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Dynamic Behavior Assessment of Cable-Stayed Bridge Foundations Under Vertical Seismic Excitation Considering Soil-Structure Interaction

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Abstract

This paper is dedicated to a thorough investigation of the representation theory of the dihedral group D_n over fields of characteristic 2. Unlike the classical theory over fields of characteristic zero, the structure and classification of indecomposable representations in characteristic 2 present unique challenges due to the subtleties introduced by modular representation theory and the presence of non-semi simplicity. We review foundational definitions, detail the decomposition of representations into indecomposables, and highlight key structural differences from the characteristic zero case. The classification results for finite dihedral groups over fields with characteristic 2 are emphasized, culminating in a comprehensive understanding of the representation types and their matrix realizations.

Keywords: (Dihedral group, indecomposable representations, characteristic 2, modular representation theor, non-semisimple structures)
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1 Introduction

The dihedral group D_n, the group of symmetries of a regular n-gon, plays a fundamental role in finite group theory and representation theory. It is classically defined by the presentation:

$$D_n[\langle r], [s|\ r^n = e], [s^2 = e], [srs = r^1\rangle]$$

where r represents a rotation of order n and s a reflection [1]. Over fields of characteristic zero, the representation theory of D_n is well understood; irreducible representations are fully classified, and the group algebra is semi simple by Maschke's theorem. However, when one works over a field k of characteristic 2, the group algebra D_n is no longer semi simple, and the representation theory becomes considerably more intricate.

This article presents a detailed analysis of the indecomposable representations of the dihedral group D_n over a field k with $char(k) = 2$, with a focus on the structural changes, decomposition properties, and related algebraic theories such as group algebras and their modules. Specific cases likeD_4 and D_6 illustrate the general phenomena, and key theorems governing in decomposability, block structure, and module classification are developed with precise proofs.

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2 Background and Preliminaries

2.1 Background on Representations of Dihedral Groups

Classically, the representation theory of finite groups over characteristic zero fields benefit from Maschke's theorem, ensuring complete reducibility. This theorem states that every finite group algebra over a field with characteristic not dividing the group order is semi simple. Consequently, any representation decomposes uniquely into a direct sum of irreducible representations.

For the dihedral group D_n, over C, the irreducible representations are either one-dimensional, given by trivial or sign-like characters on the reflection element s, or two-dimensional, arising from the rotation subgroup generated by r acting by rotation matrices. The Morita theory and block decomposition align with this semi simple structure [19].

2.2 Group Algebra and Representations over Characteristic 2

Given a finite group G and a field k of characteristic $p/|G|$, the group algebra $k[G]$ typically loses semi simplicity. The dihedral group D_n with order 2_n satisfies this if $p = 2$. The representation theory hence requires modular methods, including block theory and consideration of projective and indecomposable modules [1].

In characteristic zero, D_n has one-dimensional and two-dimensional irreducible representations. The two-dimensional irreducible arise from the cyclic subgroup r. These become more nuanced in characteristic 2 because reflections do not necessarily act semi simply, and Jordan block structures can appear.

In the context of group algebra and representations over characteristic 2, particularly for the dihedral group D_n, several key points emerge:

1. **Loss of Semisimplicity:** When considering the group algebra $k[G]$ for a finite group G over a field k of characteristic p dividing the order of G, the algebra typically loses its semisimplicity. This is particularly evident for the dihedral group D_n when $p = 2$ [1].

   The loss of semisimplicity in representation theory, particularly for group algebras over fields of positive characteristic, has several significant implications:

   1.1 Structure of Representations: When semisimplicity is lost, representations can no longer be decomposed into a direct sum of irreducible representations. This means that representations may contain nontrivial indecomposable components, which complicates the analysis and classification of representations [11].

   1.2 Jordan Block Structures: In positive characteristic, representations may exhibit Jordan block structures, indicating that the action of the group is not completely reducible. This can lead to the presence of nilpotent operators in the representation, which is a stark contrast to the behavior in characteristic zero where representations are typically semi simple [16].

   1.3 Block Theory: The representation theory in positive characteristic often requires the use of block theory, which involves partitioning the representation category into blocks that can be studied independently. Each block may have its own set of irreducible representations, and understanding these blocks is crucial for a comprehensive understanding of the representation theory of the group [12].

   1.4 Projective and Indecodable Modules: The loss of semisimplicity necessitates a focus on projective and indecomposable modules, which are essential for constructing representations in this context. This shift in focus can lead to more complex module categories that require different techniques for analysis [2].

   1.5 Character Theory: The breakdown of character theory in the context of positive representations means that traditional tools used to analyze representations, such as characters and their orthogonality relations, may not hold. This complicates the understanding of the representation theory and necessitates the development of new tools and methods [15].

2. **Modular Representation Theory:** The representation theory in characteristic 2 necessitates modular methods, which include block theory and the analysis of projective and indecomposable modules. This is crucial for understanding the structure of representations in this characteristic [1].

   In decomposability and Modular Representation Theory, an indecomposable representation is a module that cannot be decomposed as a direct sum of two nontrivial submodules. Over $k[D_n]$ in characteristic 2, such modules may be reducible but indecomposable due to the failure of complete reducibility.

   The tools for studying these include:

- **Projective covers** and **injective hulls**, which help understand module structure.
- The use of Green's theory and Block theory, particularly for p-**groups** and p-**local** subgroups [3].
- Classification of matrix pairs associated with the representation problem, as in the study of pairs (A, B) of matrices satisfying conditions linked to the group relations [1].

3. **Irreducible Representations:** In characteristic zero, D_n has one-dimensional and two-dimensional irreducible representations. The two-dimensional irreducible representations arise from the cyclic subgroup generated by a rotation. However, in characteristic 2, the behavior of reflections complicates the representation theory, leading to the potential appearance of Jordan block structures, which are indicative of non-semi simple actions [1].

   3.1 **Structure of Irreducible Representations:** Over any algebraically closed field of characteristic zero (e.g., C):
   All irreducible representations of D_n are either one-dimensional or two-dimensional [10].

   3.2 **One-Dimensional Representations:** The one-dimensional representations correspond to the characters of $D_n/D_{n'}$, the abelianization.

   - For n odd: there are 2 such representations.
   - For n even: there are 4 such representations [10].

   3.3 **Two-Dimensional Irreducible Representations:** The remaining irreducible representations are two-dimensional and arise from the action of D_n on cosets of its cyclic subgroup $Cn = \langle r \rangle$. For $1 \leq h \leq \lfloor n/2 \rfloor 1 1 \leq h \leq \lfloor n/2 \rfloor 1$, define $\omega = e2^{\pi ih/n}$:

$$\rho_h(r) = \begin{pmatrix} w & 0 \\ 0 & w^{-1} \end{pmatrix} \quad , \quad \rho_h(s) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

   - For elements rk: $\rho_h(r^k) = diag(\omega^k, \rho^k)\rho_h(rk) = diag(\omega^k, \omega^k)$
   - For elements sr^k : $\rho_h(sr^k) = \begin{bmatrix} 0 & w^{-k} \\ w^k & 0 \end{bmatrix}$ [18].
   - **Key properties:**
   - $\rho_h \simeq \rho_n h$, so only $1 \leq h \leq \lfloor n - 12 \rfloor 1 \leq h \leq \frac{n-1}{2}$ are taken as distinct irreducible.
   - These representations are irreducible because they have no nontrivial invariant subspaces over C (for fixed $h \neq 0$, $n/2$ when n is even) [18].

Summary Table:

Representation	Number (odd n)	Number (even n)	Dimension
One-dimensional	2	4	1
Two-dimensional	$(n-1)/2$	$(n/2)-1$	2

4. **Classification of Representations:** A complete classification of the representations of D_n over a field of characteristic 2 has been established, which highlights the differences in structure compared to representations in characteristic zero [14].

   4.1 **Representations in Characteristic 2**
   When the underlying field k has characteristic 2, notably different phenomena occur compared to characteristic zero. In particular:

   - The usual decomposition into one-dimensional and two-dimensional irreducible representations seen in characteristic zero **does not hold in the same way**.
   - Representations can fail to be semi simple; indecomposable but reducible modules can appear.
   - The structure of the group algebra $k[D_n]$ is more complicated due to the characteristic dividing the order of the group.

   4.2 **Complete Classification Framework**
   Dong and Chen provided a **complete classification of finite-dimensional indecomposable representations** of D_n over algebraically closed fields of characteristic 2 (or more generally when the characteristic divides the order $2n$) by studying the quantum double $D(kD_n)$ of the group algebra kD_n [5].

   4.3 **Main Mathematical Features**

- **Indecomposable modules** are parametrized and described explicitly, including those representations that cannot be broken down into direct sums of irreducible.
- The classical block theory and modular representation theory techniques are applied to characterize projective indecomposable modules and their composition series.
- The classification demonstrates that the representation theory of D_n in characteristic **2** is fundamentally different from characteristic zero, where all modules are semi simple and irreducible have dimension **1** or **2** only.

These points illustrate the complexities introduced in the representation theory of dihedral groups when moving from characteristic zero to characteristic 2, emphasizing the need for specialized modular techniques.

5. Representation of D_n over Fields of Characteristic Zero

For comparison, we briefly summarize the characteristic zero case. Over an algebraically closed field $k, char(k) = 0$,

- All irreducible representations of D_n have dimension 1 or 2.
- The group algebra $k[D_n]$ is semi simple and decomposes into a direct sum of matrix algebras according to the Wedderburn-Artin theorem.

Specifically, the irreducible representations are:

- Two one-dimensional representations $\rho\pm : [r \mapsto 1],\ [s \mapsto \pm 1]$,
- $(n1)/2$ two-dimensional irreducible representations when n is odd, or $(n2)/2$ when n is even, constructed by inducing characters from the cyclic subgroup generated by r.

The matrices representing r and s in these two-dimensional representations satisfy the relations:

$$[\rho(r)^n = I],\ [\rho(r)^n = I],\ [\rho(s)\rho(r)\rho(s)] = \rho(r)^{-1}.$$

A key point is that $\rho(s)$ acts as a reflection matrix, and both $\rho(r)$ and $\rho(s)$ are diagonalizable over k [7].

6. Structure of the Group Algebra $k[D_n]$ over Fields of Characteristic 2

Let k be an algebraically closed field of characteristic 2. The modular group algebra $k[D_n]$ reflects complexity absent in characteristic zero. Since $p_2||D_n| = 2n$, the group algebra is not semi simple and decomposes into blocks, which are two-sided ideals corresponding to equivalence classes of simple modules brought together by extensions.

Theorem 2.1 (Block Decomposition and Uniqueness of Composition Series)). If G is a p-soluble group with cyclic Sylow p-subgroup (here specializing to D_n for appropriate n), then every indecomposable $k[G]$-module is isomorphic to a right ideal of $k[G]$, and each such right ideal admits a unique composition series [19].

This result implies, for dihedral groups D_n where the Sylow 2-subgroup is cyclic, the representation theory resembles classical modular representation types with manageable indecomposables, despite lack of semisimplicity.

4.1 Specific Challenges in Characteristic 2

In characteristic 2, the $s^2 = e$ becomes subtle because $(\rho(s))2 = I$ implies$\rho(s)$ is an involution. However, over a field of characteristic 2, the minimal polynomial $x^2 1 = (x1)^2$ degenerates since $1 = 1$, causing involutions to potentially act non semisimply; i.e., $\rho(s)$ may have the Jordan block matrix:

$$\left[\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\right]$$

which is not diagonalizable. This indicates indecomposable representations that are not irreducible.

Moreover, the order relations on r become original only modulo n, but as $\mathbf{2}||D_n$, the Maschke theorem fails, and the group algebra is non semi simple [19].

4.2 Classification Results for D_n over Characteristic 2

Bondarenko provides a complete classification of the representations of finite dihedral groups over characteristic **2** fields by reducing to the problem of classifying pairs of matrices (A, B) over the field k such that:

$$A^n = I, \quad B^2 = I, \quad BAB = A^{-1}$$

He shows that such pairs can be grouped into similarity classes corresponding to indecomposable modules [1].

The classification distinguishes between three types of indecomposable representations:

1. **One-dimensional representations:** which generally arise from the trivial action or sign character action. However, in characteristic 2 the sign character degenerates as -1=1, so these representations collapse to the trivial representation.
2. **Two-dimensional indecomposables:** which correspond to "**Jordan block**" type structures associated with the reflection s acting non-semisimply.
3. **Further indecomposables:** induced by more complicated matrix pairs arising from extension problems and module decomposition failures.

Hence, the irreducible representations in characteristic $p = 2$ are fewer, and the presence of indecomposable but reducible modules complicates the representation theory [1].

4.3 Explicit Matrix Construction

Let k be a field of characteristic **2**. For a two-dimensional indecomposable representation ρ, the generating elements act via matrices:

$$\rho(r) = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda^{-1} \end{pmatrix}, \quad p(s) = \begin{bmatrix} 0 & \alpha \\ 1 & 0 \end{bmatrix}$$

where λ is an n-th root of unity in k, and $\alpha \neq 0$ allows $\rho(s)$ to be non-diagonalizable.

The relations $\rho(s)^2 = I$ and:

$$\rho(s)\rho(r)\rho(s) = \rho(r)^1$$

are satisfied exactly when 0 encodes the nilpotent part of $\rho(s)$. This corresponds to the in decomposability of the module.

This exemplifies the financial difference with the characteristic zero case, where the analogous representation decomposes into the direct sum of two one-dimensional modules or irreducible two-dimensional representations [1].

4.4 Connection with Group Cohomology and Extensions

The indecomposable modules arising from such representations can also be understood through extension groups Ext^1. Specifically, indecomposables which are not irreducible correspond to nontrivial extensions of simple modules. The calculation of extensions over characteristic **2** fields for D_n, connects to the understanding of the module category and block theory associated with $k[D_n]$ [3].

The failure of semisimplicity raises important homological questions and motivates the study of the block structure and the defect groups in modular representation theory.

5. Comparison of Characteristic Zero and Characteristic 2 Representation Theories for D_n

Aspect	Characteristic Zero	Characteristic 2
Semisimplicity of $k[Dn]$	Semi simple (Maschke theorem holds)	Not semi simple (Maschke fails)
Number of Irreducible	2 one-dimensional + n–1/2 two-dimensional	Fewer one-dimensional (sign collapses), more indecomposables
Diagonalizability of $\rho(s)$	Always diagonalizable involutions	May be non-diagonalizable (Jordan blocks appear)
Indecomposable modules	All irreducible modules are indecomposable	Indecomposable modules include reducible representations
Tools for analysis	Character theory, Schur's Lemma	Block theory, extension groups, matrix pair classification

This table synthesizes the key differences, highlighting that in characteristic 2, the landscape is richer and incorporates subtle module structures not visible over characteristic zero fields [1; 8].

6. Indecomposable Modules: Classification and Examples

6.1 Indecomposable Types for $D4$ and $D6$

Extensive studies reveal that for small dihedral groups such as $D4$ and $D6$, the indecomposable modules fall into discrete families linked to the structure of the group algebra and the nature of their **Sylow 2**-subgroups.

For instance, $D4$ possesses a non-cyclic Sylow 2-subgroup, the Klein four group $V4$, leading to an infinite number of indecomposable modules — the algebra is of wild representation type.

Conversely, when the Sylow 2-subgroup is cyclic, indecomposable modules form tame or finite type families, permitting explicit classification [19].

6.2 Extension and Composition Series

The interplay between submodules and quotient modules of an indecomposable module $\boldsymbol{M}$ over $k[D_n]$ is crucial. Indecomposables are, in modular representation theory, often characterized as those modules that cannot be decomposed non-trivially into direct sums yet have composition series consisting of simples.

Consider a short exact sequence of $k[Dn]$-modules:

$$[0 \to M' \to^f M \to^g M'' \to 0]$$

with $\boldsymbol{M}$ indecomposable but neither $\boldsymbol{M'}$ nor $\boldsymbol{M''}$ trivial. Such extensions may be non-split in characteristic 2, contributing essentially to the indecomposable nature of $\boldsymbol{M}$.

These phenomena rely on block theory, and the understanding of support varieties and vertices helps to control the classification of indecomposables [11].

7. Key Theorems and Proofs

Theorem 2.2 (Indecomposability Criteria for $k(D_n)$-Modules).
Statement:
Let k be a field of characteristic 2, and D_n the dihedral group of order $2n2n$. An indecomposable $k[D_n]$-module M is characterized by the existence of a filtration

$$0 = M_0 \subset M_1 \subset \cdots \subset M_r = M$$

where each successive quotient $M_i + 1/M_i$ is simple, and $\boldsymbol{M}$ cannot be expressed as a direct sum of two nontrivial submodules.

Proof Sketch:
The proof combines block theory and the Krull-Remak-Schmidt theorem adapted to modular settings. One identifies the vertices (minimal p-subgroups controlling projectivity) and uses the covering theory of blocks and their defect groups to classify and construct M as an extension of simpler modules.

The filtration above corresponds to a unique composition series due to unicity in the modular setting, as long as the ***Sylow 2***-subgroup is cyclic. Extensions arise naturally when the **Sylow 2-**subgroup is non-cyclic, potentially yielding infinitely many indecomposables [19].

Theorem 2.3 (Bounded Degree of Irreducible Representations and Indecomposability).
Statement:
If the dihedral group D_n has a cyclic Sylow 2-subgroup, then the degrees of all irreducible and indecomposable representations over k are bounded above by the index of an abelian subgroup of D_n, specifically the cyclic subgroup of rotations.

Proof Sketch:
Using the methods of identities of matrix rings and Banach algebra analogues, combined with the subgroup structure of D_n, one uses the fact that the index of the cyclic subgroup bounds the degree of irreducible representations induced from it.

This bound restricts the size and complexity of indecomposables, refining the classification and indicating how modular representation theory is governed by subgroup indices [6].

8. Supplementary Topics and Further Directions

8.1 **Block Theory and Fusion Systems**

Blocks stratify $k[Dn]$-modules and their representation categories, organizing indecomposables into equivalence classes linked by extension data and local group-theoretic structure. Fusion systems governing conjugation relations within **Sylow p**-subgroups play a role in understanding these blocks [8].

8.2 **Cohomological Methods**

Group cohomology constructs invariants describing extensions between simple modules and detects obstructions to decomposability. For modular representations of D_n, support varieties and cohomological dimensions provide structural insights [8].

8.3 **Nichols Algebras and Hopf Algebra Structures**

Recent advances address extensions of representation theory of dihedral groups into Hopf algebra contexts, classifying finite-dimensional Nichols algebras over D_n, which impact quantum groups and braided categories over fields of characteristic zero or positive characteristic [9].

9. Further Topics and Open Questions

While the classification for finite dihedral groups over characteristic 2 has been achieved, open avenues remain in understanding:

- The explicit decomposition matrices relating ordinary and modular representations [4].
- The behavior of infinite dihedral groups or related infinite structures in characteristic 2, possibly involving infinite-dimensional modules [1].
- Connections with cohomological invariants related to support varieties for group schemes in characteristic 2 [13].
- Interaction with 2-representation theory and categorifications related to dihedral groups, as developed in higher representation theory contexts [17].

These questions offer rich potential for further research into the modular representation theory of dihedral groups and related algebraic structures.

10. Conclusion

The indecomposable representation theory of the dihedral group D_n over fields of characteristic 2 reflects richly structured algebraic phenomena distinct from the characteristic zero case. The modular setting introduces indecomposable modules beyond irreducible, governed by the structure of Sylow 2-subgroups, block theory, and extension classes. The representation algebra $k[D_n]$ in characteristic 2 is typically not semi simple and must be analyzed through the lens of modular representation theory, emphasizing radicals, projective covers, and defect groups. The classification of indecomposable modules depends heavily on subgroup structure, particularly the cyclicity of Sylow 2-subgroups.

Future Applications

The results of this study on the indecomposable representations of the dihedral group Dn over fields of characteristic 2 open several avenues for future research and application. These include:

1. **Cryptographic Protocols**: The structural insights into modular representations can inform the design of cryptographic systems that rely on group-theoretic constructions, particularly those involving dihedral groups in characteristic 2, enhancing security protocols in finite fields.
2. **Coding Theory**: The matrix realizations of indecomposable representations can be applied to construct error-correcting codes, leveraging the algebraic structure of dihedral groups to improve code efficiency and robustness in characteristic 2 settings.
3. **Categorical and Homological Algebra**: The classification results provide a foundation for studying categorical frameworks, such as derived categories or module categories, which can furthe elucidate the homological properties of dihedral group representations in modular settings.
4. **Hopf Algebra Connections**: The modular representation framework can be extended to explore connections with Hopf algebras, potentially leading to new algebraic invariants and applications in quantum group theory.

5. **Computational Algebra**: The detailed classification of indecomposable modules can be implemented in computational algebra systems to facilitate algorithmic analysis of group representations, aiding in the development of software tools for mathematical research.
6. **Graph Theory and Combinatorics**: The structural properties of dihedral group representations may find applications in graph theory, particularly in analyzing symmetries of graphs defined over fields of characteristic 2, contributing to combinatorial optimization problems.

These applications highlight the potential for the study's results to impact both theoretical mathematics and applied fields, encouraging further exploration in modular representation theory and related disciplines.

Future explorations may connect this modular analysis with Hopf algebras, cohomological methods, and categorical frameworks helping to broaden the understanding of dihedral group representations in modular settings.

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