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GENERALIZED ADDITIVE FUNCTIONAL INEQUALITIES IN BANACH ALGEBRAS

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Dedicated to the 70th Anniversary of S.M.Ulam's Problem for Approximate Homomorphisms

Abstract. Using the Hyers-Ulam-Rassias stability method, we investigate isomorphisms in Banach algebras and derivations on Banach algebras associated with the following generalized additive functional inequality

$$\|af(x) + bf(y) + cf(z)\| \le \|f(\alpha x + \beta y + \gamma z)\|. \tag{0.1}$$

Moreover, we prove the Hyers-Ulam-Rassias stability of homomorphisms in Banach algebras and of derivations on Banach algebras associated with the generalized additive functional inequality (0.1).

1. Introduction and preliminaries

The stability problem of functional equations originated from a question of Ulam [38] concerning the stability of group homomorphisms. Hyers [9] gave a first affirmative partial answer to the question of Ulam for Banach spaces. Th.M. Rassias [28] provided a generalization of Hyers' Theorem which allows the *Cauchy difference to be unbounded.*

Theorem 1.1. (Th.M. Rassias). *Let $f : E \to E'$ be a mapping from a normed vector space E into a Banach space E' subject to the inequality*

$$\|f(x + y) - f(x) - f(y)\| \le \theta(\|x\|^p + \|y\|^p) \tag{1.1}$$

for all $x, y \in E$, where θ and p are positive real numbers with $p < 1$. Then there exists an unique additive mapping $L : E \to E'$ satisfying

$$\|f(x) - L(x)\| \le \frac{2\theta}{2 - 2^p}\|x\|^p$$

for all $x \in E$. Also, if for each $x \in E$ the function $f(tx)$ is continuous in $t \in \mathbb{R}$, then L is $\mathbb{R}$-linear.

Th.M. Rassias [29] during the 27^{th} International Symposium on Functional Equations asked the question whether such a theorem can also be proved for $p \ge 1$. Gajda [5] following the same approach as in Th.M. Rassias [28], gave an affirmative

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solution to this question for $p > 1$. It was shown by Gajda [5], as well as by Th.M. Rassias and Šemrl [35] that one cannot prove a Th.M. Rassias' type theorem when $p = 1$. The counterexamples of Gajda [5], as well as of Th.M. Rassias and Šemrl [35] have stimulated several mathematicians to invent new definitions of *approximately additive* or *approximately linear* mappings, cf. P. Găvruta [6], who among others studied the Hyers-Ulam stability of functional equations. The inequality (1.1) that was introduced for the first time by Th.M. Rassias [28] provided a lot of influence in the development of a generalization of the Hyers-Ulam stability concept. This new concept is known as *Hyers-Ulam-Rassias stability* of functional equations (cf. the books of P. Czerwik [2, 3], D.H. Hyers, G. Isac and Th.M. Rassias [10]).

P. Găvruta [6] provided a further generalization of Th.M. Rassias' Theorem. G. Isac and Th.M. Rassias [13] applied the Hyers–Ulam stability theory to prove fixed point theorems and study some new applications in Nonlinear Analysis. In [11], D.H. Hyers, G. Isac and Th.M. Rassias studied the asymptoticity aspect of Hyers-Ulam stability of mappings.

Beginning around the year 1980, the topic of approximate homomorphisms and their stability theory in the field of functional equations and inequalities was taken up by several mathematicians (cf. D.H. Hyers and Th.M. Rassias [12], Th.M. Rassias [32] and the references therein).

Theorem 1.2. [27] *Let X be a real normed linear space and Y a real complete normed linear space. Assume that $f : X \to Y$ is an approximately additive mapping for which there exist constants $\theta \geq 0$ and $p \in \mathbb{R} - \{1\}$ such that f satisfies inequality*

$$\|f(x+y) - f(x) - f(y)\| \leq \theta \cdot ||x||^{\frac{p}{2}} \cdot ||y||^{\frac{p}{2}}$$

for all $x, y \in X$. Then there exists a unique additive mapping $L : X \to Y$ satisfying

$$\|f(x) - L(x)\| \leq \frac{\theta}{|2^p - 2|} ||x||^p$$

for all $x \in X$. If, in addition, $f : X \to Y$ is a mapping such that the transformation $t \to f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in X$, then L is an $\mathbb{R}$-linear mapping.

Several papers have been published on various generalizations and applications of Hyers-Ulam stability and Hyers-Ulam-Rassias stability to a number of functional equations and mappings (see [16]–[26], [30]–[33], [37]).

Definition 1.3. Let A and B be real Banach algebras.

(i) An $\mathbb{R}$-linear mapping $H : A \to B$ is called a *algebra homomorphism* if $H(xy) = H(x)H(y)$ for all $x, y \in A$.

(ii) An $\mathbb{R}$-linear mapping $\delta : A \to A$ is called a *derivation* if $\delta(xy) = \delta(x)y + x\delta(y)$ for all $x, y \in A$.

Gilányi [7] showed that if f satisfies the functional inequality

$$\|2f(x) + 2f(y) - f(x-y)\| \leq \|f(x+y)\| \tag{1.2}$$

then f satisfies the quadratic functional equation

$$2f(x) + 2f(y) = f(x+y) + f(x-y).$$

See also [36]. Fechner [4] and Gilányi [8] proved the Hyers-Ulam-Rassias stability of the functional inequality (1.2). C. Park et al. [22] investigated the Jordan-von Neumann type Cauchy-Jensen additive mappings and prove their stability, and Cho and Kim [1] proved the Hyers-Ulam-Rassias stability of the Jordan-von Neumann type Cauchy-Jensen additive mappings.

This paper is organized as follows: In Sections 2 and 3, we investigate isomorphisms in Banach algebras and derivations on Banach algebras associated with the generalized additive functional inequality (0.1).

In Sections 4 and 5, we prove the Hyers-Ulam-Rassias stability of homomorphisms in Banach algebras and of derivations on Banach algebras associated with the generalized additive functional inequality (0.1).

Throughout this paper, we assume that A, B are real Banach algebras with norms $\|\cdot\|_A$ and $\|\cdot\|_B$, respectively, and that $a, b, c, \alpha, \beta, \gamma$ are nonzero real numbers.

2. Isomorphisms in Banach algebras

Consider a mapping $f : A \to B$ satisfying the following functional inequality

$$\|af(x) + bf(y) + cf(z)\|_B \leq \|f(\alpha x + \beta y + \gamma z)\|_B \tag{2.1}$$

for all $x, y, z \in A$.

In this section, we investigate isomorphisms in Banach algebras associated with the functional inequality (2.1).

Theorem 2.1. *Let $p \neq 1$ and θ be nonnegative real numbers, and let $f : A \to B$ be a nonzero bijective mapping satisfying* (2.1) *and $\lim_{t\in\mathbb{R}, t\to 0} f(tx) = 0$ for all $x \in A$ such that*

$$\|f(xy) - f(x)f(y)\|_B \leq \theta(\|x\|_A^{2p} + \|y\|_A^{2p}) \tag{2.2}$$

for all $x, y \in A$. Then the bijective mapping $f : A \to B$ is an isomorphism in Banach algebras.

Proof. By Theorem 2.7 of [15], the mapping $f : A \to B$ is $\mathbb{R}$-linear.

(i) Assume that $p < 1$. By (2.2),

$$\begin{aligned}\|f(xy) - f(x)f(y)\|_B &= \lim_{n\to\infty} \frac{1}{4^n}\|f(4^n xy) - f(2^n x)f(2^n y)\|_B \\ &\leq \lim_{n\to\infty} \frac{4^{np}}{4^n}\theta(\|x\|_A^{2p} + \|y\|_A^{2p}) = 0\end{aligned}$$

for all $x, y \in A$. So

$$f(xy) = f(x)f(y)$$

for all $x, y \in A$.

(ii) Assume that $p > 1$. By a similar method to the proof of the case (i), one can prove that the mapping $f : A \to B$ satisfies

$$f(xy) = f(x)f(y)$$

for all $x, y \in A$.

Therefore, the bijective mapping $f : A \to B$ is an isomorphism in Banach algebras, as desired. ∎

Theorem 2.2. *Let $p \neq 1$ and θ be nonnegative real numbers, and let $f : A \to B$ be a nonzero bijective mapping satisfying* (2.1) *and* $\lim_{t\in\mathbb{R},t\to 0} f(tx) = 0$ *for all $x \in A$ such that*

$$\|f(xy) - f(x)f(y)\|_B \leq \theta \cdot \|x\|_A^p \cdot \|y\|_A^p \tag{2.3}$$

for all $x, y \in A$. Then the bijective mapping $f : A \to B$ is an isomorphism in Banach algebras.

Proof. By Theorem 2.7 of [15], the mapping $f : A \to B$ is $\mathbb{R}$-linear.

(i) Assume that $p < 1$. By (2.3),

$$\begin{aligned} \|f(xy) - f(x)f(y)\|_B &= \lim_{n\to\infty} \frac{1}{4^n}\|f(4^n xy) - f(2^n x)f(2^n y)\|_B \\ &\leq \lim_{n\to\infty} \frac{4^{np}}{4^n}\theta \cdot \|x\|_A^p \cdot \|y\|_A^p = 0 \end{aligned}$$

for all $x, y \in A$. So

$$f(xy) = f(x)f(y)$$

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(ii) Assume that $p > 1$. By a similar method to the proof of the case (i), one can prove that the mapping $f : A \to B$ satisfies

$$f(xy) = f(x)f(y)$$

for all $x, y \in A$.

Therefore, the bijective mapping $f : A \to B$ is an isomorphism in Banach algebras, as desired. □

3. Derivations on Banach algebras

In this section, we investigate derivations on Banach algebras associated with the functional inequality (2.1).

Theorem 3.1. *Let $p \neq 1$ and θ be nonnegative real numbers, and let $f : A \to A$ be a nonzero mapping satisfying* (2.1) *and* $\lim_{t\in\mathbb{R},t\to 0} f(tx) = 0$ *for all $x \in A$ such that*

$$\|f(xy) - f(x)y - xf(y)\|_A \leq \theta(\|x\|_A^{2p} + \|y\|_A^{2p}) \tag{3.1}$$

for all $x, y \in A$. Then the mapping $f : A \to A$ is a derivation on a Banach algebra.

Proof. By Theorem 2.7 of [15], the mapping $f : A \to A$ is $\mathbb{R}$-linear.

(i) Assume that $p < 1$. By (3.1),

$$\begin{aligned} \|f(xy) - f(x)y - xf(y)\|_A &= \lim_{n\to\infty} \frac{1}{4^n}\|f(4^n xy) - f(2^n x)\cdot 2^n y - 2^n x \cdot f(2^n y)\|_A \\ &\leq \lim_{n\to\infty} \frac{4^{np}}{4^n}\theta(\|x\|_A^{2p} + \|y\|_A^{2p}) = 0 \end{aligned}$$

for all $x, y \in A$. So

$$f(xy) = f(x)y + xf(y)$$

for all $x, y \in A$.

(ii) Assume that $p > 1$. By a similar method to the proof of the case (i), one can prove that the mapping $f : A \to A$ satisfies

$$f(xy) = f(x)y + xf(y)$$

for all $x, y \in A$.

Therefore, the mapping $f : A \to A$ is a derivation on a Banach algebra. □

Theorem 3.2. *Let $p \neq 1$ and θ be nonnegative real numbers, and let $f : A \to A$ be a nonzero mapping satisfying (2.1) and $\lim_{t\in\mathbb{R},t\to 0} f(tx) = 0$ for all $x \in A$ such that*

$$\|f(xy) - f(x)y - xf(y)\|_A \leq \theta \cdot \|x\|_A^p \cdot \|y\|_A^p \tag{3.2}$$

for all $x, y \in A$. Then the mapping $f : A \to A$ is a derivation on a Banach algebra.

Proof. By Theorem 2.7 of [15], the mapping $f : A \to A$ is $\mathbb{R}$-linear.

(i) Assume that $p < 1$. By (3.2),

$$\begin{aligned}\|f(xy) - f(x)y - xf(y)\|_A &= \lim_{n\to\infty} \frac{1}{4^n}\|f(4^n xy) - f(2^n x) \cdot 2^n y - 2^n x \cdot f(2^n y)\|_A \\ &\leq \lim_{n\to\infty} \frac{4^{np}}{4^n}\theta \cdot \|x\|_A^p \cdot \|y\|_A^p = 0\end{aligned}$$

for all $x, y \in A$. So

$$f(xy) = f(x)y + xf(y)$$

for all $x, y \in A$.

(ii) Assume that $p > 1$. By a similar method to the proof of the case (i), one can prove that the mapping $f : A \to A$ satisfies

$$f(xy) = f(x)y + xf(y)$$

for all $x, y \in A$.

Therefore, the mapping $f : A \to A$ is a derivation on a Banach algebra. □

4. Stability of homomorphisms in Banach algebras

In [15], the authors introduced α-additivity of a mapping.

Definition 4.1. For a mapping $f : A \to B$, we say that f is α-additive if

$$f(x + \alpha y) = f(x) + \alpha f(y)$$

for all $x, y \in A$.

In this section, we prove the Hyers-Ulam-Rassias stability of homomorphisms in Banach algebras.

Theorem 4.2. *Let $\xi = -\frac{\alpha}{\beta}$ and $f : A \to B$ a mapping satisfying $\lim_{t\in\mathbb{R},t\to 0} f(tx) = 0$ for all $x \in A$. When $|\alpha| > |\beta|$ and $0 < p < 1$, or $|\alpha| < |\beta|$ and $p > 1$, if there exists a $\theta \geq 0$ satisfying (2.2) such that*

$$\begin{aligned}\|\alpha f(x) + \beta f(y) + \gamma f(z)\|_B &\leq \|f(\alpha x + \beta y + \gamma z)\|_B \\ &+ \theta(\|x\|_A^p + \|y\|_A^p + \|z\|_A^p)\end{aligned} \tag{4.1}$$

for all $x, y, z \in A$, then there exists a unique algebra homomorphism and ξ-additive mapping $H : A \to B$ satisfying

$$\|f(x) - H(x)\|_B \leq \frac{\theta(|\alpha|^p + |\beta|^p)}{|\alpha||\beta|(|\beta|^{p-1} - |\alpha|^{p-1})}\|x\|_A^p \tag{4.2}$$

for all $x \in A$.

Proof. By Theorem 3.6 and Corollary 3.7 of [15], there exists a unique $\mathbb{R}$-linear and ξ-additive mapping $H : A \to B$ satisfying (4.2). The mapping $H : A \to B$ is defined by $H(x) := \lim_{n\to\infty} \frac{f(\xi^n x)}{\xi^n}$ for all $x \in A$.

By (2.2),

$$\begin{aligned} \|H(xy) - H(x)H(y)\|_B &= \lim_{n\to\infty} \frac{1}{\xi^{2n}} \|f(\xi^{2n}xy) - f(\xi^n x)f(\xi^n y)\|_B \\ &\le \lim_{n\to\infty} \frac{\xi^{2np}}{\xi^{2n}} \theta(\|x\|_A^{2p} + \|y\|_A^{2p}) = 0 \end{aligned}$$

for all $x, y \in A$. So

$$H(xy) = H(x)H(y)$$

for all $x, y \in A$.

Therefore, the mapping $H : A \to B$ is an algebra homomorphism and ξ-additive mapping, as desired. □

Now we establish another stability of generalized additive functional inequalities.

Theorem 4.3. *Let $\xi = -\frac{\alpha}{\beta}$ and $f : A \to B$ a mapping satisfying $\lim_{t\in\mathbb{R}, t\to 0} f(tx) = 0$ for all $x \in A$. When $|\alpha| > |\beta|$ and $p > 1$, or $|\alpha| < |\beta|$ and $0 < p < 1$, if there exists a $\theta \ge 0$ satisfying (2.2) and (4.1), then there exists a unique algebra homomorphism and ξ-additive mapping $H : A \to B$ satisfying*

$$\|f(x) - H(x)\|_B \le \frac{\theta(|\alpha|^p + |\beta|^p)}{|\alpha||\beta|(|\alpha|^{p-1} - |\beta|^{p-1})} \|x\|_A^p \tag{4.3}$$

for all $x \in A$.

Proof. By Theorem 3.9 and Corollary 3.10 of [15], there exists a unique $\mathbb{R}$-linear and ξ-additive mapping $H : A \to B$ satisfying (4.3). The mapping $H : A \to B$ is defined by $H(x) := \lim_{n\to\infty} \xi^n f(\frac{x}{\xi^n})$ for all $x \in A$.

By (2.2),

$$\begin{aligned} \|H(xy) - H(x)H(y)\|_B &= \lim_{n\to\infty} \xi^{2n} \|f(\frac{xy}{\xi^{2n}}) - f(\frac{x}{\xi^n})f(\frac{y}{\xi^n})\|_B \\ &\le \lim_{n\to\infty} \frac{\xi^{2n}}{\xi^{2np}} \theta(\|x\|_A^{2p} + \|y\|_A^{2p}) = 0 \end{aligned}$$

for all $x, y \in A$. So

$$H(xy) = H(x)H(y)$$

for all $x, y \in A$.

Therefore, the mapping $H : A \to B$ is an algebra homomorphism and ξ-additive mapping, as desired. □

5. Stability of derivations on Banach algebras

In this section, we prove the Hyers-Ulam-Rassias stability of derivations on Banach algebras.

Theorem 5.1. *Let* $\xi = -\frac{\alpha}{\beta}$ *and* $f : A \to A$ *a mapping satisfying* $\lim_{t\in\mathbb{R},t\to 0} f(tx) = 0$ *for all* $x \in A$. *When* $|\alpha| > |\beta|$ *and* $0 < p < 1$, *or* $|\alpha| < |\beta|$ *and* $p > 1$, *if there exists a* $\theta \geq 0$ *satisfying* (3.1) *such that*

$$\begin{aligned}\|\alpha f(x) + \beta f(y) + \gamma f(z)\|_A &\leq \|f(\alpha x + \beta y + \gamma z)\|_A \\ &+ \theta(\|x\|_A^p + \|y\|_A^p + \|z\|_A^p) \end{aligned} \tag{5.1}$$

for all $x, y, z \in A$, *then there exists a unique derivation and* ξ*-additive mapping* $D : A \to A$ *satisfying*

$$\|f(x) - D(x)\|_A \leq \frac{\theta(|\alpha|^p + |\beta|^p)}{|\alpha||\beta|(|\beta|^{p-1} - |\alpha|^{p-1})}\|x\|_A^p \tag{5.2}$$

for all $x \in A$.

Proof. By Theorem 3.6 and Corollary 3.7 of [15], there exists a unique $\mathbb{R}$-linear and ξ-additive mapping $D : A \to A$ satisfying (5.2). The mapping $D : A \to A$ is defined by $D(x) := \lim_{n\to\infty} \frac{f(\xi^n x)}{\xi^n}$ for all $x \in A$.

By (3.1),

$$\begin{aligned}\|D(xy) - D(x)y - xD(y)\|_A &= \lim_{n\to\infty} \frac{1}{\xi^{2n}}\|f(\xi^{2n}xy) - f(\xi^n x)\cdot \xi^n y - \xi^n x \cdot f(\xi^n y)\|_A \\ &\leq \lim_{n\to\infty} \frac{\xi^{2np}}{\xi^{2n}}\theta(\|x\|_A^{2p} + \|y\|_A^{2p}) = 0\end{aligned}$$

for all $x, y \in A$. So

$$D(xy) = D(x)y + xD(y)$$

for all $x, y \in A$.

Therefore, the mapping $D : A \to A$ is a derivation and ξ-additive mapping, as desired. □

Theorem 5.2. *Let* $\xi = -\frac{\alpha}{\beta}$ *and* $f : A \to A$ *a mapping satisfying* $\lim_{t\in\mathbb{R},t\to 0} f(tx) = 0$ *for all* $x \in A$. *When* $|\alpha| > |\beta|$ *and* $p > 1$, *or* $|\alpha| < |\beta|$ *and* $0 < p < 1$, *if there exists a* $\theta \geq 0$ *satisfying* (3.1) *and* (5.1), *then there exists a unique derivation and* ξ*-additive mapping* $D : A \to A$ *satisfying*

$$\|f(x) - D(x)\|_A \leq \frac{\theta(|\alpha|^p + |\beta|^p)}{|\alpha||\beta|(|\alpha|^{p-1} - |\beta|^{p-1})}\|x\|_A^p \tag{5.3}$$

for all $x \in A$.

Proof. By Theorem 3.9 and Corollary 3.10 of [15], there exists a unique $\mathbb{R}$-linear and ξ-additive mapping $D : A \to A$ satisfying (5.3). The mapping $D : A \to A$ is defined by $D(x) := \lim_{n\to\infty} \xi^n f(\frac{x}{\xi^n})$ for all $x \in A$.

By (3.1),

$$\begin{aligned}\|D(xy) - D(x)y - xD(y)\|_A &= \lim_{n\to\infty} \xi^{2n}\|f(\frac{xy}{\xi^{2n}}) - f(\frac{x}{\xi^n})\cdot\frac{y}{\xi^n} - \frac{x}{\xi^n}\cdot f(\frac{y}{\xi^n})\|_A \\ &\leq \lim_{n\to\infty} \frac{\xi^{2n}}{\xi^{2np}}\theta(\|x\|_A^{2p} + \|y\|_A^{2p}) = 0\end{aligned}$$

for all $x, y \in A$. So

$$D(xy) = D(x)y + xD(y)$$

for all $x, y \in A$.

Therefore, the mapping $D : A \to A$ is a derivation and ξ-additive mapping, as desired. □

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