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Projected non-stationary simultaneous iterative methods

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Abstract

In this paper, we study Projected non-stationary Simultaneous Iterative Reconstruction Techniques (P-SIRT). Based on algorithmic operators, convergence result are adjusted with Opial's Theorem. The advantages of P-SIRT are demonstrated on examples taken from tomographic imaging.

Keywords: simultaneous iterative reconstruction techniques; convex feasibility problem; (firmly) nonexpansive operator; cutter operator.
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1. Introduction

Large-scale discretizations of ill-posed problems (as imaging problems in tomography) lead to large, sparse and ill-posed (sensitive to noise) linear systems of equations (which may be inconsistent) of the form

$$Ax = b. \tag{1.1}$$

Many problems as image reconstruction [30, 12, 13, 26, 24, 23], computed tomography [21, 22, 34], image recovery [33, 35], image restoration [36], image registration [29], seismic imaging [20], image fusion [17], radar imaging [14] lead to a linear system as (1.1).

Finding $x^* \in \mathbb{R}^n$ such that $Ax^* = b$ is a special case of convex feasibility problems (CFPs). Actually many problems in mathematics and physical sciences can be modeled as a CFP, i.e., a problem of finding a point $x \in Q = \bigcap_{i=1}^m Q_i$ where $\{Q_i\}_{i=1}^m \subseteq \mathbb{R}^n$ are closed convex sets. Using fixed point iterative methods based on algorithmic operators has been suggested by many researchers for solving CFPs, see, e.g., [2, 8]. One of the most important class of algorithmic operators is

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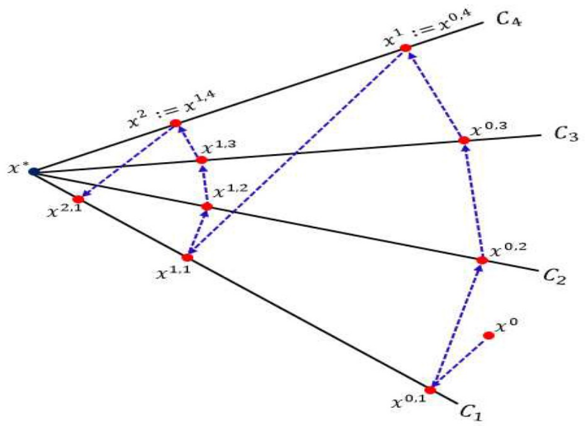

Figure 1: ART method

projection algorithms that play a main role in the area of constructive solution of CFPs. Projection algorithms are iterative algorithms that use projections onto sets. We next give some instances of such algorithms.

Let $A \in \mathbb{R}^{m\times n}$ and $b \in \mathbb{R}^m$. Let a^i and b_i denote the i-row of A and b, respectively. Therefore projection of $x \in \mathbb{R}^n$ onto the i-hyperplane, i.e. $H_i = \{x \in \mathbb{R}^n | \langle a^i, x\rangle = b_i\}$, is

$$P_{H_i}(x) = x + \frac{b_i - \langle a^i, x\rangle}{\|a^i\|^2} a^i \qquad i = 1, 2, \cdots, m \tag{1.2}$$

where $i = 1, 2, \cdots, m$. To simplifying the notation we denote $P_{H_i}(x) = P_i(x)$.

The projection operators can be used in various ways. We briefly explain a special case of sequential and simultaneous methods which use two different ways of projection operators. Algebraic Reconstruction Technique (ART) [21] is a sequential method which executes as follows. Let $x^0 \in \mathbb{R}^n$ be an arbitrary starting point. The ART algorithm projects the current iteration x^k onto a hyperplane, e.g. H_i, and puts $x^{k+1} = P_i(x^k)$. Let $T = P_m \cdots P_2P_1$ where P_i is defined in (1.2). One cycle of the ART method is performed by acting T on the staring point. In this way, we obtain a sequence of cycles which is a subsequence of iterations, see Figure 1. Simultaneous algorithms project x^k onto all hyperplanes $\{H_i\}_{i=1}^m$ simultaneously. The next iteration is performed as convex combination of m new projected points, see Figure 2.

We now explain this algorithm with more details. Let $T = \sum_{i=1}^m \omega_i P_i$, where $\sum_{i=1}^m \omega_i = 1$ and $\omega_i \geq 0$.

Using (1.2) we get

$$T(x) = \sum_{i=1}^m \omega_i P_i \tag{1.3}$$

$$= \sum_{i=1}^m \omega_i x + \omega_i \frac{b_i - \langle a^i, x\rangle}{\|a^i\|^2} a^i$$

$$= x + A^T M (b - Ax) \tag{1.4}$$

where

$$M = diag\left(\frac{\omega_1}{\|a^1\|^2}, \cdots, \frac{\omega_m}{\|a^m\|^2}\right).$$

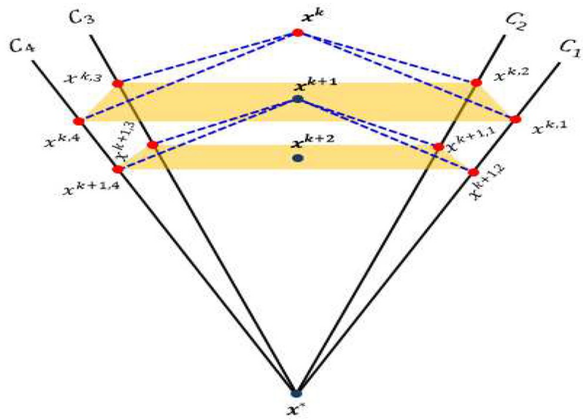

Figure 2: Simultaneous method

Therefore, using (1.3) and (1.4), the fixed point iterative method $x^{k+1} = T(x^k)$ is a special case of SIRT. In general, the SIRT is defined as the following iteration algorithm

$$x^{k+1} = x^k + \lambda_k S A^T M \left(b - Ax^k\right) \qquad k = 0, 1, 2, \dots \tag{1.5}$$

where $\lambda_k \in [\epsilon, \frac{2-\epsilon}{\sigma_1^2}]$ are relaxation parameters and σ_1 is the largest singular value of $M^{\frac{1}{2}} A S^{\frac{1}{2}}$. Also, M and S are assumed symmetric positive definite matrices. Several well-known fully simultaneous methods can be written in the form of (1.5) for appropriate choices of M and S matrices. We below give some instances

- Landweber's method [28]: $S = M = I$,
- Cimmino's method [15]: $S = I$ and $M = \frac{1}{m} diag\left(\frac{1}{\|a^i\|^2}\right)$,
- CAV (Component Averaging method) method [**?** 12]: $S = I$ and $M = \text{diag}(1/\sum_{j=1}^n N_j a_{ij}^2)$ where N_j is the number of non-zeroes in the jth column of A,
- DROP (Diagonally Relaxed Orthogonal Projection) method [**?**]: $S = \text{diag}(m/N_j)$ and M any symmetric positive definite matrix.

Furthermore, the SART method [1] and the symmetric Kaczmarz's method [6] can be rewritten as (1.5).

When solving an inverse problem, the use of constraints (like nonnegativity) and prior information are well known techniques to improve the quality of the obtained solution because incorporate prior physical knowledge about the solution leads to smaller reconstruction errors, see [5, 4, 3, 7, 25, 32, 37].

In this paper we consider the projected version of equation (1.5) in a finite dimensional Euclidean space $\mathbb{R}^n$. Let $\mathcal{C} \subseteq \mathbb{R}^n$ denote a closed convex set and $P_{\mathcal{C}}$ be the metric projection onto $\mathcal{C}$. Assume that $\{\lambda_k\}_{k=0}^{\infty}$ is a sequence of positive relaxation parameters. Now consider the following algorithm.

Algorithm 1.1. *(P-SIRT)*
Initialization: $x^0 \in \mathbb{R}^n$ *is arbitrary.*
Iterative Step: *given* x^k*, compute*

$$x^{k+1} = P_{\mathcal{C}}\left(x^k + \lambda_k S A^T M \left(b - Ax^k\right)\right) \qquad k = 0, 1, 2, \cdots .$$

In Next section, using algorithmic operators, we adjust Algorithm 1.1 with a corollary of generalization of Opial's Theorem, where relaxation parameters are changed in each iteration. The paper is organized as follows. In section 2 we recall some definitions and properties of some algorithmic operators and give the convergence analysis of Algorithm 1.1. At the end, the capability of the main result is examined in section 3 using some numerical tests form the medical imaging field.

2. Preliminaries and Notations

Throughout this section, we consider $T : H \to H$ with nonempty fixed point set, i.e., $FixT \neq \emptyset$ where H is a Hilbert space and Id denotes the identity operator on H. The following definitions, taken from [8], will be useful in our future analysis.

Definition 2.1. Let $T : H \to H$ and $\alpha \in [0, 2]$. The operator T_α defined by

$$T_\alpha := (1 - \alpha)Id + \alpha T \tag{2.1}$$

is called an α-relaxation or, shortly, relaxation of the operator T. If $\alpha \in (0, 2)$, then T_α is called a strictly (or strict) relaxation of T.

Definition 2.2. We say that an operator $T : H \to H$ is nonexpansive (NE), if

$$\|T(x) - T(y)\| \leq \|x - y\| \tag{2.2}$$

for all $x, y \in H$. Also T is an α-contraction, where $\alpha \in (0, 1)$ or, shortly, a contraction if

$$\|T(x) - T(y)\| \leq \alpha\|x - y\| \tag{2.3}$$

for all $x, y \in H$.

Another useful class of operators is the class of cutter operators, namely

Definition 2.3. An operator $T : H \to H$ with nonempty fixed point set is called *cutter* if

$$\langle x - T(x), z - T(x)\rangle \leq 0 \tag{2.4}$$

for all $x \in H$ and $z \in FixT$.

Remark 2.4. Based on [8, Remark 2.1.31] the operator T is a cutter if and only if

$$\langle T(x) - x, z - x\rangle \geq \|T(x) - x\|^2 \tag{2.5}$$

for all $x \in H$ and $z \in FixT$.

Definition 2.5. We say that an operator $T : H \to H$ is firmly nonexpansive (FNE), if

$$\langle T(x) - T(y), x - y\rangle \geq \|T(x) - T(y)\|^2 \tag{2.6}$$

for all $x, y \in H$.

Based on [8, Remark 2.1.31], an $\alpha-$relaxed cutter operator is defined as follows.

Definition 2.6. Let $T : H \to H$ has a fixed point. Then the operator T is an α-relaxed cutter, or, shortly, relaxed cutter where $\alpha \in [0, 2]$, if

$$\langle T_\alpha(x) - x, z - x\rangle = \alpha \langle T(x) - x, z - x\rangle \geq \|T(x) - x\|^2 \tag{2.7}$$

for all $x \in H$ and $z \in FixT$. If $\alpha \in (0, 2)$, then T_α is called a strictly relaxed cutter operator of T.

Definition 2.7. Let $\alpha \geq 0$ and assume that $T : H \to H$ has a fixed point. We say that T is $\alpha-$strongly quasi-nonexpansive ($\alpha-$SQNE), if

$$\|T(x) - z\|^2 \leq \|x - z\|^2 - \alpha\|T(x) - x\|^2 \tag{2.8}$$

for all $x \in H$ and $z \in FixT$. Also, the operator T satisfying (2.8) with $\alpha > 0$ is called strongly quasi-nonexpansive (SQNE) operator.

Following theorem presents the relationship between strictly relaxed cutter and SQNE operators.

Theorem 2.8. [8, Theorem 2.1.39 and Corollary 2.1.40] Assume that $T : H \to H$ has a fixed point and let $\lambda \in (0, 2]$. Then T is a λ-relaxed cutter if and only if T is $\frac{2-\lambda}{\lambda}$-SQNE, i.e.,

$$\|T_\lambda(x) - z\|^2 \leq \|x - z\|^2 - \frac{2 - \lambda}{\lambda}\|T_\lambda(x) - x\|^2 \tag{2.9}$$

for all $x \in H$ and all $z \in FixT$.

Definition 2.9. An operator $T : H \to H$ is *demi-closed* at 0 if for any weakly converging sequence $x^k \rightharpoonup y \in H$ with $T(x^k) \to 0$ we have $T(y) = 0$.

Remark 2.10. It is well known, see [31, Lemma 2], the operator $T - Id$ is demi-closed at 0 where $T : H \to H$ is a nonexpansive operator.

We now verify, using [9, Corollary 9.14.], that the sequence generated by Algorithm (1.1) converges.

Corollary 2.11. [9, Corollary 9.14.] and [8, Corollary 3.7.3] Let $T : H \to H$ be a cutter operator (e.g., a firmly nonexpansive operator having a fixed point) and $x^0 \in H$ is an arbitrary point. Assume that the sequence $\{x^k\}_{k=0}^{\infty}$ is generated by

$$x^{k+1} = P_{\mathcal{C}}\left(x^k + \lambda_k\left(T(x^k) - x^k\right)\right) \text{ for } k = 1, 2, \cdots \tag{2.10}$$

where $\lambda_k \in (0, 2)$.

(i) If $\liminf_{k\to\infty} \lambda_k(2 - \lambda_k) > 0$, then $\{x^k\}_{k=0}^{\infty}$ converges weakly to a fixed point of T.

(ii) If H is finite-dimensional and $\sum_{k=0}^{\infty} \lambda_k(2 - \lambda_k) = \infty$, then $\{x^k\}_{k=0}^{\infty}$ converges to a fixed point of T.

Let $B = S^{\frac{1}{2}}A^TMAS^{\frac{1}{2}} = (M^{\frac{1}{2}}AS^{\frac{1}{2}})^T(M^{\frac{1}{2}}AS^{\frac{1}{2}})$ then the spectral radius of B is denoted by $\rho(B) = \sigma_1^2$ where σ_1 is the largest singular value of $M^{\frac{1}{2}}AS^{\frac{1}{2}}$. We next present a useful lemma from [18].

Lemma 2.12. [18, Lemma 3.1] Let $q = \|I - \lambda B\|$. Assume that $rank(A) = n$ and $\sigma_1 > \sqrt{2}\sigma_n$. Further assume that λ fullfills $0 < \epsilon \leq \lambda \leq (2-\epsilon)/\sigma_1^2$. Then

$$q = \begin{cases} 1 - \lambda\sigma_n^2, & 0 < \lambda \leq \frac{2}{\sigma_1^2+\sigma_n^2} \\ \lambda\sigma_1^2 - 1, & \frac{2}{\sigma_1^2+\sigma_n^2} \leq \lambda < \frac{2}{\sigma_1^2}. \end{cases} \tag{2.11}$$

Remark 2.13. It should be noted that for inverse problems $\sigma_n \ll \sigma_1$ and hence $2/(\sigma_1^2+\sigma_n^2) \approx 2/\sigma_1^2$. Therefore, we will consider only the case $q = 1 - \lambda\sigma_n^2$. Furthermore, one can avoid the assumption $rank(A) = n$ and consider the rank-deficient case using [18, Lemma 3.9].

We next present the convergence analysis of Algorithm 1.1.

Theorem 2.14. *The sequence generated by Algorithm* 1.1*, where* $\lambda_k \in [\epsilon, \frac{2-\epsilon}{\sigma_1^2}]$*, converges to a solution* x^* *of* $\min \|Ax - b\|_M$.

Proof . Since $\lambda_k \in [\epsilon, \frac{2-\epsilon}{\sigma_1^2}]$ we can rewrite the Algorithm 1.1 as below

$$\begin{aligned} x^{k+1} = U(x^k) &= P_{\mathcal{C}}\left(x^k + \frac{\lambda_k}{\rho(S^{\frac{1}{2}}A^TMAS^{\frac{1}{2}})}SA^TM(b - Ax^k)\right) && (2.12)\\ &= P_{\mathcal{C}}\left(x^k + \lambda_k\left(T(x^k) - x^k\right)\right) && (2.13)\\ &= P_{\mathcal{C}}T_{\lambda_k}(x^k) && (2.14) \end{aligned}$$

where

$$T(x) = x + \frac{1}{\rho(S^{\frac{1}{2}}A^TMAS^{\frac{1}{2}})}SA^TM(b - Ax). \tag{2.15}$$

Furthermore, we have

$$\begin{aligned} \|T(x) - T(y)\| &= \left\|(x-y) - \frac{1}{\rho(S^{\frac{1}{2}}A^TMAS^{\frac{1}{2}})}SA^TMA(x-y)\right\| \\ &= \left\|\left(I - \frac{1}{\rho(S^{\frac{1}{2}}A^TMAS^{\frac{1}{2}})}SA^TMA\right)(x-y)\right\| \\ &= \left\|S^{\frac{1}{2}}\left(I - \frac{1}{\rho(S^{\frac{1}{2}}A^TMAS^{\frac{1}{2}})}S^{\frac{1}{2}}A^TMAS^{\frac{1}{2}}\right)S^{\frac{-1}{2}}(x-y)\right\| \\ &\leq \left\|S^{\frac{1}{2}}\left(I - \frac{1}{\rho(S^{\frac{1}{2}}A^TMAS^{\frac{1}{2}})}S^{\frac{1}{2}}A^TMAS^{\frac{1}{2}}\right)S^{\frac{-1}{2}}\right\| \|(x-y)\| \\ &= \left\|\left(I - \frac{1}{\rho(S^{\frac{1}{2}}A^TMAS^{\frac{1}{2}})}S^{\frac{1}{2}}A^TMAS^{\frac{1}{2}}\right)\right\| \|(x-y)\|. \end{aligned}$$

Based on Lemma 2.12, Remark 2.13 and setting $\bar{A} = S^{\frac{1}{2}}A$, we have

$$\alpha = \|I - \frac{1}{\rho(\bar{A}^TM\bar{A})}\bar{A}^TM\bar{A}\| = 1 - \sigma_n^2 < 1.$$

Thus operator T is an α-contraction operator. Also based on [8, Theorem 2.2.34], T is a $(1+\alpha)$-relaxed firmly nonexpansive operator. Using [8, Corollary 2.2.11] T is a firmly nonexpansive and consequently based on the first part of [8, Theorem 2.2.5] T is a cutter operator. Using Remark 2.10 we know that the operator $T-I$ is demi-closed at 0. Therefore based on Corollary 2.11 the sequence $\{x^k\}$ converges weakly to a fixed point of T. Since we are using finite dimensional space $\mathbb{R}^n$ we obtain $x^k \to x^*$ such that $T(x^*) = x^*$. It gives $A^T M(b - Ax^*) = 0$ which is equivalent to the fact that x^* is a minimizer of $\|Ax - b\|_M$. □

3. Numerical Result

In this section we report some numerical results in field of medical imaging. Our numerical results show the effect of using projection operator after each iteration. Furthermore we suggest a rule for picking relaxation parameters.

In following two tables we show error histories for Landweber, Cimmino, CAV and DROP algorithms without constraint ($\mathcal{C} = \mathbb{R}^n$), with non-negativity constraints ($\mathcal{C} = \mathbb{R}^n_+$), and with box constraints ($\mathcal{C} = [0,1]^n$) within 40 iterations. For all of algorithms, we use the following strategy for picking relaxation parameters that were proposed in [18, 19].

$$\lambda_k = \begin{cases} \sqrt{2}\sigma_1^{-2} & \text{for} \quad k = 0, 1 \\ 2\sigma_1^{-2} \dfrac{1-\zeta_k}{(1-\zeta_k^k)^2}, & \text{for} \quad k \geq 0 \end{cases} \tag{3.1}$$

where σ_1 is largest singular value of $M^{\frac{1}{2}} A S^{\frac{1}{2}}$ and ζ_k are roots of a certain polynomial such that $0 < \zeta_k < \zeta_{k+1}$ and $\lim_{k\to\infty} \zeta_k = 1$.

The test is taken from the field of image reconstruction from projections using the SNARK93 software package [27]. We work with the standard head phantom from [21]. The phantom is discretized into 63×63 pixels, and 16 projections (evenly distributed between 0 and 174 degrees) with 99 rays per projection are used. The resulting matrix A has dimension 1584×3969, so that the system of equations is highly underdetermined. In addition to A, the software also produces a noise-free right-hand side b_{snark} and a phantom (translated into vector form) x^*. Using SNARK93's right-hand side b_{snark}, which is not generated as the product Ax^*, we avoid committing an inverse crime where the exact same model is used in the forward and reconstruction models. Apart from using noise-free data we also added additive independent Gaussian noise of mean 0 and relative noise-level ($\|\delta b\|/\|b_{snark}\|$) 5% where $b_{noisy} = b_{snark} + \delta b$.

Table 1: The smallest relative error with noiseless (top) and noisy data (down) using Algorithm 1.1

Algorithm	$\mathcal{C} = \mathbb{R}^n$	$\mathcal{C} = \mathbb{R}^n_+$	$\mathcal{C} = [0,1]^n$
Landweber	0.2623	0.2571	0.2571
Cimmino	0.2338	0.2218	0.2218
CAV	0.2207	0.2014	0.2014
DROP	0.2379	0.2379	0.2379
Landweber	0.2713	0.2621	0.2621
Cimmino	0.2686	0.2316	0.2316
CAV	0.2665	0.2157	0.2157
DROP	0.2665	0.2157	0.2157

In second test we give a strategy for picking relaxation parameters. Assume that the linear system (1.1) is consistent. This strategy is based on picking λ_k such that the error $\|x^k - x^*\|$ is minimized in each iteration where x^* is any solution of (1.1). The cases $S = Id$ and $S \neq Id$ were studied in [16] and [18], respectively. Let $r^k = b - Ax^k$. It is easy to show that the following relaxation parameter

$$\lambda_k = \frac{\langle r^k, Mr^k \rangle}{\|A^T M(b - Ax)\|_S^2}$$

minimizes $\|x^k - x^*\|$. Simple calculation show that $\lambda_k \geq 1/\sigma_1^2$. Therefore, to preserve our convergence analysis we suggest following strategy for picking relaxation parameters

$$\lambda_k = min\left\{\frac{\langle r^k, Mr^k \rangle}{\|A^T M(b - Ax)\|_S^2}, \frac{2}{\sigma_1^2}\right\} \quad for \quad k = 1, 2, \ldots. \tag{3.2}$$

In Table 2, we demonstrate the effect of using this strategy.

Table 2: The smallest relative error with noiseless (top) and noisy data (down) using Algorithm 1.1 with relaxation parameters (3.2).

Algorithm	$\mathcal{C} = \mathbb{R}^n$	$\mathcal{C} = \mathbb{R}^n_+$	$\mathcal{C} = [0, 1]^n$
Landweber	0.2005	0.1642	0.1642
Cimmino	0.1902	0.1424	0.1424
CAV	0.1903	0.1425	0.1425
DROP	0.1902	0.1424	0.1424
Landweber	0.2488	0.2022	0.2022
Cimmino	0.2757	0.1975	0.1975
CAV	0.2756	0.1974	0.1974
DROP	0.2757	0.1975	0.1975

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