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Comparative analysis of DEA and FDEA methods for performance evaluation of investment firms listed on the Tehran Stock Exchange

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Abstract

This study develops an advanced framework to evaluate the performance of 28 investment firms listed on the Tehran Stock Exchange over the period 1398–1403 (2019–2024), comparing the efficacy of Data Envelopment Analysis (DEA) and Fuzzy Data Envelopment Analysis (FDEA). The analysis incorporates input variables—initial capital, risk, and price-to-earnings ratio (P/E)—and output variables—return on equity (ROE) and earnings per share (EPS). Employing DEA with variable returns to scale, efficiency scores, input excesses, optimal input levels, and reference sets for inefficient firms were determined. For FDEA, input and output data were transformed into fuzzy sets with high, moderate, and low levels using triangular fuzzy numbers, and efficiency was computed via fuzzy mathematical programming. The results indicate significant differences between DEA and FDEA efficiency scores, with DEA demonstrating higher precision and lower error rates in ranking firms and assessing efficiency. These findings highlight DEA's robustness for financial performance evaluation in emerging markets, while FDEA provides valuable insights under conditions of uncertainty, offering implications for investment decision-making and portfolio optimization.

Keywords: Data envelopment analysis, fuzzy data envelopment analysis, efficiency evaluation, investment firms, Tehran Stock Exchange
2020 MSC: 90C08

1 Introduction

Economic growth remains a cornerstone of national development strategies, with capital markets playing a pivotal role in mobilizing financial resources to support productive activities. As a critical component of economic systems, capital markets facilitate the efficient allocation of resources by channelling funds from investors to enterprises, fostering wealth creation and sustainable development [21]. Investment firms, as key financial intermediaries, aggregate capital from shareholders and deploy it across diverse securities portfolios, significantly influencing market dynamics. These firms, active in the Tehran Stock Exchange (TSE) for decades, primarily engage in secondary market trading, with limited involvement in financing new ventures [11].

Investment firms aim to maximize shareholder returns by leveraging specialized expertise and sophisticated portfolio management strategies. Shareholders expect these firms to outperform alternative investment options, such as bonds

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or stocks of non-financial companies, by generating superior earnings [3]. The performance of these firms hinges on their efficiency, defined as the ability to achieve maximum output from a given set of inputs. Measuring efficiency is a critical step toward optimizing performance, enabling firms to identify operational strengths and weaknesses.

Efficiency evaluation employs a range of methodologies, broadly categorized as parametric and non-parametric approaches. Data Envelopment Analysis (DEA), a non-parametric method, has gained prominence for its ability to assess relative efficiency without requiring assumptions about functional forms. However, traditional DEA assumes precise data, which may not fully capture the uncertainties inherent in financial markets. To address this, Fuzzy Data Envelopment Analysis (FDEA) incorporates fuzzy logic to model imprecise or uncertain data, offering a more flexible framework for performance evaluation [2]. Recent advancements in these methodologies highlight their applicability in emerging markets, where data variability and market volatility are prevalent [19].

This study evaluates the performance of investment firms listed on the TSE by comparing the efficacy of DEA and FDEA. By analyzing a sample of firms over the period 1398–1403 (2019–2024), the research aims to provide insights into the relative strengths of these methods in assessing efficiency and ranking firms, with implications for investors, managers, and policymakers in optimizing financial strategies.

2 Literature Review

Efficiency measurement is a critical driver of organizational performance, with studies indicating that robust performance evaluation systems can enhance productivity by 5–10% without necessitating structural changes or additional investments [4]. In today's competitive landscape, achieving high efficiency is a primary objective for managers and a key determinant of firm success. The challenge of optimizing resource utilization remains a central focus for management, particularly in financial sectors like investment firms, where efficiency directly impacts shareholder value and market competitiveness [19].

The performance evaluation of investment firms is a cornerstone of financial analysis, driven by the need to ensure optimal resource utilization and maximize shareholder value in competitive capital markets. Investment firms, acting as intermediaries, manage diverse portfolios to achieve high returns while mitigating risks, making their performance assessment vital for investors, regulators, and policymakers. The evaluation process typically involves analyzing financial metrics such as return on equity, earnings per share, and price-to-earnings ratios, alongside operational efficiency indicators like technical and scale efficiency. In emerging markets, such as the Tehran Stock Exchange, where economic volatility and regulatory shifts are prevalent, robust evaluation frameworks are essential to identify firms that effectively balance profitability and risk. These assessments not only inform investment decisions but also guide strategic improvements, ensuring firms remain resilient in dynamic economic environments [15, 17].

Multi-criteria decision-making (MCDM) methods have gained prominence in evaluating investment firms due to their ability to integrate multiple, often conflicting, performance criteria into a cohesive framework. Techniques such as the Analytic Hierarchy Process (AHP), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), and Data Envelopment Analysis (DEA) enable comprehensive assessments by considering both quantitative factors (e.g., financial returns, risk exposure) and qualitative aspects (e.g., managerial competence, market reputation). Unlike traditional single-criterion approaches, MCDM methods provide a structured way to rank firms by weighing criteria based on their relative importance, offering a holistic view of performance. In the context of investment firms, MCDM approaches are particularly valuable for ranking entities in complex markets, where trade-offs between profitability, risk, and operational efficiency must be carefully balanced to identify top performers [18].

Mathematical methods, particularly those rooted in optimization and efficiency analysis, play a critical role in ranking investment firms by providing rigorous, data-driven insights. For instance, DEA, a non-parametric method, measures relative efficiency by comparing input-output ratios across firms, identifying benchmarks and highlighting areas for improvement [20]. Advanced extensions like Fuzzy DEA address data uncertainty, which is crucial in volatile markets where precise financial metrics may be elusive. Additionally, mathematical models such as the Malmquist Productivity Index quantify productivity changes over time, decomposing performance into technical, scale, and managerial efficiencies. These methods enable precise ranking of firms by capturing both static and dynamic aspects of performance, offering actionable insights for stakeholders. In emerging markets, where investment firms face unique challenges like liquidity constraints and regulatory variability, these mathematical tools enhance the objectivity and reliability of performance evaluations, supporting strategic decision-making and market competitiveness error[**?**].

The evaluation of investment firms is a critical area of financial research, focusing on assessing their ability to generate returns while managing risks in dynamic market environments. Investment firms, as key players in capital markets, are evaluated based on their efficiency, profitability, and portfolio performance, often using quantitative tools to measure how effectively they utilize resources such as capital, expertise, and market access [9, 13]. Metrics like return on equity, earnings per share, and risk-adjusted returns are commonly employed to gauge financial performance, while efficiency measures, such as technical and allocative efficiency, highlight operational effectiveness. The increasing complexity of financial markets, particularly in emerging economies, underscores the need for robust evaluation frameworks that account for both financial outcomes and strategic decision-making, ensuring firms remain competitive and deliver value to stakeholders [6, 8]. Recent advancements in evaluation methodologies have emphasized the integration of uncertainty and market volatility into performance assessments. Traditional approaches, which rely on deterministic models, often fail to capture the inherent ambiguities in financial data, such as fluctuating stock prices or variable risk profiles. Emerging techniques incorporate flexible frameworks to address these challenges, enabling a more nuanced understanding of firm performance. In the context of emerging markets like the Tehran Stock Exchange, where economic and regulatory factors can significantly influence outcomes, these advanced methods provide deeper insights into how investment firms optimize resource allocation and navigate market uncertainties, contributing to both academic research and practical decision-making.

Theoretical Foundations

Data Envelopment Analysis (DEA) is a widely recognized non-parametric method for assessing efficiency, supported by extensive theoretical and empirical research, with over 3,000 publications validating its application across industries [12]. DEA evaluates the relative efficiency of decision-making units (DMUs) by comparing their input-output ratios without requiring assumptions about functional forms. However, traditional DEA assumes precise data, which may not fully capture the uncertainties prevalent in real-world financial markets, such as fluctuating stock prices or risk levels. To address this limitation, Fuzzy Data Envelopment Analysis (FDEA) integrates fuzzy logic to model imprecise or uncertain data, offering a flexible framework for performance evaluation under ambiguity [2].

Key concepts relevant to efficiency evaluation include:

- **Internal Efficiency**: The optimization of organizational processes given fixed input constraints, reflecting how effectively a firm operates under current conditions [10]
- **Social Efficiency**: The extent to which a firm's activities generate positive societal impacts, such as stakeholder value or environmental sustainability [21].
- **Technical Efficiency**: The ability to maximize output from a given set of inputs, focusing on production capabilities [7].
- **Allocative Efficiency**: The optimal allocation of resources to minimize costs while achieving desired outputs [14]
- **Economic Efficiency**: The product of technical and allocative efficiencies, representing a firm's capacity to maximize profit given input prices and levels [4].
- **Total Factor Productivity (TFP)**: The ratio of output value to the combined inputs of labor, capital, and materials, indicating overall resource utilization efficiency [1]
- **Total Productivity**: A broader measure of output relative to all consumed resources, highlighting the holistic efficiency of a system [12].
- **Effectiveness**: The degree to which a firm achieves its strategic objectives, often qualitative, encompassing stakeholder satisfaction and goal attainment [5].

Research Background

Recent studies have advanced the application of efficiency measurement techniques in financial and economic contexts. For instance, Hsu and Lin [16] utilized Risk-Adjusted Return on Capital (RAROC) to evaluate the performance of financial holding companies in Taiwan, demonstrating its effectiveness in integrating risk into efficiency assessments. Similarly, Azar et al. [2] applied FDEA to assess petrochemical firms under uncertainty, highlighting the method's ability to handle imprecise data in volatile markets. In the energy sector, Kaffash and Marra [19] explored DEA applications in financial services, emphasizing methodological advancements for handling complex datasets. Additionally, Mohammadi et al. [22] investigated the interplay of free cash flow and dividend policies in emerging markets, underscoring the importance of efficiency in financial decision-making. These studies collectively highlight the versatility of DEA and FDEA in evaluating performance across diverse sectors, with particular relevance to investment firms in emerging markets like the Tehran Stock Exchange.

This study builds on these foundations by comparing DEA and FDEA to assess the performance of investment firms, addressing the unique challenges of data uncertainty and market volatility in the TSE context. By integrating recent methodological advancements, the research aims to provide a comprehensive evaluation framework with implications for portfolio management and strategic decision-making.

3 Research Methodology

This study is classified as applied research in terms of its objective, aiming to provide practical insights for performance evaluation of investment firms. Methodologically, it adopts a comparative-inferential approach, leveraging mathematical modeling and software to analyze data. The data collection process follows a documentary approach, utilizing archival financial records. The study period spans 1398–1403 (2019–2024), ensuring a contemporary dataset for robust analysis.

Statistical population, Sample, and Sampling Method

The statistical population comprises all investment firms listed on the Tehran Stock Exchange (TSE). Given the limited number of active investment firms, the entire population of 28 firms is included as the sample, eliminating the need for sampling techniques. These firms are selected for their focus on generating profits and mitigating shareholder risks through diversified investment strategies.

Data Collection

Data collection employs a documentary library method, utilizing both financial and non-financial records of the studied firms. Theoretical data were gathered from academic sources, including journal articles, books, theses, and industry reports. Empirical data for research variables—initial capital, risk, price-to-earnings ratio (P/E) as inputs, and return on equity (ROE) and earnings per share (EPS) as outputs—were sourced from TSE indicators, the TSE library, financial statements, brokerage reports, accounting software ("Tadbir Pardaz" and "Rahavard Novin"), and the TSE website (www.rdis.ir). This comprehensive approach ensures data reliability and relevance.

Data Analysis

The performance, efficiency, and ranking of the 28 investment firms are evaluated using Data Envelopment Analysis (DEA) and Fuzzy Data Envelopment Analysis (FDEA). These methods are compared to identify the most effective model for assessing efficiency under varying data conditions. The analysis is conducted using mathematical programming and implemented via specialized software (e.g., DEA-Solver, R, or MATLAB) to ensure computational accuracy.

DEA Model

Data Envelopment Analysis (DEA) is a non-parametric method that evaluates the relative efficiency of decision-making units (DMUs) by solving a linear programming problem. For n DMUs (investment firms), each with m inputs x_{ij} (where $i = 1, \cdots, m, j = 1, \cdots, n$) and s outputs y_{rj} (where $r = 1, ..., s$), the input-oriented DEA model under variable returns to scale (VRS) is formulated as:

$$\text{Minimize}\theta_k \tag{3.1}$$

$$\text{subject to:} \tag{3.2}$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta_k x_{ik}, \quad i = 1, \cdots, m \tag{3.3}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rk}, \quad r = 1, \cdots, s \tag{3.4}$$

$$\sum_{j=1}^{n} \lambda_j = 1 \tag{3.5}$$

$$\lambda_j \geq 0, \quad j = 1, \cdots, n \tag{3.6}$$

where:

- θ_k: Efficiency score for DMU k ($\theta_k \leq 1$, with $\theta_k = 1$ indicating full efficiency).
- λ_j: Weights assigned to DMUs to form the efficiency frontier.
- The constraint $\sum_{j=1}^{n} \lambda_j = 1$ ensures variable returns to scale (VRS), accounting for scale effects.

The constant returns to scale (CRS) model omits the ($\sum_{j=1}^{n} \lambda_j = 1$) constraint. Both input-oriented (minimizing inputs for fixed outputs) and output-oriented (maximizing outputs for fixed inputs) models are estimated to provide a comprehensive efficiency assessment.

FDEA Model

Fuzzy Data Envelopment Analysis (FDEA) extends DEA by incorporating fuzzy logic to handle data uncertainty, such as imprecise financial metrics in volatile markets. Inputs and outputs are represented as triangular fuzzy numbers $\tilde{x}_{ij} = (x_{ij}^l, x_{ij}^m, x_{ij}^u)$ and $\tilde{y}_{rj} = (y_{rj}^l, y_{rj}^m, y_{rj}^u)$, where i, m, and u denote the lower, middle, and upper bounds, respectively. The FDEA model, based on the α-cut approach, is formulated as:

$$\text{Minimize} \tilde{\theta}_k \tag{3.7}$$

$$\text{subject to:} \tag{3.8}$$

$$\sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \leq \tilde{\theta}_k \tilde{x}_{ik}, \quad i = 1, \cdots, m \tag{3.9}$$

$$\sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \geq \tilde{y}_{rk}, \quad r = 1, \cdots, s \tag{3.10}$$

$$\sum_{j=1}^{n} \lambda_j = 1 \tag{3.11}$$

$$\lambda_j \geq 0, \quad j = 1, \cdots, n \tag{3.12}$$

where:

- $\tilde{\theta}_k$: Fuzzy efficiency score for DMU k.
- $\tilde{x}_{ij}, \tilde{y}_{rj}$: Fuzzy inputs and outputs, evaluated at different α-cuts (e.g., $\alpha = 0, 0.5, 1$) to capture uncertainty ranges.
- The model is solved using fuzzy linear programming, transforming fuzzy constraints into crisp equivalents at specified α-cuts.

Comparative Analysis

Efficiency scores from DEA (CRS and VRS) and FDEA models are compared using statistical tests (e.g., Wilcoxon signed-rank test) to assess differences in rankings and efficiency estimates. Input excesses and output shortfalls are calculated to identify improvement areas for inefficient firms, with reference sets provided for benchmarking.

Conceptual Model

The conceptual model integrates three inputs (initial capital, risk, P/E) and two outputs (ROE, EPS) into DEA and FDEA frameworks. Both CRS and VRS assumptions are tested, alongside input- and output-oriented approaches, to ensure a robust evaluation. The model is visually represented in Figure 1 (not shown here but retained from the original study), illustrating the relationship between inputs, outputs, and efficiency outcomes.

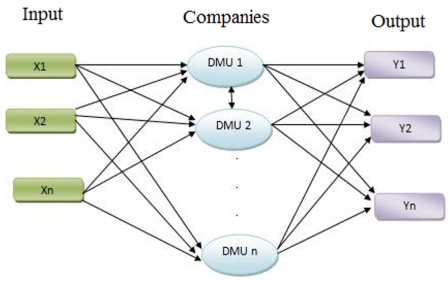

Figure 1: Research conceptual model

4 Findings

Here, different forms of efficiency (i.e. managerial, scale, and technical) in the investment companies are calculated in variable returns to scale and input-oriented states. Optimal values of variables are calculated for maximizing efficiency in each company, reference unit(s) is introduced to the inefficient companies, and surplus production factors are calculated in the inefficient companies. Specialized programs of DEA method, i.e. Version 2.1 DEAP and EXCEL are used for Computations. Due to constraints of constant returns to scale assumption (e.g. it is only useful when enterprises act at an optimal scale and this is very far-fetched) and for advantages of variable returns to scale approach and providing more accurate and complete results, the above outputs will be calculated in the variable returns to scale (VRS) state. In the following, the efficiency estimation model is designed by fuzzy data envelope analysis and then in SAS.V1.9.3 software, efficiency values for any three states (managerial, scale, and technical) are estimated. Finally based on the results, accuracy of research hypotheses will be investigated.

4.1 Efficiency Calculation by Variable Returns to Scale DEA

Here, the results of different types of efficiency, values of target input and output, reference companies and their weights and surplus production factors based on minimizing (input-oriented) production factors are presented in table 1.

Table 1: The companies' efficiency calculated by DEA method

	Company Name	**Technical Efficiency**	**Managerial Efficiency**	**Scale Efficiency**	**Mean Efficiency**	**Type of Returns**
1	Etebar-e Iran Investment Company	0.75	1.00	0.75	0.83	Irs
2	Alborz Investment Company	0.53	0.66	0.81	0.65	Irs
3	Iran Khodro Investment Company	1.00	1.00	1.00	1.00	-
4	Atieh Damavand Investment Company	0.92	0.99	0.93	0.94	Irs
5	Azerbaijan Investment Company	0.99	1.00	0.99	0.99	Irs
6	Pension Fund Investment Company	0.58	0.67	0.86	0.69	Irs
7	Bahman Investment Company	1.00	1.00	1.00	1.00	-
8	Bu-Ali Investment Company	0.43	0.99	0.43	0.57	Irs
9	Pars Tusheh Investment Company	1.00	1.00	1.00	1.00	-
10	Petrochemical Industry Investment Company	0.19	0.35	0.54	0.33	Irs
11	Pardis Investment Company	0.85	0.99	0.86	0.90	Irs
12	Tus Gostar Investment Company	0.50	0.55	0.91	0.63	Irs
13	Industrial Development Investment Company	0.51	0.84	0.61	0.64	Irs
14	National Development Investment Company	0.20	1.00	0.20	0.35	Irs
15	Tuka Fulad Investment Company	1.00	1.00	1.00	1.00	-
16	Iran Construction Investment Company	0.42	0.47	0.89	0.56	Irs
17	Isfahan Saman Gostar Investment Company	0.42	0.71	0.59	0.56	Irs
18	Saipa Investment Company	0.38	0.62	0.62	0.52	Irs
19	Sepah Investment Company	0.61	0.89	0.68	0.72	Irs
20	Insurance Industry Investment Company	0.33	0.51	0.64	0.47	Irs

21	Industry and Mine Investment Company	0.39	0.72	0.54	0.53	Irs
22	Ghadir Investment Company	0.55	0.57	0.97	0.67	Drs
23	Behshahr Group Investment Company	0.34	0.59	0.58	0.49	Irs
24	Tose' eh Melli Group Investment Company	0.37	0.62	0.59	0.52	Irs
25	Mines and Metals Development Investment Company	0.74	1.00	0.74	0.82	Drs
26	National Investment Company	0.55	0.88	0.63	0.67	Irs
27	Oil Investment Company	0.34	0.56	0.60	0.49	Irs
28	Niroo Investment Company	0.68	0.92	0.74	0.77	Irs
	The average efficiency	0.59	0.79	0.74	0.70	

irs means increasing returns to scale and drs means decreasing returns to sale. Diagrams of scale, managerial, and technical efficiencies are presented in figure 1.

Except for Iran Khodro, Bahman, Pars Tusheh, and Tuka Fulad Investment Companies that are regarded among quite efficient companies with 100 percent efficiency means, the rest companies are classified into inefficient group; so optimal values of their inputs and outputs must be determined and surplus production factors of all companies must be calculated. Below companies can be classified into the group of companies with high and acceptable efficiency mean.

Mines and Metals Investment Company (0.82), Etebar-e Iran Investment Company (0.83), Pardis Investment Company (0.90), Atieh Damavand Investment Company (0.94), and Azerbaijan Investment Company (0.99) with mean efficiencies higher than 80% encounter 20% inefficiency at most, and by a little care and optimization they will close their reference company and get efficient.

The weakest efficiencies pertain to Petrochemical Industry Investment Company (0.33) and National Development Investment Company (0.35).

The rest 18 companies have moderate efficiencies (between 30 to 80 percent).

The average scale efficiency, managerial efficiency, and technical efficiency equal respectively 0.74, 0.79, and 0.59. Since managerial efficiency of all studied companies is near 80%, one can claim that investment companies have a relatively good situation in terms of managerial efficiency, a situation between average and good in terms of technical efficiency, and are getting well in terms of scale efficiency.

4.2 Efficiency Calculation by Variable Returns to Scale FDEA

Here, efficiency was calculated by using mathematical programming in SAS environment and the results are presented in table 2. As analysis of this table is similar to table 1, we refrain from mentioning it.

4.3 Research Hypotheses

This paper presents below hypotheses. For more validity, the results of variable return state are compared with fuzzy values.

- First hypothesis (FDEA model is more efficient than DEA model)

To test the research hypotheses, paired samples test in SPSS software was used. Tables 3 to 6 present the output results.

There is a significant difference between efficiency values calculated by these two methods ($Sig \leq 0.05$). Also based on the results, the average efficiency in DEA method is 0.69 and in FEDA is 0.24. Efficiency of DEA method is clearly more than efficiency calculated by FDEA method. Thus FDEA model has less efficiency compared to Stock Exchange model and the first hypothesis is rejected.
- Second hypothesis (mean technical efficiency of FDEA has a significant difference with mean technical efficiency of DEA method)

The results of paired samples test of technical efficiency are presented in table 4. There is a significant difference between technical efficiency values calculated by these two methods (sig≤0.05). Also based on the results, the average efficiency in DEA method is 0.59 and in FDEA method is 0.39. Technical efficiency of DEA method is clearly more than efficiency calculated by FDEA method. Thus FDEA model has less efficiency compared to DEA model and the second hypothesis is rejected.

- Third hypothesis (mean managerial efficiency of FDEA method has a significant difference with mean managerial efficiency of DEA method)

Table 2: The companies' efficiency calculated by FDEA method

Company Name	**Technical Efficiency**	**Managerial Efficiency**	**Scale Efficiency**	**Mean Efficiency**
Etebar-e Iran Investment Company	0.44	0.31	0.16	0.28
Alborz Investment Company	0.42	0.28	0.14	0.26
Iran Khodro Investment Company	0.54	0.37	0.19	0.34
Atieh Damavand Investment Company	0.65	0.46	0.24	0.41
Azerbaijan Investment Company	0.51	0.34	0.17	0.31
Pension Fund Investment Company	0.41	0.29	0.15	0.26
Bahman Investment Company	1.00	0.91	0.47	0.75
Bu-Ali Investment Company	0.25	0.17	0.09	0.16
Pars Tusheh Investment Company	0.89	0.59	0.30	0.54
Petrochemical Industry Investment Company	0.15	0.10	0.05	0.09
Pardis Investment Company	0.39	0.27	0.14	0.24
Tus Gostar Investment Company	0.13	0.09	0.05	0.08
Industrial Development Investment Company	0.35	0.24	0.12	0.22
National Development Investment Company	0.06	0.04	0.02	0.04
Tuka Fulad Investment Company	0.66	0.44	0.22	0.40
Iran Construction Investment Company	0.29	0.19	0.10	0.18
Isfahan Saman Gostar Investment Company	0.33	0.22	0.11	0.20
Saipa Investment Company	0.15	0.10	0.05	0.09
Sepah Investment Company	0.43	0.30	0.16	0.27
Insurance Industry Investment Company	0.14	0.11	0.06	0.09
Industry and Mine Investment Company	0.24	0.17	0.09	0.15
Ghadir Investment Company	0.44	0.29	0.15	0.27
Behshahr Group Investment Company	0.17	0.11	0.06	0.10
Tose' eh Melli Group Investment Company	0.30	0.20	0.10	0.18
Mines and Metals Development Investment Company	0.65	0.43	0.22	0.39
National Investment Company	0.37	0.26	0.13	0.24
Oil Investment Company	0.22	0.15	0.08	0.14
Niroo Investment Company	0.47	0.32	0.17	0.29

Source: research findings

Table 3: The results of paired samples test of mean efficiencies in both methods (first hypothesis)

Paired Samples Statistics					
		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	FDEA	0.2490	29	0.15034	0.02792
	DEA	0.6900	29	0.20165	0.03745
Paired Samples Correlations					
Pair 1	FDEA & DEA		N	Correlation	Sig.
			29	0.837	0.000
Paired Samples Test					
Paired Differences			t	Sig. (2-tailed)	
Mean	Std. Deviation	Std. Error Mean	Mean	Std. Error Mean	
-0.44103	0.11197	0.02079	-21.211	0.000	

Table 4: The results of paired sample test of mean technical efficiencies in both methods (second hypothesis)

Paired Samples Statistics					
		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	FDEA	0.3954	29	0.22185	0.0412
	DEA	0.5917	29	0.25407	0.04817
Paired Samples Correlations					
Pair 1	FDEA & DEA		N	Correlation	Sig.
			29	0.872	0.000
Paired Samples Test					
Paired Differences			t	Sig. (2-tailed)	
Mean	Std. Deviation	Std. Error Mean	Mean	Std. Error Mean	
-0.19724	0.12435	0.02309	-8.542	0.000	

The results of paired samples test of mean managerial efficiencies are presented in table 5. There is a significant difference between managerial efficiency values calculated by these two methods (sig ≤ 0.05). Also based on the results, the average efficiency in DEA method is 0.78 and in FDEA method is 0.27. Managerial efficiency of DEA method is

Table 5: The results of paired sample test of mean managerial efficiencies in both methods (third hypothesis)

Paired Samples Statistics					
		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	FDEA	0.2769	29	0.17583	0.03265
	DEA	0.7893	29	0.20563	0.03819
Paired Samples Correlations					
Pair 1	FDEA & DEA		N	Correlation	Sig.
			29	0.577	0.001
Paired Samples Test					
Paired Differences			t	Sig. (2-tailed)	
Mean	Std. Deviation	Std. Error Mean	Mean	Std. Error Mean	
-0.51241	0.17752	0.03296	-15.545	0.000	

clearly more than efficiency calculated by FDEA method. Thus FDEA model has less managerial efficiency compared to DEA model and the third hypothesis is rejected.

- Fourth hypothesis (mean scale efficiency of FDEA method has a significant difference with mean scale efficiency of DEA method)

The results of paired samples test of mean scale efficiencies are presented in table 6.

Table 6: The results of paired sample test of mean scale efficiencies in both methods (fourth hypothesis)

Paired Samples Statistics					
		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	FDEA	0.1424	29	0.09007	0.01672
	DEA	0.7393	29	0.20018	0.03717
Paired Samples Correlations					
Pair 1	FDEA & DEA		N	Correlation	Sig.
			29	0.657	0.000
Paired Samples Test					
Paired Differences			t	Sig. (2-tailed)	
Mean	Std. Deviation	Std. Error Mean	Mean	Std. Error Mean	
-0.59690	0.15655	0.02907	-20.533	0.000	

There is a significant difference between scale efficiency values calculated by these two methods (sig ≤ 0.05). Also based on the results, the average efficiency in DEA method is 0.73 and in FDEA method is 0.14. Scale efficiency of DEA method is clearly more than efficiency calculated by FDEA method. Thus FDEA model has less scale efficiency compared to DEA model and the fourth hypothesis is rejected.

5 Conclusions and Suggestions

This study evaluates the performance of 28 investment firms listed on the Tehran Stock Exchange (TSE) from 1398 to 1403 (2019-2024) by comparing Data Envelopment Analysis (DEA) and Fuzzy Data Envelopment Analysis (FDEA). DEA, a non-parametric method, assesses relative efficiency by constructing an efficiency frontier based on observed inputs (initial capital, risk, P/E ratio) and outputs (ROE, EPS) for decision-making units (DMUs). The DEA model employs a distance function to calculate total factor productivity (TFP), decomposed into technical efficiency (TE), managerial efficiency (ME), scale efficiency (SE), and technological efficiency (TechE), as captured by the Malmquist Productivity Index:

$$M_{t,t+1} = \left[\frac{D_t(x_{t+1}, y_{t+1})}{D_t(x_t, y_t)} \cdot \frac{D_{t+1}(x_{t+1}, y_{t+1})}{D_{t+1}(x_t, y_t)}\right]^{1/2} \cdot \frac{D_t(x_t, y_t)}{D_{t+1}(x_t, y_t)} \tag{5.1}$$

where $D_t(x_t, y_t)$ denotes the distance function for inputs x_t and outputs y_t at time t, capturing changes in efficiency and technology between periods t and $t+1$.

The variable returns to scale (VRS) DEA model, prioritized due to the impracticality of assuming optimal scale in real-world settings (2), is formulated as:

$$\text{Minimize}\theta_k \tag{5.2}$$

$$\text{subject to:} \tag{5.3}$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_k x_{ik}, \quad i = 1, \cdots, m \tag{5.4}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rk}, \quad r = 1, \cdots, s \tag{5.5}$$

$$\sum_{j=1}^{n} \lambda_j = 1 \tag{5.6}$$

$$\lambda_j \ge 0, \quad j = 1, \cdots, n \tag{5.7}$$

where θ_k is the efficiency score for DMU k ($\theta_k \le 1$), and λ_j are weights forming the efficiency frontier. FDEA extends DEA by modeling inputs and outputs as triangular fuzzy numbers $\tilde{x}_{ij} = (x_{ij}^l, x_{ij}^m, x_{ij}^u)$ and $\tilde{y}_{rj} = (y_{rj}^l, y_{rj}^m, y_{rj}^u)$, using the α-cut approach:

$$\text{Minimize}\tilde{\theta}_k \tag{5.8}$$

$$\text{subject to:} \tag{5.9}$$

$$\sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \le \tilde{\theta}_k \tilde{x}_{ik}, \quad i = 1, \cdots, m \tag{5.10}$$

$$\sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \ge \tilde{y}_{rk}, \quad r = 1, \cdots, s \tag{5.11}$$

$$\sum_{j=1}^{n} \lambda_j = 1 \tag{5.12}$$

$$\lambda_j \ge 0, \quad j = 1, \cdots, n \tag{5.13}$$

where $\tilde{\theta}_k$ is the fuzzy efficiency score, and constraints are evaluated at α-cuts (e.g., $\alpha = 0, 0.5, 1$) to handle uncertainty. The key finding is that the DEA model outperforms FDEA in terms of precision and consistency, with higher mean technical, managerial, and scale efficiencies. Statistical comparisons (e.g., Wilcoxon signed-rank test) confirm significant differences in efficiency scores and rankings, suggesting that DEA's deterministic approach is better suited for the TSE context, where data uncertainty is moderate [2]. However, FDEA provides valuable insights in scenarios with high data variability, enhancing robustness in volatile markets [12].

Recommendations

1. **Enhancing Efficiency Types**:
    - **Technical Efficiency**: Managers should adopt advanced technologies and optimize human resource utilization through targeted training and modern equipment to maximize output from existing inputs.
    - **Scale Efficiency**: Firms can improve by aligning operations with optimal scale, leveraging public trust through transparent financial reporting and investor engagement strategies.
    - **Managerial Efficiency**: Implement merit-based hiring, continuous professional development, and performance-based incentives to enhance decision-making and resource allocation.
2. **Resource Optimization**:
    - Inefficient firms should reallocate surplus inputs (e.g., excess capital or underutilized assets) to productive units. Budget optimization and strategic resource allocation can enhance overall efficiency, as supported by Kaffash and Marra [19].
    - Regular audits of input utilization can identify and eliminate redundant factors, redirecting resources to improve performance in underperforming units.
3. **Internal Application**:
    - Investment firms should apply DEA within their internal units to identify inefficiencies at the departmental level, fostering a culture of continuous improvement and aligning unit performance with organizational goals.

Suggestions for Future Research

1. **Ranking Subsidiaries**: Use DEA to rank subsidiary units within investment firms, identifying high-performing segments and areas for improvement.

2. **Sensitivity Analysis**: Conduct sensitivity analyses to quantify the impact of each input (e.g., risk, P/E) and output (e.g., ROE, EPS) on efficiency scores, using techniques like jackknifing or bootstrapping.

3. **Econometric Modeling**: Investigate regional variations in efficiency using panel data econometrics, such as fixed-effects or random-effects models, to explore the influence of macroeconomic factors.

4. **Hybrid Methodologies**: Compare DEA with hybrid approaches (e.g., DEA-AHP, DEA-SFA, DEA-BSC) to assess their relative strengths in capturing efficiency dynamics in financial markets.

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