Int. J. Nonlinear Anal. Appl. 17 (2026) 1, 143–152
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/ijnaa.2024.34418.5142

Introducing a model for selecting a lower-cost optimal portfolio using smart beta

Amir Nasrollahi, Behzad Parvizi*, Ataollah Mohammadi Molgharani

Faculty of Financial Management and Accounting, Sanandaj Branch, Islamic Azad University, Sanandaj, Iran

(Communicated by Sirous Moradi)

Abstract

In recent years, selecting optimal portfolios with higher returns and lower costs compared to a benchmark portfolio has drawn the attention of researchers to examine smart beta in choosing the investment portfolio. Due to the significance of the issue, the current research aims to provide a model to select the optimal portfolio at a lower cost using smart beta and compare its performance with the benchmark portfolio in the Tehran Stock Exchange. The present applied research is quantitative regarding its data type. The statistical population includes all active companies on the Tehran Stock Exchange. Regarding the many companies accepted on the stock exchange, the asset records were examined from 2014 to 2019; a statistical sample of 148 companies with 15 shares was obtained from systematic elimination sampling. To optimize the portfolio and get the model weights based on the defined models, the genetic algorithm was used, and to solve the algorithm, MATLAB and SPSS22 software were used. The results obtained from the algorithm iteration and the optimization of the objective function are equal to 1.23. In consecutive iterations for various rates of mutation and intersection, the obtained values of the objective function are very close to each other, which indicates that the genetic algorithm is suitable to solve this model and that no scattered or outlier solutions exist. The smart beta model provides a better risk-return tradeoff and better performance than the benchmark portfolio in the Iranian stock market.

Keywords: optimal portfolio, smart beta, lower cost, genetic algorithm
2020 MSC: 90B50, 68W50, 91G30

1 Introduction

Assessing the existing financial markets to build an optimal portfolio of assets is a measure taken by most real and legal investors according to the characteristics of the stock market, such as the market value of the company, the market-to-book ratio (price-to-book ratio) or the previous year return about the expected return of the shares, the variance and covariance of a single stock with other stocks [11]. Nevertheless, it is challenging to incorporate all of the above-mentioned issues into portfolio management. Most mutual funds, private investors, and investment funds measure the performance of the assets held for investment based on structural indicators based on the market value-weighted method and have the benefits of using readjustment, easy implementation, low asset turnover, and the possibility of building a profitable and large investment portfolio [12].

*Corresponding author
Email addresses: nasrolahi1986@gmail.com (Amir Nasrollahi), parvizibehzadaccounting@yahoo.com (Behzad Parvizi), attta.mm68@gmail.com (Ataollah Mohammadi Molgharani)

Received: May 2024 *Accepted:* July 2024

Although implementing this method is easy, its portfolio does not lead to large-cap stocks and brings about inefficiency since the stock selection criteria and weights of stocks are based on price, not mean-variance optimization [6]. Markowitz's mean-variance approach [9], as the creator of modern portfolio management theory (MPT), requires modelling the expected return, variance, and covariance for all available stocks as a function of their characteristics.

This is an incomprehensible economic problem in the presence of a large number of parameters involved and requires positive covariance between the stocks in the portfolio, the results of which are often intermingled with a lot of mistakes and instability. That is why portfolio optimization with the traditional Markowitz approach is less commonly presented in the real world. For this reason, the researchers always sought to optimize the portfolio based on the asset characteristics that would meet the expectations of investors and bring those benefits [1].

Kordian [8] claims that when risk measures are used theoretically in portfolio optimization, they bring similar results. In spite of the fact that, in practical terms, the comparison between different risk measures gives particularly diverse results, these discrepancies can be the result of other components, such as fat-tailed return distribution or asymmetry within the data, which lead to changes in the results of portfolio optimization in practice. Thus, the premise of evaluating the performance of distinctive risk measures in response to questions concerning the selection of appropriate measures for diverse investor strategies, the relative benefits of utilizing each risk measure compared to other measures, and the impact of other components such as the size and investment horizon on the efficiency of optimization models, is essential [5].

Recently, investors have selected alternative weighting strategies called smart beta strategies to allocate assets in their stock portfolios. This strategy encompasses the systematic selection and allocation of assets to a portfolio based on the particular characteristics of stocks [11]. The research unravels that smart beta is a strategy in the two forms of passive and active portfolio management.

Passive portfolio management is a strategy in which investments are made with the objective of long-term gain with negligible scrutiny. In active portfolio management strategies, investments are made, and portfolios are continuously monitored using data and research methods to exploit profitable opportunities [10]. The financial literature records the superior performance of smart beta compared to the normal portfolio. Recent inquiries have also provided theoretical and empirical evidence that supports the superiority of smart beta over weighted market value [2]. Thus, providing a model to select a lower-cost optimal portfolio using smart beta is one of the most significant points of the present research. The other significant points of the research are as follows:

Managing existing or alternative (other) risks

Smart beta, as a new approach in the field of optimal portfolio selection, brings better market return along with the advantages of traditional strategies associated with other portfolio selection models, including broad market exposures, rule-based implementation, transparency, high capacity, and low cost. Using the advantages of active and passive strategies, smart beta can increase efficiency and productivity and reduce the cost of portfolio selection. Anchoring factors other than price, these strategies seek to exploit market inefficiencies. In other words, smart beta strategies break the relationship between price and portfolio weight to provide better returns than the market [11].

Using novel and more precise models to select a portfolio can increase investment return, meet the needs of rapid and dynamic changes in the external environment, and show a suitable response [1]. As such, the model presented in the current research increases the productivity of stock portfolios and reduces the transaction cost and risk to avoid costly portfolios.

2 Literature review

2.1 Theoretical background

Due to the research purpose, an overview of smart beta, a portfolio with lower cost, an optimal portfolio, and a research model are explored.

Smart Beta: This is an investment strategy that operates on a multi-factor investment model, which refers to a model that considers multiple factors or characteristics of investment factors systematically and transparently. In the world of indexing, smart beta is classified into two key categories: fundamental and risk-oriented (A risk-oriented strategy has been used in the current research). A risk-oriented strategy has been used in the current research. The fundamental method, like the Fama and French [4] model, considers the weight of a company in the portfolio based on characteristics such as sales, cash flow, and dividends. However, the risk-oriented method takes advantage of risk anomaly. Equal weighting is the simplest smart beta weighting strategy. An equal weighting strategy assigns equal weight to assets in every investment in the investment universe, regardless of the stock size or price. On the other hand, the goal of the portfolio strategy with the low-risk anomaly is to build a portfolio of stocks that can reduce the overall portfolio risk [10]. The risk anomaly index is calculated according to equation (2.1).

$$w_i = \frac{\frac{B_i}{\sum_{i=1}^{N} B_i} + \frac{S_i}{\sum_{i=1}^{N} S_i} + \frac{C_i}{\sum_{i=1}^{N} C_i} + \frac{DY_i}{\sum_{i=1}^{N} DY_i}}{4} \times Cap_i w_i = \frac{\frac{B_i}{\sum_{i=1}^{N} B_i} + \frac{S_i}{\sum_{i=1}^{N} S_i} + \frac{C_i}{\sum_{i=1}^{N} C_i} + \frac{DY_i}{\sum_{i=1}^{N} DY_i}}{4} \times Cap_i \tag{2.1}$$

in the above model, B is the stock value of the last year, S is the 5-year sales volume, C is the 5-year cash flow, DY is the 5-year dividend, and Cap is the market share of the i^{th} company in the portfolio.

A lower-cost portfolio: In this type of portfolio, the presumption is that the risk tolerance level of the investor is minimal. That is, the investor is assumed to have a lower level of risk aversion than the standard level. In this case, the optimization of the objective function is equivalent to the optimization of the portfolio variance [12]. The lower-cost portfolio model is calculated according to equation (2.2):

$$\min \frac{1}{2}(X^T H X) \tag{2.2}$$

where $X$is the weight vector, and H is the covariance of daily returns. Optimal portfolio and research model: In general terms, an optimal portfolio maximizes return and minimizes risk. In the mean-variance portfolio optimization model introduced by Markowitz [9], the portfolio return is measured by the expected portfolio return, and risk is measured by the return variance [3]. The portfolio optimization of Markowitz's model [9] is defined below. In this model, the stock return is represented by r, the percentage of investment in each share in each period is represented by w, and the total percentage of investment at any time is equal to 1. The objective function is in the form of equation (2.3).

$$\begin{aligned} &\min \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j cov(r_i, r_j) \\ &s.t. \sum_{i=1}^{n} w_i = 1, \quad w_i \geq 0, \quad i = 1, 2, ..., n \end{aligned} \tag{2.3}$$

The purpose of this model is to minimize the portfolio risk at a certain rate of return (RoR). The return of each share is represented by r_i, and the portfolio risk is obtained from:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j cov(r_i, r_j)$$

Therefore, the amount of investment in each asset (investment weight) is based on the amount of return and risk that the asset carries. The more reasonable the ratio of return to risk of an asset, the more weight is assigned to that asset. Taking into account the optimization of the lowest-cost portfolio, the current research model is described in equation (2.4):

$$\begin{aligned} &\min(W^T H W) \\ &\sum_{i=1}^{n} w_i = 1 \\ &w_i \geq 0, \quad i = 1, 2, ..., n \end{aligned} \tag{2.4}$$

where W is the weight vector, and H is the covariance of daily returns. In this model, the weights are defined based on the smart beta risk anomaly index, and the weights are not calculated using linear methods or simulation methods. In other words, the presumption is that the weights obtained from portfolio optimization are not only dependent on asset returns. However, the characteristics of the company whose assets are invested should also be taken into account in obtaining the weights [8].

2.2 Experimental background

Following the studies by Almahdi [2] and Wajid Raza and Ashraf [11], the current research used the following indexes to define the beta index: stock value indicators, cash flow, sales growth, dividend, and market share of the company. Attempting to maximize the welfare of their owners (stockholders) through investment decisions and policies, companies decide on the Dividend Payout Ratio. The higher the dividend ratio, the higher the company's profit allocation to investors. The company's dividend policy becomes a long-term financial strategy option during the decision-making process for investment [7].

The dividend policy that has a high return in the form of dividends and instead of distributing profits in cash, provides a significant amount of money to the stockholders in the form of shares, will be more trustworthy for the stockholders. Fons et al. [6], in a research entitled "A novel dynamic asset allocation system using Feature Saliency Hidden Markov models for smart beta investing", emphasize that the 2008 financial crisis engaged the attention in more transparent, rules-based strategies to build a portfolio. In the meantime, smart beta strategies as an emerging trend were more appropriate compared to the hidden Markov models among investors.

Nazaire et al. [10] emphasize that different sources of return first drive beta-based investment strategies and that both betas and variance properties explain trait-based strategies. They conclude that beta variance is a more effective risk management tool than specific variance. They also emphasize that variance in smart-beta funds is beneficial and less risky with more return. Monte Carlo simulation (A mathematical technique that predicts possible outcomes of an uncertain event) confirms these results.

Ding et al. [3], in another study entitled "High Dimensional Minimum Variance Portfolio Estimation under Statistical Factor Models," suggested a generalized composite estimator of portfolio variance as a statistical factor model and showed that the estimated portfolio has high-risk coherence and relies on the integration of constraints on portfolio weighting.

Extensive simulation and empirical studies on the stocks that make up the S&P100 index show the favourable performance of the proposed estimator compared to the benchmark portfolio. Kohansal Kafshgari et al. [7], in research entitled "Presentation of intelligent Meta-heuristic Hybrid models (ANFIS-MGGP) to predict stock returns with more accuracy and speed than other Meta-heuristic methods," concluded that using the hybrid method (ANFIS-MGGP) has higher accuracy and speed than other innovative models in predicting stock returns because, firstly, the most optimal input variables are selected through the ANFIS technique. Then, the prediction is made using the meta-heuristic MGGP model.

As no research has been conducted to explore smart beta within the country, to have a brief summary regarding the comparison of the current research with other international research, it can be declared that all the studies conducted in this field are effectively included in the present research, and each one is a part of it. The present study has provided a specialized model for choosing the optimal portfolio using smart beta at a lower cost. It has been widely evaluated against the benchmark portfolio in the Iranian stock market. Thus, the current study is the primary reference for smart beta.

3 Research methodology

The current study is quantitative in terms of data type, as it concerns numerical data and applied in terms of research direction because it takes into account a specific time and spatial domain, the change of which also changes the results. Regarding the method of reasoning, the current research is inductive because the results are generalized to the entire statistical population by examining the statistical sample under study.

The statistical population of the current research encompasses all companies listed on the Tehran Stock Exchange that have the following conditions:

1. Companies whose fiscal year ends on March 29 every year.
2. Do not exit the stock market during the research period.
3. Wait to stop the symbol for more than six months.
4. Do not belong to financial and credit institutions, banks, insurance, and investment funds.
5. The required information of the companies should be available in databases.

To choose the appropriate statistical sample, it was assumed that the investor is rational and chooses stocks with the highest risk-return ratio. Therefore, to select the sample companies, once the beta of the companies was calculated based on the capital asset pricing model (CAPM), and once again, the sample companies were selected for investment based on the smart beta.

Using systematic elimination sampling, 148 companies were selected as a statistical sample for six consecutive years (2015-2020). Finally, 15 stocks have been chosen based on the risk-return tradeoff and the portfolio optimization strategies with the highest level of performance.

4 Research results

Feedback test (validity check of calculated risk)

Table 1 presents the results of the feedback test and stock portfolio risk in the studied period.

The hypotheses are tested as follows.

H_0: The calculated risk (β) using stock returns is correct.

H_1: The calculated risk (β) using stock returns is incorrect.

Table 1: Feedback test results

Optimization model	The amount of variance calculated for e	Feedback test result
Smart beta	0.953150	Hypothesis H_0 is accepted

According to Table 1, the variance values are calculated for each portfolio, and if these variances are smaller than 2, the null hypothesis is confirmed. The calculations of the stock portfolio confirm the null hypothesis; that is, the risk measured using the stock returns is correct and stable over time (the tests are valid).

4.1 Checking the stability of algorithm solutions

One essential test for optimizing the stock portfolio is checking the algorithm's stability—that is, whether a unique and identical optimal solution has been obtained by algorithm iteration. According to the results presented in Table 2, the algorithms have been iterated thousands of times in each implementation, and there is a series of errors each time. There is convergence if the errors are close to each other in each series. The results indicate an insignificant difference between the solutions obtained by repeating the algorithm many times. The variance of the algorithm's solutions in 20 repetitions is close to zero (0.0000046238).

Table 2: Checking the stability of algorithm solutions (iteration 20 times)

Algorithm implementation number	Optimal value of the objective function	Algorithm implementation number	Optimal value of the objective function
1	0.11023	11	0.115426
2	0.11032	12	0.10237
3	0.10128	13	0.11638
4	0.09852	14	0.11203
5	0.10230	15	0.09986
6	0.11452	16	0.11653
7	0.11411	17	0.11202
8	0.09329	18	0.09451
9	0.11000	19	0.10987
10	0.09632	20	0.11020
Variance: 0.0000046238			

4.2 Stock portfolio optimization

The present research sample includes fifteen high-yield dividend stocks of the Tehran Stock Exchange. The stocks are diversified in the sense that the portfolio's components consist of different industries. Besides, they have been active in the Iran Stock Exchange since 2010. In the current study, researchers use return, risk, and financial indicators data to determine the selected weights of different assets.

4.3 Portfolio optimization using smart beta

To optimize the stock portfolio using smart beta according to table 3, in this model, the total selected percentage is equal to 1 at any time.

Table 3: Uncertain rate of return of the examined shares

Share	Average return	SD of returns	Value at risk (VAR)	Utility value
1	0.103	0.014	0.0036	6.173E-05
2	0.223	0.030	0.0061	1.119E-02
3	0.049	0.003	0.0000	1.678E-09
4	1.869	0.231	0.0008	5.856E-01
5	0.363	0.040	1.4293	6.347E-02
6	0.413	0.024	0.0020	8.905E-02
7	0.173	0.022	0.0053	2.958E-03
8	0.289	0.032	0.0372	3.155E-02
9	0.233	0.020	0.0032	1.358E-02
10	0.376	0.039	0.0378	6.993E-02
11	0.144	0.018	0.104	9.504E-04
12	0.076	0.005	0.0026	1.887E-06
13	0.119	0.017	0.0158	2.241E-04
14	0.136	0.009	0.0021	6.369E-04
15	0.093	0.011	0.0042	2.097E-05

To calculate the weight using the smart beta method, sales volume, stock value, stock exchange, cash, and dividends are also involved, as well as the attractive characteristics of return and risk. Thus, examining how the variables are closely related might be better. Dependence usually denotes any statistical relationship between two variables or data sets in statistics. In another definition, correlation is any class of statistical relationships dealing with dependence. Correlations are functional because they can predict the relationship to be used in practice. The results of the correlation test are presented in Table 4.

Table 4: Correlation matrix between the returns of the examined shares

	Sales	Cash flow from operating activities (CFO)	market-to-book ratio	Dividends	Fund	Returns
Sales	1	0.595	-0.067	0.610	-0.102	0.106
Cash flow from operating activities (CFO)		1	0.144	0.484	-0.234	0.094
market-to-book ratio			1	0.069	-0.351	-0.066
Dividends				1	-0.440	0.019
Fund					1	-0.173
Returns						1

The results indicate that the increase in the company's sales brings about an increase in cash flows. Besides, the companies with more sales and cash flows have more dividends, and companies with a higher share in the stock market exchange encounter a decrease in cash flows, company value (corporate value or core value), and dividends. There is also no significant relationship between the efficiency level and the sales variables, cash flows, value, dividend, and market share. Thus, companies with a sound financial position do not necessarily have high-yield dividends.

4.4 Determining the coefficients of the market index model related to company stocks

Table 5 demonstrates the expected return values for the market index and each state's probability of occurrence. It shows that the estimated returns of the stock market participants from the market index till the end of the year were between two percent to positive 59 percent and that the 28 percent return of the market index has the highest probability of occurrence, i.e., 0.148 percent. Once we've determined the discrete distribution of the estimated return of the market index, the next step is to calculate the cumulative distribution of the market index's return. This process is crucial in understanding the market index's return distribution, and the resulting values are displayed in Table 6.

Table 5: Values of market return and its probability of occurrence

Market index	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15
Returns	-0.02	-0.12	-0.03	0.025	0.84	0.143	0.207	0.281	0.358	0.434	0.498	0.59	0.414	0.193	0.189
Probability	0.68	0.142	0.23	0.313	0.394	0.486	0.591	0.733	0.836	0.903	0.951	1	0.648	0.442	0.596

The data in Table 7 can be used to estimate the coefficients of the market model for each stock using the linear regression and the least squares methods. The results are provided in Table 9.

Table 6: Values of market return and its cumulative probability

Market index	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15
Returns	-0.02	-0.12	-0.03	0.025	0.84	0.143	0.207	0.281	0.358	0.434	0.498	0.59	0.414	0.193	0.189
Probability	0.066	0.075	0.087	0.083	0.080	0.091	0.105	0.148	0.096	0.066	0.048	0.049	0.069	0.071	0.074

Table 7: Coefficient values of the single-factor model

Stock	intercept elevation A	gradient of line B	share	intercept elevation A	gradient of line B
1	0.027	1.3427	9	-0.0377	1.5383
2	-0.118	1.3088	10	-0.0212	1.1654
3	-0.0035	0.7912	11	-0.0231	1.5757
4	-0.277	1.0118	12	-0.0211	1.5755
5	-0.0257	1.2476	13	-0.0218	1.4212
6	-0.036	1.2248	14	-0.02247	1.5971
7	-0.0323	1.1498	15	-0.02640	1.5638
8	-0.386	1.1088			

4.5 Calculating the parameters of the market index return rate distribution

To calculate the market index return rate distribution parameters, the F values are in Table 7 and the parameters ($\hat{e}$ and $\bar{\sigma}$) must be obtained. Therefore, firstly, the probability of occurrence given by financial market experts should be converted into a cumulative probability (The obtained values are in Table 8). As we have the cumulative distribution of F, the conditions are:

$$(F_1, a_1), (F_2, a_2), ..., (F_p, a_p), \quad F_1 \leq F_2 \leq ... \leq F_p, \quad 0 \leq a_1 \leq a_2 \leq ... \leq a_p \leq 1 \tag{4.1}$$

it is all true. Now, we can calculate the parameters of the non-deterministic normal distribution of market returns.

$$\hat{e} = 0.11, \quad \bar{\sigma} = 0.02. \tag{4.2}$$

4.6 Solving the model using genetic algorithm

After determining all the values of the model parameters, i.e., market model coefficients for each stock and parameters of the non-deterministic normal distribution of the market index, we can enter the model step. The current study used meta-heuristic methods, specifically the genetic algorithm method, to solve the desired model. Building upon the parameters derived from the previous step, the model requires the investor's input to be fully resolved. These parameters, such as the expected return level, the maximum acceptable chance of not achieving the desired return, the minimum number of stocks in the optimal portfolio, and the investment limits for each asset, are crucial in shaping the model's outcomes. In this study, the minimum expected return of the portfolio was set at 20%, and the maximum acceptable chance was capped at 2%.

In other words, the chance of obtaining a portfolio with a return of less than 20% can be equal to two percent at most. The ceiling of investment in each asset is equal to 40%, the floor is equal to 10%, and the minimum number of assets in the portfolio is equal to 5. After adding the parameters to the research model and solving it using the genetic algorithm, the optimal solution of the model, which is the same as the optimal portfolio, is obtained, as shown in Table 8.

Table 8: Optimal coefficients of investment in stocks

Shotoran	Haseena	Vbsader	Vtejarat	Sheeran
X_2	X_3	X_4	X_5	X_6
0.0869	0.3358	0.3207	0.0143	0.2423

The optimal efficiency corresponding to the above solution (the same objective function value) equals 36.7%. Ultimately, we will compare the results obtained from the designed model with the random portfolio. To compare the optimal portfolio formed by current research, which is given in Table 8, with the random portfolio, we have created ten random portfolios in the MATLAB environment, presented in Table 9. It should be noted that given the random portfolio values in the objective function, their yield was calculated, and its results are presented in the last column of Table 9. The average return of the random portfolio is 0.02336 or 2.33%. Thus, it is unravelled that the model and the program written to solve the model have the necessary efficiency because the optimal portfolio it provides leads to a 36.7% return.

Table 9: Optimal portfolio based on smart beta

Stock	Risk anomaly index	Standard weight	Stock	Risk anomaly index	Standard weight
1	0.0002	0.0038	9	0.0038	0.0810
2	0.0010	0.0211	10	0.0006	0.0116
3	0.0322	0.6786	11	0.0003	0.0062
4	0.0006	0.0134	12	0.0009	0.0179
5	0.0002	0.0049	13	0.0001	0.0026
6	0.0012	0.0243	14	0.0042	0.0890
7	0.0010	0.0217	15	0.0006	0.0124
8	0.0005	0.0114			

$$N \geq -2^{k-1} \ln(\alpha) \left(\sigma_{bb} \sqrt{\frac{\pi(m-1)}{d}} \right). \tag{4.3}$$

To solve the genetic algorithm, first, the model parameters encompassing the number of population members, algorithm iteration frequency, mutation probability, and combination probability must be set.

The number of the initial population is obtained from equation (4.3), where N is the number of the initial population, k is the order of the constructed boxes (between 1 to 5), α is the failure probability (less than 5%), $\sigma_{bb}\sqrt{\frac{\pi(m-1)}{d}}$ is the standard error of the quality of the solutions for a random population, and d is the difference between the quality of the first and the second solution. The combination probability, mutation probability, and algorithm iteration frequency values are respectively obtained from Equation (4.4), where L is the number of chromosomes.

$$P_c \leq \frac{S-1}{S}$$
$$P_m \sim \frac{1}{N} \tag{4.4}$$

and

$$t \sim 2L$$

To adjust the parameters, a population with a volume of 100 is generated three consecutive times, the values of the initial parameters are calculated, and then the genetic algorithm is implemented with the generated parameters. The results obtained from solving the model using the genetic algorithm and weight calculation are presented in Table 10.

Table 10: Optimal portfolio in 4 different iterations

Stock	Investment percentage	Investment percentage	Investment percentage	Investment percentage	Total in four iterations
1	0.125	0.009	0.014	0.001	0.149
2	0.016	0.054	0.066	0.005	0.141
3	0.127	0.161	0.102	0.305	0.694
4	0.985	0.995	0.993	0.998	0.972
5	0.893	0.949	0.029	0.027	1.898
6	0.969	0.989	0.957	0.985	3.900
7	0.104	0.093	0.079	0.000	0.277
8	0.092	0.031	0.045	0.011	0.178
9	0.061	0.025	0.078	0.026	0.190
10	0.937	0.081	0.978	0.987	2.983
11	0.051	0.140	0.010	0.022	0.223
12	0.215	0.095	0.059	0.084	0.453
13	0.054	0.060	0.084	0.006	0.205
14	0.050	0.070	0.070	0.045	0.235
15	0.045	0.102	0.186	0.020	0.353

According to the results, the most selected stocks are for stocks 4 and 6, and the least selected stocks are for stocks 1 and 2. The results of algorithm iteration and optimizing the objective function are presented in Figure 1. The value of the objective function in this case is equal to 1.23. In consecutive iterations for different rates of mutation and intersection, the values of the obtained objective function are very close.

The obtained values indicate that the genetic algorithm is suitable for solving this model and that scattered and outlier solutions are absent.

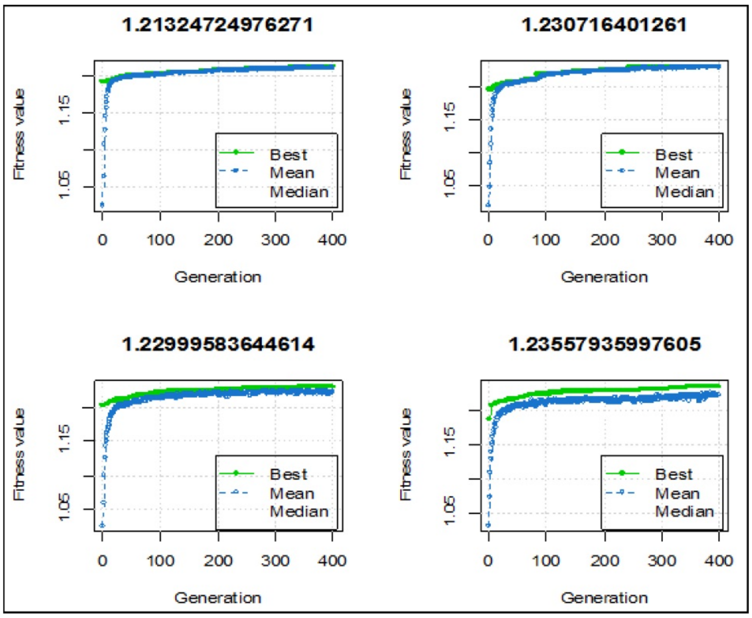

Figure 1: Algorithm iteration to get the optimal objective function

5 Discussion and conclusion

One of the most crucial and complicated issues in the field of investment is selecting the optimal portfolio. The investor encounters many different and diverse options. In other words, selecting the outstanding stock that deserves to be chosen and included in the investment portfolio and how to allocate capital between the bonds, is a complex issue. Theoretically, minimizing risk while keeping returns constant can make portfolio selection a solvable issue, according to the Markowitz model. However, one criticism of Markowitz's model is that he only considers the two criteria of risk and return and does not consider other criteria. That is why researchers have proposed other methods to measure portfolio investment risk. However, the question of which one of these strategies is outstanding remains. Many investors hold that smart beta strategies consider the best of both worlds and provide another option that increases portfolio-building opportunities. Smart beta strategies are used to select value by systematically selecting, distributing, and balancing portfolio resources based on other factors or characteristics of the market capitalization. According to the smart beta approach, considering companies' financial indicators instead of focusing only on the risk-return tradeoff can be effective in getting an optimal portfolio.

In the current study, an optimal portfolio model based on a genetic algorithm was used to solve the introduced model, and finally, a numerical example was provided to show the efficiency of the suggested methods. The analyzed information included the stock returns of 15 active companies in the Tehran Stock Exchange with the highest level of performance. Using the suggested model of smart beta, the portfolio was optimized. The results indicate that the proposed model has a superior potential in driving profitability and trading decisions. The suggested model can be used as an auxiliary and even a primary tool in decision-making. The present research results indicate that this model can bring about significant profitability from the stock exchange and reduce risk in performance. The proposed model is a reality-based and flexible model in which several different stocks and several limitations, preferences, and experiences of decision-makers can be considered. Therefore, this paper can be considered the primary reference for smart beta.

Limitations are an integral part of any research because they provide the context for future and novel research, and this research is no exception. The most important limitation of this research is the need for a clear and transparent model in the field of smart beta. However, the American company BlackRock claims to publish a leading and flexible indicator for calculating smart beta based on all desired factors shortly, whose correct use will bring about a better performance than the weighted market value index in all situations. Reviewing several international articles in the field reveals that there needs to be specific modelling for smart beta analysis, and that they have resorted to the most innovative analyses. The present research on the effectiveness of the smart beta model denotes the following suggestions: According to the smart beta approach, financial indicators of companies compared to the risk-return tradeoff can build a more optimal portfolio. Therefore, investors and financial institutions are recommended to select stocks with the lowest ratio of price to profit, price to book value, price to cash flow, and price to sales.

The obtained results indicate that the expected return (risk) of the smart beta portfolio is more (less) than that of the benchmark portfolio. As such, it is suggested to use the existing statistical models to understand the function of investors' preferences in the stock market and to consider investment according to this function. It is also suggested to consider the rate of return and risk, and the characteristics of companies when investing. However, the rate of return may not be absolute, and in this case, the fuzzy distributions should be considered for portfolio selection.

References

[1] L. Abdollahzadeh, F. Hanifi and M. Fallah, *Provide a model for forecasting the stock price crash risk in Tehran Stock Exchange on the basis of Hutton & Chen models*, Quart. Financ. Eng. Secur. Maneg. **12** (2022), no. 46, 146–170. [In Persian]

[2] S. Almahdi, *Smart beta portfolio optimization*, J. Math. Financ. **5** (2015), no. 2, 2162–2442.

[3] Y. Ding, Y. Li, and X. Zheng, *High dimensional minimum variance portfolio estimation under statistical factor models*, J. Economet. **222** (2020), no. 1, 502–515.

[4] E.F. Fama and K.R. French, *The cross-section of expected stock returns*, J. Finance, **47** (1992), 427–465.

[5] Y. Fang and T. Post, *Optimal portfolio choice for higher-order risk averters*, J. Bank. Finance **137** (2022), no. c, 327–349.

[6] E. Fons, D. Paula, Y. Jeffrey, Z. Xiao-Jun, and K. John, *A novel dynamic asset allocation system using feature saliency hidden Markov models for smart beta investing*, Expert Syst. Appl. **163** (2021), no. 7, 210–223.

[7] M. Kohansal Kafshgari, Z. Zarei, and R. Behmanesh, *Presentation of intelligent Meta-heuristic Hybrid models (ANFIS -MGGP) to predict stock returns with more accuracy and speed than other Meta-heuristic methods*, Quart. Financ. Eng. Secur. Maneg. **12** (2021), no. 47, 390–417. [In Persian]

[8] E. Kordian, *Investigating the basics of stock market efficiency, company's performance, fluctuations in abnormal stock returns and board independence*, 2nd Int. Conf. Challeng. New Solut. Ind. Engin. Manag. Account., 2022. [In Persian]

[9] H. Markowitz, *Portfolio selection*, J. Finance **7** (1952), no. 1, 77–91.

[10] G. Nazaire, M. Pacurar and O. Sy, *Factor investing and risk management: is smart-beta diversification smart?*, J. Empir. Finance **9** (2021), no. 56, 94–104.

[11] M. Wajid Raza and D. Ashraf, *Does the application of smart beta strategies enhance portfolio performance? the case of Islamic equity investments*, Int. Rev. Econ. Financ. **60** (2019), no. 3, 46–61.

[12] H. Zhang, R. An, and Q. Zhong, *Anti-corruption, government subsidies, and investment efficiency*, China J. Account. Res. **14** (2019), no. 4, 113–133.