



Decomposition of supra soft locally closed sets and supra SLC-continuity

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Abstract

In this paper, we introduce two different notions of generalized supra soft sets namely supra A -soft sets and supra soft locally closed sets in supra soft topological spaces, which are weak forms of supra open soft sets and discuss their relationships with each other and other supra open soft sets [*International Journal of Mathematical Trends and Technology (IJMTT)*, (2014) Vol. 9 (1):37–56] like supra semi open soft sets, supra pre open soft sets, supra α -open sets and supra β -open sets. Furthermore, the soft union and intersection of two supra soft locally closed sets have been obtained. We also introduce two different notions of generalized supra soft continuity namely supra soft A -continuous functions and supra SLC-continuous functions. Finally, we obtain decompositions of supra soft continuity: f_{pu} is a supra soft A -continuous if it is both supra soft semi-continuous and supra SLC-continuous, and also f_{pu} is a supra soft continuous if and only if it is both supra soft pre-continuous and supra SLC-continuous. Several examples are provided to illustrate the behavior of these new classes of supra soft sets and supra soft functions.

Keywords: supra soft topological space; supra A -soft sets; supra soft locally closed sets; supra SLC-continuous functions.

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1. Introduction

The notion of supra topological spaces was introduced in [16] as a generalization to the topological spaces, which is generalized to the minimal spaces in [18]. An application on the minimal spaces was introduced in [9]. El-Sheikh et al. [10] introduced the notion of supra soft topological spaces, which

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is recently extended in [[1]–[6], [14]]. In [8], the authors introduced the concept of soft locally closed sets in soft topological spaces.

In our recent research we introduce the notions of supra A -soft sets and supra soft locally closed sets in supra soft topological spaces and study their relationships with each other and other supra open soft sets [10] in detail, supported by counterexamples. Furthermore, the decompositions of the supra soft A -continuous functions, supra SLC-continuous functions and other types of supra continuous functions [10] were introduced and studied in detail. Especially, we show that f_{pu} is supra soft continuous if and only if it is both supra soft α -continuous and supra SLC-continuous, and also f_{pu} is supra soft A -continuous if it is both supra soft semi-continuous and supra SLC-continuous, furthermore f_{pu} is supra soft continuous if and only if it is both supra soft pre-continuous and supra SLC-continuous. Finally, these relations were summarized in two diagrams.

2. Preliminaries

In this section we present the basic definitions and theorems which shall be needed later in this paper.

Definition 2.1. (Molodtsov [17]) Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E . A pair (F, A) denoted by F_A is called a soft set over X , where F is a mapping given by $F : A \rightarrow P(X)$ i.e. $(F, A) = \{(e, F(e)) : e \in A \subseteq E, F : A \rightarrow P(X)\}$. The family of all these soft sets denoted by $SS(X)_A$.

Definition 2.2. (Maji et. al. [15], Shabir and Naz [19]) Let $(F, E), (G, E) \in SS(X)_E$. Then

- (1) If $F(e) = \varphi$ for each $e \in E$, then (F, E) is said to be a null soft set, denoted by $\tilde{\varphi}$.
- (2) If $F(e) = X$ for each $e \in E$, then (F, E) is said to be absolute soft set, denoted by \tilde{X} .
- (3) (F, E) is soft subset of (G, E) , denoted by $(F, E) \tilde{\subseteq} (G, E)$, if $F(e) \subseteq G(e)$ for each $e \in E$.
- (4) $(F, E) = (G, E)$, if $(F, E) \tilde{\subseteq} (G, E)$ and $(G, E) \tilde{\subseteq} (F, E)$.
- (5) Soft union of (F, E) and (G, E) , denoted by $(F, E) \tilde{\cup} (G, E)$, is a soft set over X and defined by $(F, E) \tilde{\cup} (G, E) : E \rightarrow P(X)$ such that $((F, E) \tilde{\cup} (G, E))(e) = F(e) \cup G(e)$ for each $e \in E$.
- (6) Soft intersection of (F, E) and (G, E) , denoted by $(F, E) \tilde{\cap} (G, E)$, is a soft set over X and defined by $(F, E) \tilde{\cap} (G, E) : E \rightarrow P(X)$ such that $((F, E) \tilde{\cap} (G, E))(e) = F(e) \cap G(e)$ for each $e \in E$.
- (7) Soft complement of (F, E) is denoted by $(F^{\tilde{c}}, E) = (F, E)^{\tilde{c}}$ and defined by $F^{\tilde{c}} : E \rightarrow P(X)$ such that $F^{\tilde{c}}(e) = X \setminus F(e)$ for each $e \in E$.
- (8) Soft different of (F, E) and (G, E) is denoted by $(F, E) - (G, E)$ and defined by $(F, E) - (G, E) = (F, E) \tilde{\cap} (G, E)^{\tilde{c}}$.

Definition 2.3. (Shabir and Naz [19]) Let τ be a collection of soft sets over a universe X with a fixed set of parameters E , then $\tau \subseteq SS(X)_E$ is called a soft topology on X if

- (1) $\tilde{X}, \tilde{\varphi} \in \tau$, where $\tilde{\varphi}(e) = \varphi$ and $\tilde{X}(e) = X$ ($\forall e \in E$),
- (2) The union of any number of soft sets in τ belongs to τ ,

(3) The intersection of any two soft sets in τ belongs to τ .

The triplet (X, τ, E) is called a soft topological space over X .

Definition 2.4. (Hussain and Ahmad [11], Kandil et. al. [13], Shabir and Naz [19]) The soft set $(F, E) \in SS(X)_E$ is called a soft point in X if there exist $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e') = \varphi$ for each $e' \in E - \{e\}$, and the soft point (F, E) is denoted by x_e .

The soft point x_e is said to be belonging to the soft set (G, E) , denoted by $x_e \tilde{\in} (G, E)$, if for the element $e \in E$, $F(e) \subseteq G(e)$.

Definition 2.5. (El-Sheikh and Abd El-latif [10]) Let τ be a collection of soft sets over a universe X with a fixed set of parameters E , then $\mu \subseteq SS(X)_E$ is called supra soft topology on X with a fixed set E if

(1) $\tilde{X}, \tilde{\varphi} \in \mu$,

(2) The union of any number of soft sets in μ belongs to μ .

The triplet (X, μ, E) is called supra soft topological space (or supra soft spaces) over X .

Definition 2.6. (El-Sheikh and Abd El-latif [10]) Let (X, τ, E) be a soft topological space and (X, μ, E) be a supra soft topological space. We say that, μ is a supra soft topology associated with τ if $\tau \subset \mu$.

Definition 2.7. (El-Sheikh and Abd El-latif [10]) Let (X, μ, E) be a supra soft topological space over X , then the members of μ are said to be supra open soft sets in X . We denote the set of all supra open soft sets over X by supra- $OS(X, \mu, E)$, or when there can be no confusion by supra- $OS(X)$ and the set of all supra closed soft sets by supra- $CS(X, \mu, E)$ or supra- $CS(X)$.

Definition 2.8. (El-Sheikh and Abd El-latif [10]) Let (X, μ, E) be a supra soft topological space over and $(F, E) \in SS(X)_E$. Then, the supra soft interior of (G, E) , denoted by $int^s(G, E)$, is the soft union of all supra open soft subsets of (G, E) . Also, the supra soft closure of (F, E) , denoted by $cl^s(F, E)$, is the soft intersection of all supra closed super soft sets of (F, E) .

Definition 2.9. (Abd El-latif and S. Karataş [1], El-Sheikh and Abd El-latif [10]) Let (X, μ, E) be a supra soft topological space and $(F, E) \in SS(X)_E$. Then (F, E) is said to be

(1) Supra pre open soft set if $(F, E) \tilde{\subseteq} int^s(cl^s(F, E))$.

(2) Supra semi open soft set if $(F, E) \tilde{\subseteq} cl^s(int^s(F, E))$.

(3) Supra α -open soft set if $(F, E) \tilde{\subseteq} int^s(cl^s(int^s(F, E)))$.

(4) Supra β -open soft set if $(F, E) \tilde{\subseteq} cl^s(int^s(cl^s(F, E)))$.

(5) Supra b-open soft set if $(F, E) \tilde{\subseteq} cl^s(int^s(F, E)) \tilde{\cup} int^s(cl^s(F, E))$.

The set of all supra pre open (resp. semi open, α -open, β -open, b-open) soft sets is denoted by supra- $POS(X)$ (resp. supra- $SOS(X)$, supra- $\alpha OS(X)$, supra- $\beta OS(X)$, supra- $BOS(X)$) and the set of all supra pre closed (resp. semi closed, α -closed, β -closed, b-closed) soft sets is denoted by supra- $PCS(X)$ (resp. supra- $SCS(X)$, supra- $\alpha CS(X)$, supra- $\beta CS(X)$, supra- $BCS(X)$).

Definition 2.10. (Ergul and Yuksel [20]) Let (X, μ, E) be a supra soft topological space and $(F, E) \in SS(X)_E$. Then (F, E) is called supra regular open soft set (resp. supra regular closed soft set) if $(F, E) = int^s(cl^s(F, E))$ (resp. $(F, E) = cl^s(int^s(F, E))$).

Definition 2.11. (Ergul and Yuksel [20]) A soft set (F, E) is called soft supra regular generalized closed (soft supra rg-closed) in a supra soft topological space (X, μ, E) if $cl^s(F, E) \tilde{\subseteq} (G, E)$ whenever $(F, E) \tilde{\subseteq} (G, E)$ and (G, E) is supra regular open soft in X .

Definition 2.12. (Ahmad and Kharal [7]) Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets, $u : X \rightarrow Y$ and $p : A \rightarrow B$ be mappings. Let $f_{pu} : SS(X)_A \rightarrow SS(Y)_B$ be a mapping.

(1) If $(F, A) \in SS(X)_A$, then the image of (F, A) under f_{pu} , written as $f_{pu}(F, A) = (f_{pu}(F), p(A))$, is soft set in $SS(Y)_B$ such that

$$f_{pu}(F)(b) = \begin{cases} \cup_{a \in p^{-1}(b) \cap A} u(F(a)), & p^{-1}(b) \cap A \neq \emptyset, \\ \emptyset, & \text{otherwise} \end{cases}$$

for all $b \in B$.

(2) If $(G, B) \in SS(Y)_B$, then the inverse image of (G, B) under f_{pu} , written as $f_{pu}^{-1}(G, B) = (f_{pu}^{-1}(G), p^{-1}(B))$, is soft set in $SS(X)_A$ such that

$$f_{pu}^{-1}(G)(a) = \begin{cases} u^{-1}(G(p(a))), & p(a) \in B, \\ \emptyset, & \text{otherwise} \end{cases}$$

for all $a \in A$.

The soft function f_{pu} is called surjective if p and u are surjective, also is said to be injective if p and u are injective.

Definition 2.13. (Abd El-latif and S. Karataş [1], El-Sheikh and Abd El-latif [10]) Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces. Let μ_1 be an associated supra soft topology with τ_1 . Let $u : X \rightarrow Y$ and $p : A \rightarrow B$ be mappings. Let $f_{pu} : SS(X)_A \rightarrow SS(Y)_B$ be a function. Then f_{pu} is called:

- (1) Supra soft continuous if $f_{pu}^{-1}(G, B) \in \mu_1 \forall (G, B) \in \tau_2$.
- (2) Supra soft pre-continuous if $f_{pu}^{-1}(G, B) \in \text{supra-}POS(X) \forall (G, B) \in \tau_2$.
- (3) Supra soft semi-continuous if $f_{pu}^{-1}(G, B) \in \text{supra-}SOS(X) \forall (G, B) \in \tau_2$.
- (4) Supra soft α -continuous if $f_{pu}^{-1}(G, B) \in \text{supra-}\alpha OS(X) \forall (G, B) \in \tau_2$.
- (5) Supra soft β -continuous if $f_{pu}^{-1}(G, B) \in \text{supra-}\beta OS(X) \forall (G, B) \in \tau_2$.
- (6) Supra soft B-continuous if $f_{pu}^{-1}(G, B) \in \text{supra-}BOS(X) \forall (G, B) \in \tau_2$.

3. Supra A-Soft Sets

In this section, we introduce the notion of supra A-soft sets in supra soft topological spaces and discuss its relationships with other supra open soft sets [10] in detail, supported by counterexamples.

Definition 3.1. A soft subset (F, E) of a supra soft topological space (X, μ, E) is called supra A-soft set if $(F, E) = (G, E) - (H, E)$ where (G, E) is supra open soft and (H, E) is supra regular open soft set in X .

In other words, (F, E) is supra A-soft set if $(F, E) = (W, E) \tilde{\cap} (S, E)$ where (W, E) is supra open soft and (S, E) is supra regular closed soft set in X . We will denote the family of all supra A-soft sets of a supra soft topological space X by supra- $AS(X)$.

Proposition 3.2. *In a supra soft topological space (X, μ, E) , every supra open soft set is a supra A-soft set.*

Proof . The proof is clear. \square

Remark 3.3. The converse of the above theorem is not true in general as shall shown in the following example.

Example 3.4. Suppose that there are four jobs in the universe X given by $X = \{j_1, j_2, j_3, j_4\}$. Let $E = \{e_1, e_2\}$ be the set of decision parameters which stand for "position" and "salary", respectively. Let $(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E), (F_8, E), (F_9, E), (F_{10}, E)$ be ten soft sets over the common universe X which describe the importance of the jobs defined as follows:

$$\begin{aligned} F_1(e_1) &= \{j_1\}, & F_1(e_2) &= \{j_1\}, \\ F_2(e_1) &= \{j_2\}, & F_2(e_2) &= \{j_2\}, \\ F_3(e_1) &= \{j_1, j_2\}, & F_3(e_2) &= \{j_1, j_2\}, \\ F_4(e_1) &= \{j_1, j_2, j_3\}, & F_4(e_2) &= \{j_1, j_2, j_3\}, \\ F_5(e_1) &= \{j_1, j_2\}, & F_5(e_2) &= \{j_1\}, \\ F_6(e_1) &= \{j_1, j_2, j_3\}, & F_6(e_2) &= \{j_1, j_3\}, \\ F_7(e_1) &= \{j_1, j_2, j_4\}, & F_7(e_2) &= \{j_1, j_2, j_3\}, \\ F_8(e_1) &= \{j_1, j_2\}, & F_8(e_2) &= \{j_1, j_2, j_3\}, \\ F_9(e_1) &= \{j_1, j_2\}, & F_9(e_2) &= \{j_1, j_3\}, \\ F_{10}(e_1) &= X, & F_{10}(e_2) &= \{j_1, j_2, j_3\}. \end{aligned}$$

Hence, $\mu = \{\tilde{X}, \tilde{\varphi}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E), (F_8, E), (F_9, E), (F_{10}, E)\}$ is a supra soft topology over X . Therefore, the soft set (K, E) is a supra A-soft set in (X, μ, E) , but not supra open soft, where $K(e_1) = \{j_2, j_3\}$, $K(e_2) = \{j_3\}$. Since $(K, E) = (F_6, E) - (F_1, E)$, where (F_6, E) is supra open soft and (F_1, E) is supra regular open soft set.

There are no prior relation between the class supra- $AS(X)$ and the class supra- $SOS(X)$ (resp. supra- $\alpha OS(X)$). The following examples support our claim.

Example 3.5. (1) Assume that there are three houses in the universe X given by $X = \{h_1, h_2, h_3\}$ and consider $E = \{e_1, e_2\}$ be the set of decision parameters which stand for "quality of houses" and "green surroundings", respectively. Let

$$\mu = \{\tilde{X}, \tilde{\varphi}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E)\},$$

where $(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E)$ are six soft sets over X describe the compositions of the houses, defined as follows:

$$\begin{aligned} F_1(e_1) &= \{h_1\}, & F_1(e_2) &= \{h_1\}, \\ F_2(e_1) &= \{h_1\}, & F_2(e_2) &= \{h_2\}, \\ F_3(e_1) &= \{h_2\}, & F_3(e_2) &= \{h_2\}, \\ F_4(e_1) &= \{h_1, h_2\}, & F_4(e_2) &= \{h_1, h_2\}, \\ F_5(e_1) &= \{h_1\}, & F_5(e_2) &= \{h_1, h_2\}, \\ F_6(e_1) &= \{h_1, h_2\}, & F_6(e_2) &= \{h_2\}. \end{aligned}$$

Then μ defines a supra soft topology on X . Hence, the soft set (G, E) which defined by $G(e_1) = \{h_1\}$ and $G(e_2) = \{h_1, h_3\}$ is a supra semi open soft set but not supra A-soft.

- (2) In Example 3.4, the soft set (K, E) is supra A-soft set but not supra semi-open soft set.
- (3) In Example 3.4, the soft set (H, E) is supra A-soft set but not supra α -open soft set, where $H(e_1) = \{j_2, j_3\}$ and $H(e_2) = \{j_3\}$.
- (4) Suppose that there are three cars in the universe X given by $X = \{c_1, c_2, c_3\}$ and $E = \{e_1, e_2\}$ be the set of decision parameters which stand for "color" and "price". Let $(F_1, E), (F_2, E)$ and (F_3, E) be three soft sets over the common universe X which describe the goodness of the cars defined as follows:

$$\begin{aligned} F_1(e_1) &= \{c_1, c_2\}, & F_1(e_2) &= \{c_2\}, \\ F_2(e_1) &= \{c_1, c_3\}, & F_2(e_2) &= \{c_2, c_3\}, \\ F_3(e_1) &= X, & F_3(e_2) &= \{c_2, c_3\}. \end{aligned}$$

Then μ defines a supra soft topology on X . Hence, the soft set (K, E) , where $K(e_1) = \{c_1, c_2\}$ and $F_1(e_2) = \{c_2, c_3\}$, is supra α -open soft set but not supra A-soft.

4. Supra soft locally closed Sets

Our main aim of this section, is to introduce the notion supra soft locally closed sets in supra soft topological spaces and discuss its relationships with other supra open soft sets [10] in detail, supported by counterexamples.

Definition 4.1. A soft set (F, E) is called supra soft locally closed in a supra soft topological space (X, μ, E) if $(F, E) = (G, E) \tilde{\cap} (H, E)$ where (G, E) is supra open soft and (H, E) is supra closed soft in X . We will denote the family of all supra soft locally closed sets of a supra soft topological space X by supra- $SLC(X)$.

Remark 4.2. A soft subset (F, E) of (X, μ, E) is supra soft locally closed if its relative complement $(F, E)^{\tilde{c}}$ is the soft union of a supra open soft set and a supra closed soft set.

Theorem 4.3. Let (X, μ, E) be a supra soft topological space. Then, (F, E) is supra soft locally closed if and only if $(F, E) = (G, E) \tilde{\cap} cl^s(F, E)$ for some supra open soft set (G, E) .

Proof . (Necessity): Let (F, E) be a supra soft locally closed set in X . Then, $(F, E) = (G, E) \tilde{\cap} (H, E)$ where (G, E) is supra open soft and (H, E) is supra closed soft in X . It follows

$$cl^s(F, E) = cl^s[(G, E) \tilde{\cap} (H, E)] \tilde{\subseteq} cl^s(G, E) \tilde{\cap} cl^s(H, E) = cl^s(G, E) \tilde{\cap} (H, E).$$

Hence $cl^s(F, E) \tilde{\subseteq} (H, E)$. Therefore $(F, E) \tilde{\subseteq} (G, E) \tilde{\cap} cl^s(F, E) \tilde{\subseteq} (G, E) \tilde{\cap} (H, E) = (F, E)$. Thus, $(F, E) = (G, E) \tilde{\cap} cl^s(F, E)$.

(Sufficiently): It is clear. \square

Remark 4.4. The relative complement of a supra soft locally closed set need not to be supra soft locally closed. The following example supports our claim.

Example 4.5. In Example 3.4, the soft set (G, E) is supra soft locally closed sets in (X, μ, E) , where $G(e_1) = \{j_3, j_4\}$ and $G(e_2) = \{j_2\}$, but its relative complement $(G, E)^{\tilde{c}}$ is not supra soft locally closed, where $G^{\tilde{c}}(e_1) = \{j_1, j_2\}$ and $G^{\tilde{c}}(e_2) = \{j_1, j_3, j_4\}$.

Theorem 4.6. In a supra soft topological space (X, μ, E) , every supra A -soft set is a supra soft locally closed.

Proof . Follows from the fact that, every supra regular closed soft set in a supra soft topological space (X, μ, E) is a supra closed soft set [20, Remark 3.2]. \square

Remark 4.7. The converse of the above theorem is not true in general as shall shown in the following example.

Example 4.8. In Example 3.4, the soft set (G, E) is supra soft locally closed set, where $G(e_1) = \{j_3, j_4\}$, $G(e_2) = \{j_2\}$. Since $(G, E) = (F_{10}, E) \tilde{\cap} (H, E)$, where (F_{10}, E) is supra open soft and (H, E) is supra closed soft in X defined by $H(e_1) = \{j_3, j_4\}$, $H(e_2) = \{j_2, j_4\}$. On the other hand, it is not supra A -soft.

Theorem 4.9. In a supra soft topological space (X, μ, E) , every supra open soft set is a supra soft locally closed.

Proof . It is obvious. \square

Remark 4.10. The converse of the above theorem is not true in general as shall shown in the following example.

Example 4.11. In Example 3.4, the soft set (G, E) is supra soft locally closed set in (X, μ, E) , but not supra open soft, where $G(e_1) = \{j_3, j_4\}$ and $G(e_2) = \{j_2\}$.

Remark 4.12. The finite soft intersection (resp. soft union) of supra soft locally closed subsets need not to be supra soft locally closed in general as shown in the following examples.

Example 4.13. (1) In Example 3.4, the soft sets (H_1, E) and (H_2, E) are supra soft locally closed sets in (X, μ, E) , where

$$H_1(e_1) = \{j_3, j_4\}, \quad H_1(e_2) = \{j_2\},$$

$$H_2(e_1) = \{j_1, j_2, j_3\}, \quad H_2(e_2) = \{j_1, j_2, j_3\}.$$

But, their soft intersection $(H_1, E) \tilde{\cap} (H_2, E) = (H_3, E)$, where $H_3(e_1) = \{j_3\}$ and $H_3(e_2) = \{j_2\}$ is not supra soft locally closed.

(2) In Example 3.4, the soft sets (K_1, E) and (K_2, E) are supra soft locally closed sets in (X, μ, E) , where

$$K_1(e_1) = \{j_1\}, \quad K_1(e_2) = \{j_1\},$$

$$K_2(e_1) = \{j_3, j_4\}, \quad K_2(e_2) = \{j_2\}.$$

But, their soft union $(K_1, E) \tilde{\cup} (K_2, E) = (K_3, E)$, where $K_3(e_1) = \{j_1, j_3, j_4\}$ and $K_3(e_2) = \{j_1, j_2\}$ is not supra soft locally closed.

Definition 4.14. Two non-null soft subsets (G, E) and (H, E) of a supra soft topological space (X, μ, E) are said to be supra soft separated sets if $(G, E) \tilde{\cap} cl^s(H, E) = \tilde{\varphi}$ and $cl^s(G, E) \tilde{\cap} (H, E) = \tilde{\varphi}$.

The following theorem gives the necessary condition for the finite soft union (resp. soft intersection) of supra soft locally closed sets to be supra soft locally closed.

Theorem 4.15. Let (F, E) and (G, E) be supra soft locally closed subsets of (X, μ, E) . If (F, E) and (G, E) are supra soft separated, then $(F, E) \tilde{\cup} (G, E) \in \text{supra-}SLC(X)$ and $(F, E) \tilde{\cap} (G, E) \in \text{supra-}SLC(X)$.

Proof . Let (F, E) and (G, E) be supra soft locally closed subsets of (X, μ, E) . Then, there exist open soft sets (A, E) and (B, E) such that $(F, E) = (A, E) \tilde{\cap} cl^s(F, E)$ and $(G, E) = (B, E) \tilde{\cap} cl^s(G, E)$ from Theorem 4.3. Since (F, E) and (G, E) are supra soft separated we may assume that

$$(A, E) \tilde{\cap} cl^s(G, E) = (B, E) \tilde{\cap} cl^s(F, E) = \tilde{\varphi}.$$

Therefore $(F, E) \tilde{\cup} (G, E) = [(A, E) \tilde{\cup} (B, E)] \tilde{\cap} cl^s[(F, E) \tilde{\cup} (G, E)]$. Hence $(F, E) \tilde{\cup} (G, E)$ is supra soft locally closed. On the other hand

$$\begin{aligned} (F, E) \tilde{\cap} (G, E) &= [(A, E) \tilde{\cap} cl^s(F, E)] \tilde{\cap} [(B, E) \tilde{\cap} cl^s(G, E)] \\ &= [(A, E) \tilde{\cap} cl^s(G, E)] \tilde{\cap} [(B, E) \tilde{\cap} cl^s(F, E)] \\ &= \tilde{\varphi}. \end{aligned}$$

Hence $(F, E) \tilde{\cap} (G, E) \in \text{supra-}SLC(X)$. \square

Definition 4.16. A soft subset (F, E) of a supra soft topological space (X, μ, E) is called supra soft dense set if $cl^s(F, E) = \tilde{X}$.

Proposition 4.17. A supra soft dense set (F, E) is supra open soft set if and only if it is supra soft locally closed.

Proof . Let (F, E) be a supra open soft set. Since (F, E) is supra soft dense set

$$(F, E) \tilde{\cap} \tilde{X} = (F, E) = (F, E) \tilde{\cap} cl^s(F, E).$$

Hence (F, E) is supra soft locally closed from Theorem 4.3. Conversely, if (F, E) is a supra soft locally closed set then $(F, E) = (G, E) \tilde{\cap} cl^s(F, E)$ for some supra open soft set (G, E) from Theorem 4.3. Since (F, E) is supra soft dense set, $(F, E) = (G, E) \tilde{\cap} \tilde{X} = (G, E)$. Thus (F, E) is supra open soft set. \square

Definition 4.18. A supra soft topological space (X, μ, E) is called supra soft submaximal if every supra soft dense subset of (X, μ, E) is supra open soft.

Corollary 4.19. A supra soft topological space (X, μ, E) is supra soft submaximal if and only if every soft dense subset of (X, μ, E) is supra soft locally closed.

Proof . Immediate from Proposition 4.17 and Definition 4.18. \square

Theorem 4.20. Let (X, μ, E) be a supra soft topological space. Then the following are equivalent with respect to a soft set (F, E) over X :

- (1) (F, E) is supra open soft set.
- (2) (F, E) is both supra α -open soft and supra soft locally closed in X .
- (3) (F, E) is both supra pre open soft and supra soft locally closed in X .

Proof .

(1) \Rightarrow (2) Follows from Theorem 4.9 and [10, Theorem 5.1 (3)].

(2) \Rightarrow (3) Follows from [10, Theorem 5.2 (4)].

(3) \Rightarrow (1) Let (F, E) be a supra pre open soft set and supra soft locally closed in X . Then $(F, E) \tilde{\subseteq} int^s(cl^s(F, E))$ and $(F, E) = (G, E) \tilde{\cap} cl^s(F, E)$ where (G, E) is supra open soft from Theorem 4.15. It follows $(F, E) \tilde{\subseteq} (G, E) \tilde{\cap} int^s(cl^s(F, E)) = int^s[(G, E) \tilde{\cap} cl^s(F, E)] = int^s(F, E)$. Therefore (F, E) is supra open soft.

\square

Theorem 4.21. Let (X, μ, E) be a supra soft topological space over X . If (F, E) be both supra semi open soft and supra soft locally closed in X , then it is supra A -soft.

Proof . Let (F, E) be semi open soft set and supra soft locally closed, so that $(F, E) \tilde{\subseteq} cl^s(int^s(F, E))$ and $(F, E) = (M, E) \tilde{\cap} cl^s(F, E)$, where (M, E) is supra open soft from Theorem 4.15. Now $cl^s(F, E) \tilde{\subseteq} cl^s(int^s(F, E)) \tilde{\subseteq} cl^s(int^s(cl^s(F, E))) \tilde{\subseteq} cl^s(F, E)$. Therefore $cl^s(F, E) = cl^s(int^s(cl^s(F, E)))$. This means that $cl^s(F, E)$ is supra regular closed soft and hence (F, E) is a supra A-soft set. \square

Corollary 4.22. on account of Theorem 4.20 and Theorem 4.21 , for any supra soft topological space (X, μ, E) we have the following fundamental relationships between the classes of supra soft sets over X :

- (1) Supra- $OS(X) = \text{supra-}\alpha OS(X) \cap \text{supra-}SLC(X)$.
- (2) Supra- $AS(X) \supseteq \text{supra-}SOS(X) \cap \text{supra-}SLC(X)$.
- (3) Supra- $OS(X) = \text{supra-}POS(X) \cap \text{supra-}SLC(X)$.

For a supra soft topological space (X, μ, E) we have the following implications from Proposition 3.2, Theorems 4.6, 4.9, [1, Corollary 4.1] and [10, Corollary 4.1]. These implications are not reversible.

$$\begin{array}{ccccc}
 & \text{supra-}SLC(X) & \longleftarrow & \text{supra-}AS(X) & \\
 \circ & \nearrow & & \nwarrow & \\
 \text{supra-}OS(X) & \longrightarrow & \text{supra-}\alpha OS(X) & \longrightarrow & \text{supra-}SOS(X) \\
 \circ & \downarrow & & \swarrow & \\
 \text{supra-}POS(X) & & \longrightarrow & \text{supra-}BOS(X) & \longrightarrow & \text{supra-}\beta OS(X)
 \end{array}$$

5. Supra SLC-continuity

In this section, we introduce two different notions of generalized supra soft continuity, namely supra soft A-continuous functions and supra SLC-continuous functions. Furthermore, we obtain decompositions of supra soft continuity. Finally, Several examples are provided to illustrate the behavior of these new classes of soft functions.

Definition 5.1. Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces. Let μ_1 be an associated supra soft topology with τ_1 . Let $u : X \rightarrow Y$ and $p : A \rightarrow B$ be mappings. Let $f_{pu} : SS(X)_A \rightarrow SS(Y)_B$ be a function. Then f_{pu} is called:

- (1) Supra soft A-continuous function if $f_{pu}^{-1}(G, B) \in \text{supra-}AS(X) (\forall (G, B) \in \tau_2)$.
- (2) Supra soft locally closed continuous function (supra SLC-continuous) if $f_{pu}^{-1}(G, B) \in \text{supra-}SLC(X) (\forall (G, B) \in \tau_2)$.

Theorem 5.2. Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces. Let μ_1 be an associated supra soft topology with τ_1 . Let $u : X \rightarrow Y$ and $p : A \rightarrow B$ be mappings. Let $f_{pu} : SS(X)_A \rightarrow SS(Y)_B$ be a function. Then

- (1) Every supra soft continuous function is supra soft A-continuous.
- (2) Every supra soft A-continuous function is supra SLC-continuous.
- (3) Every supra soft continuous function is supra SLC-continuous.

Proof . It is obvious from Proposition 3.2 and Theorems 4.6, 4.9. \square

Remark 5.3. The converse of Theorem 5.2 is not true in general, as shown in the following examples.

Example 5.4. (1) Let $X = \{j_1, j_2, j_3, j_4\}$, $Y = \{x, y, z\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u : X \rightarrow Y$ and $p : A \rightarrow B$ as follows:

$$u(a) = \{z\}, u(b) = \{y\}, u(c) = \{x\}, u(d) = \{z\}$$

and

$$p(e_1) = \{k_2\}, p(e_2) = \{k_1\}.$$

Let (X, τ_1, A) be a soft topological space over X where $\tau_1 = \{\tilde{X}, \tilde{\varphi}, (F_1, A)\}$, and (F_1, A) is a soft set over X defined as follows:

$$F(e_1) = \{a, b\} \quad \text{and} \quad F(e_2) = \{a, b\}.$$

Consider the supra soft topology $\mu_1 = \{\tilde{X}, \tilde{\varphi}, (F_1, A), (F_2, A), \dots, (F_{10}, A)\}$ in Example 3.4. Let (Y, τ_2, B) be a soft topological space over Y where $\tau_2 = \{\tilde{Y}, \tilde{\varphi}, (G, B)\}$, and (G, B) is a soft set over Y defined by $G(k_1) = \{x, y\}$ and $G(k_2) = \{x\}$. Let $f_{pu} : (X, \tau_1, A) \rightarrow (Y, \tau_2, B)$ be a soft function. Then $f_{pu}^{-1}((G, B)) = \{(e_1, \{j_2, j_3\}), (e_2, \{j_3\})\}$ is a supra A-soft set, but it is not supra open soft. Hence f_{pu} is a supra soft A-continuous function, but it is not supra soft continuous.

(2) Let $X = \{j_1, j_2, j_3, j_4\}$, $Y = \{x, y, z, w\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u : X \rightarrow Y$ and $p : A \rightarrow B$ as follows:

$$u(a) = \{z\}, u(b) = \{w\}, u(c) = \{x\}, u(d) = \{y\}$$

and $p(e_1) = \{k_2\}$, $p(e_2) = \{k_1\}$. Let (X, τ_1, A) be a soft topological space over X where $\tau_1 = \{\tilde{X}, \tilde{\varphi}, (F_1, A)\}$, and (F_1, A) is a soft set over X defined as $F(e_1) = \{a, b\}$, $F(e_2) = \{a, b\}$. Consider the supra soft topology μ_1 in Example 3.4, $\mu_1 = \{\tilde{X}, \tilde{\varphi}, (F_1, A), (F_2, A), \dots, (F_{10}, A)\}$. Let (Y, τ_2, B) be a soft topological space over Y where $\tau_2 = \{\tilde{Y}, \tilde{\varphi}, (G, B)\}$, and (G, B) is a soft set over Y defined by $G(k_1) = \{x, y\}$, $G(k_2) = \{w\}$. Let $f_{pu} : (X, \tau_1, A) \rightarrow (Y, \tau_2, B)$ be a soft function. Then $f_{pu}^{-1}((G, B)) = \{(e_1, \{j_3, j_4\}), (e_2, \{j_2\})\}$ is a supra soft locally closed, but it is not supra A-soft. Hence f_{pu} is a supra SLC-continuous, but it is not supra soft A-continuous.

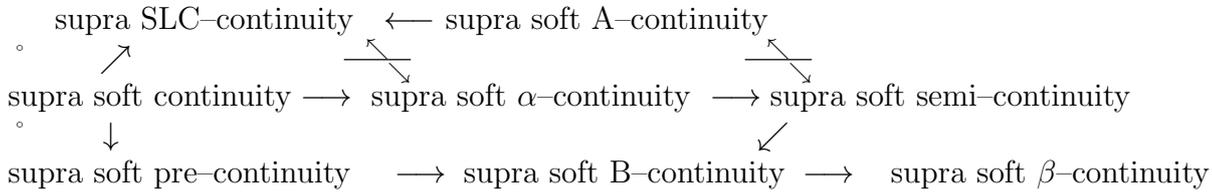
(3) In (1), f_{pu} is a supra SLC-continuous, but it is not supra soft continuous .

Theorem 5.5. Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces. Let μ_1 be an associated supra soft topology with τ_1 . Let $u : X \rightarrow Y$ and $p : A \rightarrow B$ be mappings. Let $f_{pu} : SS(X)_A \rightarrow SS(Y)_B$ be a function. Then the function f_{pu} is:

- (1)** Supra soft continuous if and only if it is both supra soft α -continuous and supra SLC-continuous.
- (2)** Supra soft A-continuous if it is both supra soft semi-continuous and supra SLC-continuous.
- (3)** Supra soft continuous if and only if it is both supra soft pre-continuous and supra SLC-continuous.

Proof . It follows from Theorems 4.20 and Theorem 4.21. \square

For a supra soft topological space (X, μ, E) we have the following implications from Theorems 5.2, [1, Corollary 6.1] and [10, Corollary 6.1]. These implications are not reversible.



6. Conclusion

In this work, we introduce supra A-soft sets and supra soft locally closed sets in supra soft topological spaces and discuss their relationships with each other and other supra open soft sets, supported by examples and counterexamples. We also introduce the concepts of supra soft A-continuous functions and supra SLC-continuous functions. Finally, the decompositions of the supra soft A-continuous functions, supra SLC-continuous functions and other types of supra continuous functions [10] were introduced and studied in detail. In future, the generalization of these concepts to fuzzy supra soft topological spaces will be introduced.

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