z_{\lambda^*}^{\frac{1-\alpha}{1+\alpha}} - \left(\frac{1-\alpha}{1+\alpha}\right)\frac{|\nabla z_{\lambda^*}|^2}{z_{\lambda^*}^{\frac{2\alpha}{1+\alpha}}}\right\} \\ &= K(\frac{2}{1+\alpha})\lambda^* z_{\lambda^*}^{\frac{2}{1+\alpha}} - \frac{1}{2}K(\frac{2}{1+\alpha})z_{\lambda^*}^{\frac{1-\alpha}{1+\alpha}} \\ &\quad - \frac{1}{2}K(\frac{2}{1+\alpha})z_{\lambda^*}^{\frac{1-\alpha}{1+\alpha}} \\ &\quad - K(\frac{2}{1+\alpha})\left(\frac{1-\alpha}{1+\alpha}\right)\frac{|\nabla z_{\lambda^*}|^2}{z_{\lambda^*}^{\frac{2\alpha}{1+\alpha}}} \\ &\leq a\Big(Kz_{\lambda^*}^{\frac{2}{1+\alpha}}\Big) - b\Big(Kz_{\lambda^*}^{\frac{2}{1+\alpha}}\Big)^{\gamma} - f\Big(Kz_{\lambda^*}^{\frac{2}{1+\alpha}}\Big) - \frac{c}{\Big(Kz_{\lambda^*}^{\frac{2}{1+\alpha}}\Big)^{\alpha}} \\ &= a\psi - b\psi^{\gamma} - f(\psi) - \frac{c}{\psi^{\alpha}}, \quad x \in \overline{\Omega}_\delta. \end{aligned}$$

Next in $\Omega \setminus \overline{\Omega}_\delta$, for $c \leq c^*$ from the choice of c^* and K, we know that

$$\frac{c}{K^{\alpha}} \leq \frac{1}{3}Kz_{\lambda^*}^2\left(a - (\frac{2}{1+\alpha})\lambda^*\right), \quad (2.7)$$

and

$$bK^{\gamma-1}z_{\lambda^*}^{\frac{2(\gamma-1)}{1+\alpha}} \leq \frac{1}{3}\left(a - (\frac{2}{1+\alpha})\lambda^*\right) \quad (2.8)$$

$$AK^{\beta-1}z_{\lambda^*}^{\frac{2(\beta-1)}{1+\alpha}} \leq \frac{1}{3}\left(a - (\frac{2}{1+\alpha})\lambda^*\right). \quad (2.9)$$

By combining (2.7),(2.8) and (2.9) we have

$$\begin{aligned}
-\Delta\psi &= K(\frac{2}{1+\alpha})\left\{\lambda^* z_{\lambda^*}^{\frac{2}{1+\alpha}} - z_{\lambda^*}^{\frac{1-\alpha}{1+\alpha}} - \left(\frac{1-\alpha}{1+\alpha}\right)\frac{|\nabla z_{\lambda^*}|^2}{z_{\lambda^*}^{\frac{2\alpha}{1+\alpha}}}\right\} \\
&\le K(\frac{2}{1+\alpha})\lambda^* z_{\lambda^*}^{\frac{2}{1+\alpha}} \\
&= \frac{1}{z_{\lambda^*}^{\frac{2\alpha}{1+\alpha}}}\sum_{i=1}^{3}\left(\frac{1}{3}K(\frac{2}{1+\alpha})\lambda^* z_{\lambda^*}^2\right) \\
&\le \frac{1}{z_{\lambda^*}^{\frac{2\alpha}{1+\alpha}}}\Big\{\left(\frac{1}{3}Kz_{\lambda^*}^2 a - \frac{c}{K^\alpha}\right) + Kz_{\lambda^*}^2\left(\frac{1}{3}a - bK^{\gamma-1}z_{\lambda^*}^{\frac{2(\gamma-1)}{1+\alpha}}\right) \\
&\qquad + Kz_{\lambda^*}^2\left(\frac{1}{3}a - AK^{\beta-1}z_{\lambda^*}^{\frac{2(\beta-1)}{1+\alpha}}\right)\Big\} \\
&\le aKz_{\lambda^*}^{\frac{2}{1+\alpha}} - bK^\gamma z_{\lambda^*}^{\frac{2\gamma}{1+\alpha}} - AK^\beta z_{\lambda^*}^{\frac{2\beta}{1+\alpha}} - \frac{c}{K^\alpha z_{\lambda^*}^{\frac{2\alpha}{1+\alpha}}} \\
&\le a\Big(Kz_{\lambda^*}^{\frac{2}{1+\alpha}}\Big) - b\Big(Kz_{\lambda^*}^{\frac{2}{1+\alpha}}\Big)^\gamma - f\Big(Kz_{\lambda^*}^{\frac{2}{1+\alpha}}\Big) - \frac{c}{\Big(Kz_{\lambda^*}^{\frac{2}{1+\alpha}}\Big)^\alpha} \\
&= a\psi - b\psi^\gamma - f(\psi) - \frac{c}{\psi^\alpha}, \quad x\in\Omega\setminus\overline{\Omega}_\delta.
\end{aligned}$$

Thus ψ is a positive subsolution of (1.1). From (H1) and $\gamma > 1$, it is obvious that $Z = M$ where M is sufficiently large constant is a supersolution of (1.1) with $Z \ge \psi$. Thus, by Lemma 1.1 there exists a solution u of (1.1) with $\psi \le u \le Z$. This completes the proof of Theorem 2.1. □

3. An extension to system (3.1)

In this section, we consider the extension of (1.1) to the following system:

$$\begin{cases} -\Delta u = a_1 u - b_1 u^\gamma - f_1(u) - \dfrac{c_1}{v^\alpha}, & x\in\Omega, \\ -\Delta v = a_2 v - b_2 v^\gamma - f_2(v) - \dfrac{c_2}{u^\alpha}, & x\in\Omega, \\ u = 0 = v, & x\in\partial\Omega, \end{cases} \tag{3.1}$$

where Δ denotes the Laplacian operator, $\gamma > 1$, $\alpha \in (0,1)$, a_1, a_2, b_1, b_2, c_1 and c_2 are positive constants, Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$, and $f_i : [0,\infty) \to \mathbb{R}$ is a continuous function for $i = 1, 2$. We make the following assumptions:

(H3) $f_i : [0,+\infty) \to \mathbb{R}$ is continuous functions such that $\lim_{s\to+\infty} f_i(s) = \infty$ for $i = 1, 2$.

(H4) There exist $A > 0$ and $\beta > 1$ such that $f_i(s) \le As^\beta$, $i = 1, 2$, for all $s \ge 0$.

We prove the following result by finding sub-super solutions to infinite semipositone system (3.1).

Theorem 3.1. *Let* (H3) *and* (H4) *hold, If* $\min\{a_1, a_2\} > (\frac{2}{1+\alpha})\lambda_1$, *Then there exists positive constant* $c^* := c^*(a_1, a_2, b_1, b_2, A, \Omega)$ *such that for* $\max\{c_1, c_2\} \le c^*$, *problem* (3.1) *has a positive solution.*

Proof . Let σ be as in section 2, $\tilde{a}=\min\{a_1,a_2\}$ and $\tilde{b}=\max\{b_1,b_2\}$. Choice $\lambda^*\in(\lambda_1,\min\{\lambda_1+\sigma,(\frac{1+\alpha}{2})\tilde{a}\})$. Define

$$K:=\min\Big\{\Big(\frac{(2/1+\alpha)}{2\tilde{b}\,\|z_{\lambda^*}\|_\infty^{\frac{2\gamma-\alpha+1}{1+\alpha}}}\Big)^{\frac{1}{\gamma-1}},\Big(\frac{\tilde{a}-(\frac{2}{1+\alpha})\lambda^*}{3\tilde{b}\,\|z_{\lambda^*}\|_\infty^{\frac{2(\gamma-1)}{1+\alpha}}}\Big)^{\frac{1}{\gamma-1}},\Big(\frac{(2/1+\alpha)}{2A\,\|z_{\lambda^*}\|_\infty^{\frac{2\beta-\alpha+1}{1+\alpha}}}\Big)^{\frac{1}{\beta-1}},\Big(\frac{\tilde{a}-(\frac{2}{1+\alpha})\lambda^*}{3A\,\|z_{\lambda^*}\|_\infty^{\frac{2(\beta-1)}{1+\alpha}}}\Big)^{\frac{1}{\beta-1}}\Big\},$$

and

$$c^*:=K^{1+\alpha}\min\Big\{(\frac{2}{1+\alpha})\Big(\frac{1-\alpha}{1+\alpha}\Big)m^2,\frac{1}{3}\mu^2\Big(\tilde{a}-(\frac{2}{1+\alpha})\lambda^*\Big)\Big\}.$$

By the same argument as in the proof of theorem 2.1, we can show that $(\psi_1,\psi_2):=(Kz_{\lambda^*}^{\frac{2}{1+\alpha}},Kz_{\lambda^*}^{\frac{2}{1+\alpha}})$ is a positive subsolution of (3.1) for $\max\{c_1,c_2\}\le c^*$, i.e.

$$\begin{cases}-\Delta\psi_1\le a_1\psi_1-b_1{\psi_1}^\gamma-f_1(\psi_1)-\frac{c_1}{{\psi_2}^\alpha}, & x\in\Omega,\\ -\Delta\psi_2\le a_2\psi_2-b_2{\psi_2}^\gamma-f_2(\psi_2)-\frac{c_2}{{\psi_1}^\alpha}, & x\in\Omega,\\ (\psi_1,\psi_2)\le(0,0), & x\in\partial\Omega.\end{cases}$$

Also it is easy to check that constant function $(Z_1,Z_2):=(M,M)$ is a supersolution of (3.1) for M large, i.e.

$$\begin{cases}-\Delta Z_1\ge a_1Z_1-b_1{Z_1}^\gamma-f_1(Z_1)-\frac{c_1}{{Z_2}^\alpha}, & x\in\Omega,\\ -\Delta z_2\ge a_2Z_2-b_2{Z_2}^\gamma-f_2(Z_2)-\frac{c_2}{{Z_1}^\alpha}, & x\in\Omega,\\ (Z_1,Z_2)\ge(0,0), & x\in\partial\Omega.\end{cases}$$

Further M can be chosen large enough so that $(Z_1,Z_2)\ge(\psi_1,\psi_2)$ on Ω. Hence for $\max\{c_1,c_2\}\le c^*$, (3.1) has a positive solution and the proof is complete. □

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