

Estimation of the Discharge Coefficient of a Lateral Piano Key Weir in Bends of Erodible Channels Using Gene Expression Programming and Support Vector Regression

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Abstract

Lateral weirs are widely used in open channels to divert excess discharge and, in some cases, serve as protective hydraulic structures upstream of facilities such as inverted siphons. In recent years, Piano Key Weirs (PKWs) have attracted considerable attention because of their enhanced hydraulic performance. In this study, Gene Expression Programming (GEP) and Support Vector Regression (SVR) were applied to estimate the discharge coefficient of lateral Piano Key Weirs located in bends of erodible channels. Five model configurations were developed using different combinations of the governing dimensionless parameters. The complete experimental dataset was randomly divided into common 70% training and 30% testing subsets, and identical subsets were used for GEP and SVR to ensure a fair comparison. Model performance was assessed using the coefficient of determination (R^2), correlation coefficient (CC), root mean square error (RMSE), and mean absolute error (MAE). The SVR model incorporating all dimensionless parameters achieved $R^2 = 0.85$ and $RMSE = 0.157$. The best GEP performance was obtained for Model 2, in which the relative sediment-size ratio d_{50}/h_1 was excluded, yielding $R^2 = 0.92$ and $RMSE = 0.061$. Overall, the reported R^2 , RMSE, and MAE values indicate that GEP provided better predictive performance than SVR within the experimental domain considered.

Keywords: (Discharge coefficient, lateral weir, curved channel, gene expression programming, support vector regression)

2022 MSC: O20, O10

1 Introduction

Lateral weirs are widely used in open channels to divert excess discharge and, in some cases, function as protective hydraulic structures upstream of facilities such as inverted siphons. Therefore, accurate estimation of the discharge capacity of lateral weirs is of considerable importance. This issue becomes even more critical in channels with erodible beds, where bed deformation influences flow patterns and discharge characteristics. Extensive research has been conducted on the hydraulic performance of lateral weirs. De Marchi [19] developed equations for estimating the

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discharge of rectangular lateral weirs under different upstream heads. Subramanya & Awasthy [23] investigated the hydraulic behavior of non-linear weirs under various flow conditions. Hager [14], evaluated the discharge coefficient of weirs with different geometries by incorporating the concepts of tailwater depth and flow drop depth. Tullis et al. [24] examined trapezoidal weirs and demonstrated their advantages in terms of increased effective crest length and enhanced discharge capacity.

Johnson [16] proposed discharge coefficient equations for both sharp-crested and broad-crested rectangular weirs. Lempérière and Ouamane [18] were the first to systematically develop the concept of Piano Key Weirs (PKWs) at the University of Biskra, France, and highlighted their advantages over conventional weirs. They classified PKWs into four configurations (Types A, B, C, and D), emphasizing their ability to provide up to four times greater discharge capacity while requiring less maintenance than conventional weirs. Erpicum et al. [8] reported that the discharge capacity of PKWs can be approximately three times greater than that of ogee weirs. They further demonstrated that PKWs perform more efficiently under low-head conditions, whereas at higher heads their efficiency decreases because of the formation and expansion of a recirculation zone near the inlet, which reduces the effective inlet width.

Rosier [21] investigated the influence of bed variations on the outflow discharge of lateral weirs in alluvial channels and reported that the unique flow pattern formed in the vicinity of the weir, particularly near its downstream end, causes significant changes in the bed profile. These changes are primarily characterized by bed aggradation downstream of the weir, leading to flow backwater effects and modifications in the local flow pattern, which ultimately increase the diverted discharge through the lateral weir. Faramarz et al. [1] examined bed variations adjacent to lateral weirs and found that increasing discharge in channels with erodible beds significantly intensifies water surface fluctuations and the curvature of streamlines near the downstream end of the lateral weir due to increased bed roughness along the weir. Furthermore, these changes result in non-uniform approach velocity distribution at the weir entrance.

In recent decades, machine learning techniques have attracted considerable attention for predicting complex hydraulic phenomena. Giustolisi [12] used Genetic Programming to determine the Chezy resistance coefficient in corrugated channels. Bilhan et al. [3] predicted the discharge coefficient of lateral weirs in straight channels using different ANN techniques, including feed-forward neural networks and kernel-based methods. Emiroglu et al. [6] applied ANN models to estimate the discharge coefficient of non-linear lateral weirs in straight channels and compared their performance with linear and nonlinear regression models. Kişi [17] employed several ANN architectures, including artificial neural networks, radial basis function (RBF), and generalized regression neural networks (GRNN), to predict monthly streamflow in the Goksudere River, located in the Black Sea region, concluding that the proposed ANN model outperformed the other approaches.

Dursun et al. [4] developed an Adaptive Neuro-Fuzzy Inference System (ANFIS) model for estimating the discharge coefficient of lateral weirs in rectangular channels. Azamathulla et al. [2] utilized Gene Expression Programming (GEP) to derive an equation for predicting discharge through rectangular sluice gates based on the Froude number and upstream water depth. Haghiabi et al. [15] investigated head losses over weirs using Genetic Algorithms (GA) and Support Vector Machines (SVM), demonstrating that the GA model with a radial basis kernel outperformed the SVR approach. Parsaie [20] compared Multi-Layer Perceptron (MLP) and Radial Basis Function (RBF) neural networks for estimating lateral-weir discharge coefficients and concluded that the MLP model with ten hidden neurons achieved the highest prediction accuracy. Ebtehaj et al. [5] applied Gene Expression Programming to predict the discharge coefficient of sharp-crested rectangular lateral weirs. Roushangar et al. [22] compared a hybrid Support Vector Machine–Genetic Programming (SVM–GP) model with the Gene Expression Programming (GEP) approach for evaluating trapezoidal and rectangular lateral weirs and reported superior performance of the hybrid SVM–GP model. Emiroglu et al. [7] also evaluated the discharge coefficient of non-linear lateral weirs using an ANFIS model and compared its performance with nonlinear multiple regression models.

2 Laboratory Model

The experiments were carried out at the Soil Conservation and Watershed Management Research Institute using a curved laboratory flume with a radius of 2.5 m, a width of 0.5 m, a depth of 0.5 m, and a total length of 14 m (Figure 1). The lateral weir employed in this study was a Type C Piano Key Weir (PKW), which was installed along the curved channel at a central angle of 120°, as shown in Figure 2. Since the experiments were conducted in a channel with an erodible sediment bed, sediment with a median grain size (d_{50}) of 0.6 mm was selected as the bed material.

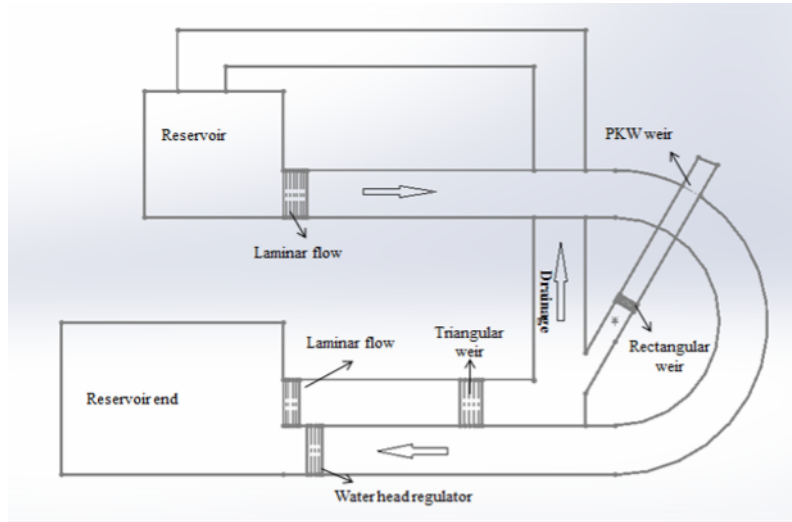


Figure 1: Curved laboratory flume used in the experiments

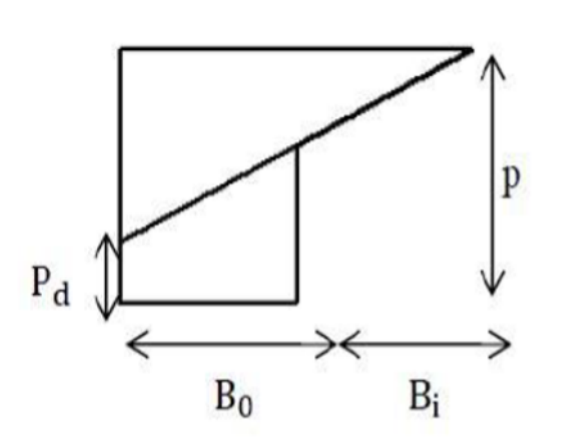


Figure 2: Geometric parameters of the Type C Piano Key Weir

2.1 Dimensional Analysis

The characteristic parameters considered to influence the discharge coefficient of the lateral weir in the present study are expressed as follows:

$$C_d = f(b, h_1, V_1, L, g, r_c, d_{50}, P, p_d, B, \rho) \quad (1)$$

Using the identified governing variables and selecting the repeating variables, dimensional analysis was performed based on the Buckingham π theorem. The resulting dimensionless relationship is given in Equation (2).

$$C_d = f(Fr, L/b, L/r_c, p_d/B, P/h_1, d_{50}/h_1) \quad (2)$$

In Equations (1) and (2), C_d is the discharge coefficient; ρ is the fluid density; V_1 is the approach-flow velocity; g is gravitational acceleration; b is the channel width; h_1 is the flow depth at the lateral-weir location; B is the weir width; L is the characteristic weir length used in the experimental geometry; r_c is the radius of the curved channel; d_{50} is the median sediment particle size; and P and p_d are the PKW geometric height parameters indicated in Figure 2. The Froude number is defined as $Fr = V_1/(gh_1)^{1/2}$. Each variable is defined here once and used consistently thereafter.

3 Numerical Models

3.1 Gene Expression Programming

Gene Expression Programming (GEP) is an advanced form of Genetic Programming (GP) based on the evolutionary computation paradigm, first introduced by Ferreira [10]. According to Ferreira [11], the GEP model combines both

simple linear chromosomes of fixed length, similar to those employed in Genetic Algorithms (GAs), and expression tree (ET) structures used in Genetic Programming (GP). Consequently, the GEP phenotype consists of expression tree structures analogous to those in GP; however, unlike GP, the expression trees generated by GEP are completely independent of the genome.

The primary objective of GEP is to establish an optimal relationship between a given dataset by employing mathematical operators through gene expression programming. As illustrated in the GEP flowchart, the modeling process begins with a predefined number of chromosomes representing the initial population. These chromosomes are then evaluated according to their fitness, after which the fittest individuals are selected. The selected individuals subsequently undergo evolutionary operations, including gene expression, environmental interaction, reproduction, and genetic modification. This evolutionary cycle is repeated over a specified number of generations until a satisfactory solution is obtained [9].

The GEP chromosome consists of two distinct regions, namely the head and the tail. The head contains mathematical operators, variables, and constants (e.g., +, -, ×, ÷, sin, cos, 1, a, b, c), which together form the mathematical expression. The tail, on the other hand, contains only variables and constants (e.g., 1, a, b, c). Whenever the symbols available in the head are insufficient to complete a valid expression, the symbols in the tail are utilized to ensure syntactic correctness of the generated expression [13].

3.2 Support Vector Regression

The Support Vector Machine (SVM) is an efficient machine learning technique founded on statistical learning theory and optimization principles. It employs the Structural Risk Minimization (SRM) principle to achieve a globally optimal solution rather than a locally optimal one.

In the regression framework, Support Vector Regression (SVR) estimates a continuous response using an ϵ -insensitive loss function. For a kernel-based SVR model, the prediction function can be written in the standard dual form shown in Equation (1):

$$\hat{Y}(X) = \sum_{i=1}^n (\alpha_i \alpha_i^*) K(X_i, X) + b \quad (1)$$

where $\hat{Y}(X)$ is the predicted response; X is the input vector; X_i denotes a training/support vector; α_i and α_i^* are the dual Lagrange multipliers; $K(X_i, X)$ is the kernel function; and b is the bias term. The regularization parameter C controls the penalty for deviations outside the ϵ -insensitive tube, ϵ specifies the tube width, and γ controls the kernel scale where applicable.

Three kernel functions were evaluated for the SVR models: linear, polynomial, and radial basis function (RBF). For each model configuration, the most appropriate kernel was selected during model development, and the corresponding hyperparameters C , γ , and ϵ were tuned where applicable. The kernel functions are expressed in Equations (2)-(4).

$$K(x_i, x_j) = x_i^T x_j \quad (2)$$

$$K(x_i, x_j) = (\gamma x_i^T x_j + r)^d, \quad \gamma > 0 \quad (3)$$

$$K(x_i, x_j) = \exp[-\gamma \|x_i - x_j\|^2], \quad \gamma > 0 \quad (4)$$

In these equations, γ is the kernel scale parameter, r is the polynomial-kernel offset (coefficient term), and d is the polynomial degree. The same five input configurations used for GEP were evaluated with SVR, as summarized in Table 2.

AUTHOR VERIFICATION REQUIRED: The reviewer requested a short table reporting the selected kernel and final C , γ , and ϵ values for SVR Models 1–5. These numerical hyperparameters are not contained in the supplied manuscript and must be inserted from the original SVM/SVR computational output before submission.

3.3 Performance Criteria

To evaluate predictive performance, four complementary statistical indices were used: mean absolute error (MAE), correlation coefficient (CC), coefficient of determination (R^2), and root mean square error (RMSE). They are defined as follows:

$$MAE = (1/n) \sum_{i=1}^n |O_i - P_i| \quad (5)$$

$$CC = \left[\frac{\sum_{i=1}^n (O_i - \bar{O})(P_i - \bar{P})}{\left[\sum_{i=1}^n (O_i - \bar{O})^2 \right] \left[\sum_{i=1}^n (P_i - \bar{P})^2 \right]^{1/2}} \right] \quad (6)$$

$$R^2 = CC^2 \quad (7)$$

$$RMSE = \left[\frac{1}{n} \sum_{i=1}^n (O_i - P_i)^2 \right]^{1/2} \quad (8)$$

Lower MAE and RMSE values indicate smaller prediction errors, whereas larger CC and R^2 values indicate stronger agreement between measured and predicted discharge coefficients.

In Equations (5)-(8), O_i and P_i are the observed and predicted discharge coefficients for observation i , respectively; $\bar{O}$ and $\bar{P}$ are the corresponding sample means; and n is the total number of observations.

4 Results and Discussion

The primary objective of this study is to predict the discharge coefficient of a lateral Piano Key weir installed in a curved channel with an erodible bed using the extracted dimensionless parameters and two machine learning approaches, Gene Expression Programming (GEP) and Support Vector Regression (SVR), and to compare their predictive performance. The results are presented and discussed in the following subsections.

4.1 GEP Results

The development of the GEP model for predicting the discharge coefficient involved several stages, including data preprocessing, selection of the GEP model type, specification of the GEP structure, selection of the transfer function, and determination of the training algorithm. During the data preprocessing stage, the dataset was divided into training and testing subsets, and all data were normalized to the range of 0-1. Data normalization improves the capability of the GEP model to identify an accurate relationship between the input and output variables while preventing misleading or unrealistic prediction accuracy.

The GEP model parameters selected in GENEXPROTOOLS V5.0 are presented in Table 1. Sensitivity analysis for the target discharge coefficient was performed using the Gene Expression Programming module. The available experimental dataset was randomly divided into 70% training and 30% testing subsets. The identical training and testing observations were used for the GEP and SVR models to ensure a fair comparison, and all variables were normalized to the range 0-1 before model development.

AUTHOR VERIFICATION REQUIRED: Please insert the total number of experimental observations (N), the exact training and testing sample sizes, and the random seed used for the 70:30 split (if a fixed seed was used). These values are not reported in the supplied manuscript.

Table 1: Parameters defined for the GEP model

Number of chromosomes	30
Head size	8
Number of genes	
Linking function	addition
Fitness function	RMSE
Mutation	0.03
Inversion	0.1
One-point recombination	0.3
Two-point recombination	0.3
Gene recombination	0.1
Gene transposition	0.1
Functions	+, −, *, /, power

The model inputs consisted of the six dimensionless parameters Fr , L/b , L/r_c , p_d/B , P/h_1 , and d_{50}/h_1 . Five configurations were defined to examine the contribution of these inputs, as presented in Table 2.

Table 2: Dimensionless input parameters for Models 1–5

Model number	Dimensionless parameters
Model 1	$Fr, L/b, L/r_c, p_d/B, P/h_1, d_{50}/h_1$
Model 2	$Fr, L/b, L/r_c, p_d/B, P/h_1$
Model 3	$L/b, L/r_c, p_d/B, P/h_1, d_{50}/h_1$
Model 4	$Fr, L/b, L/r_c, p_d/B, d_{50}/h_1$
Model 5	$Fr, P/h_1$

Since numerous parameters influence the discharge coefficient in curved channels, dimensional analysis was performed in this study to identify the most significant dimensionless parameters. Based on the extracted parameters, five different model configurations were developed to evaluate the influence of each parameter on the prediction of the discharge coefficient.

As shown in Figure (3-a), Model 1 incorporates all of the identified dimensionless parameters and achieved a coefficient of determination (R^2) of 0.90. Figure (3-b) presents the results of Model 2, in which the parameter d_{50}/h_1 was excluded. The prediction accuracy did not decrease after excluding this parameter; instead, the R^2 increased from 0.90 for Model 1 to 0.92 for Model 2, indicating that d_{50}/h_1 had a relatively limited contribution to the prediction of the discharge coefficient.

Figure (3-c) illustrates the results of Model 3, in which the Froude number (Fr) was omitted. In this case, the prediction accuracy decreased to an R^2 value of 0.67, representing a noticeable reduction compared with Model 1. This result indicates that the Froude number is an important parameter in estimating and predicting the discharge coefficient.

The results of Model 4 are presented in Figure (3-d), where the relative weir-height parameter (P/h_1) was excluded. Model performance decreased substantially, with R^2 falling to approximately 0.43. Within the present experimental range and modeling framework, this deterioration indicates that P/h_1 is an influential predictor of the discharge coefficient.

Finally, Figure (3-e) presents Model 5, which uses only Fr and P/h_1 and achieved $R^2 = 0.72$. The satisfactory performance obtained with this reduced input set indicates that Fr and P/h_1 carry substantial predictive information within the present dataset; this interpretation is limited to the tested hydraulic and geometric conditions.

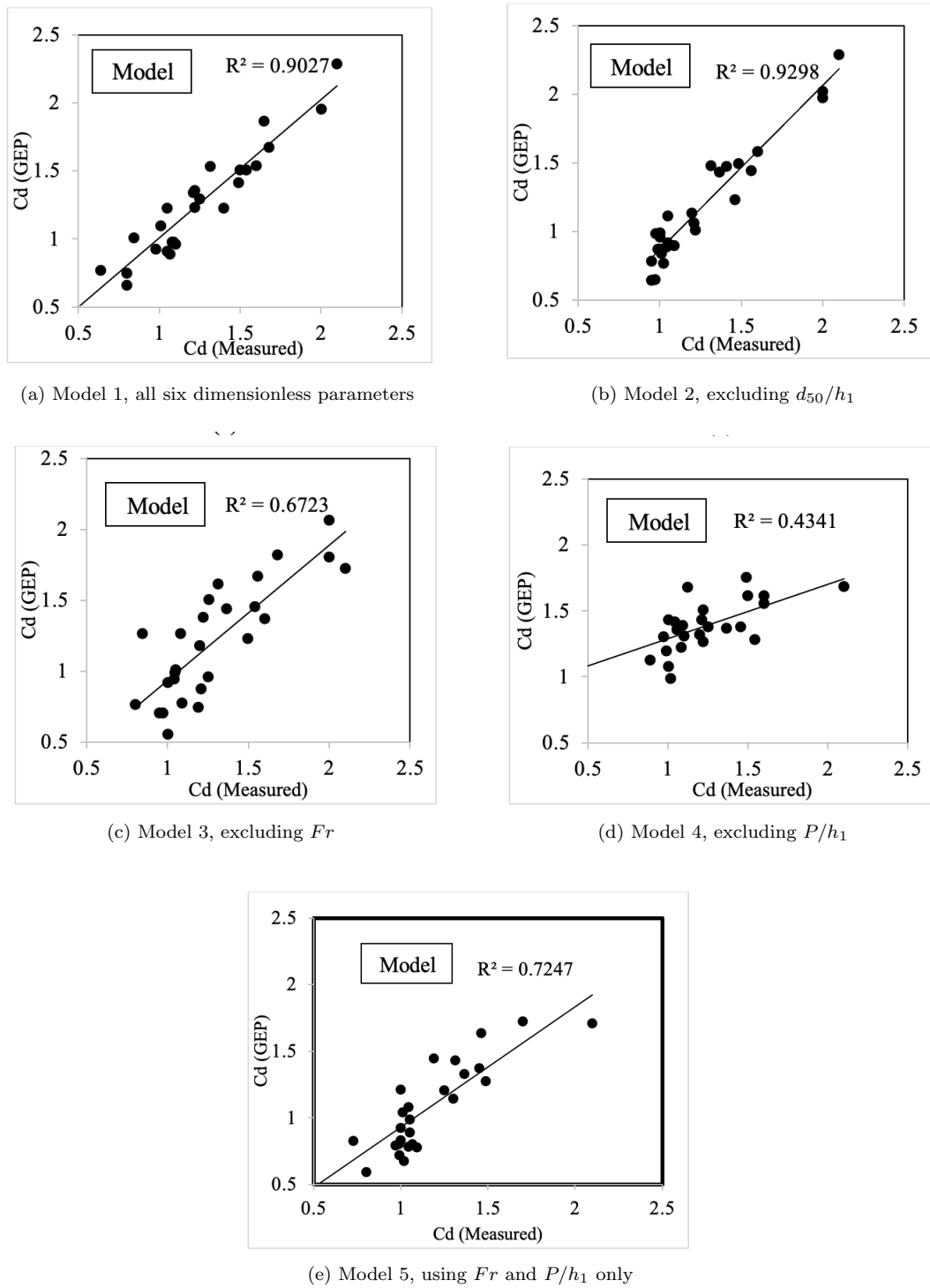


Figure 3: Measured versus GEP-predicted discharge coefficients (C_d) for the five input configurations

4.2 SVR Results

For development of the Support Vector Regression (SVR) model, the dataset was prepared using the same procedure as for Gene Expression Programming (GEP). The identical training and testing subsets used for GEP were also used for SVR to ensure a consistent and fair comparison. After preprocessing, the linear, polynomial, and radial basis function (RBF) kernels described previously were evaluated within the SVR framework, with kernel-specific hyperparameters tuned during model development.

Similar to the GEP model, the SVR model was developed by considering the effects of the extracted dimensionless parameters. Five different model configurations were established to evaluate the influence of these parameters on the prediction of the discharge coefficient. The results of these models are presented in Figures (4-a)–(4-e), demonstrating different levels of predictive performance for each configuration.

As shown in Figure (4-d), Model 4, in which P/h_1 was excluded, produced the lowest SVR prediction accuracy among the five configurations. Model 5 [Figure (4-e)], which used only Fr and P/h_1 , retained satisfactory predictive performance. Together, these results indicate that P/h_1 and Fr are influential inputs within the present experimental dataset and model structure, rather than establishing a universal ranking for all lateral PKW configurations.

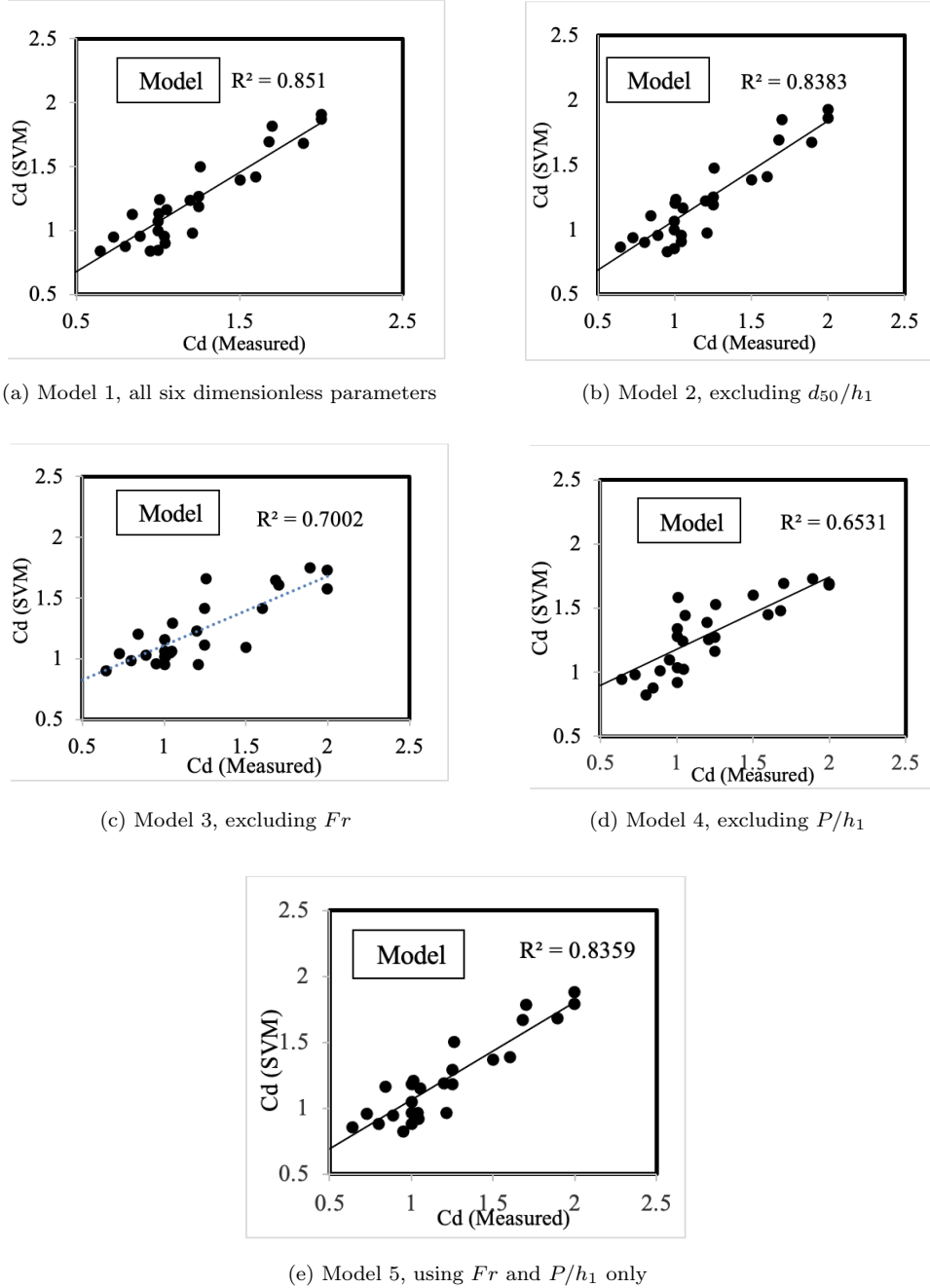


Figure 4: Measured versus SVM-based SVR predictions of the discharge coefficient (C_d) for the five input configurations

4.3 Comparison of GEP and SVR Results

Table 3 summarizes the predictive performance of the GEP and SVR models across all five input configurations. In both approaches, the discharge coefficient (C_d) was the target variable, and identical input configurations and train-test subsets were used to ensure a consistent comparison.

As shown in Table 3, Model 5 retained satisfactory predictive performance using only the Froude number (Fr) and relative weir height (P/h_1). Conversely, Model 4, in which P/h_1 was omitted, showed the lowest or among the lowest predictive performance in both approaches. Within the tested experimental range, this pattern supports the importance of P/h_1 for estimating the discharge coefficient without implying universal dominance outside the present hydraulic and geometric conditions.

Considering the overall reported R^2 , RMSE, and MAE values, the GEP approach demonstrated better predictive performance than the SVR approach. In particular, GEP Model 2 achieved the highest R^2 (0.92) and the lowest RMSE (0.061) among the developed models.

Table 3: Performance of the GEP and SVR models

Model	inputs	target	SVR				GEP			
			R^2	CC	RMSE	MAE	R^2	CC	RMSE	MAE
Model 1	$Fr, \frac{l}{b}, \frac{l}{rc}, \frac{pd}{B}, p/h_1, d_{50}/h_1$	Cd	0.85	0.92	0.157	0.133	0.90	0.95	0.092	0.073
Model 2	$Fr, \frac{l}{b}, \frac{l}{rc}, \frac{pd}{B}, p/h_1$	Cd	0.83	0.91	0.164	0.026	0.92	0.96	0.061	0.051
Model 3	$\frac{l}{b}, \frac{l}{rc}, \frac{pd}{B}, p/h_1, d_{50}/h_1$	Cd	0.70	0.83	0.226	0.178	0.67	0.81	0.138	0.114
Model 4	$Fr, \frac{l}{b}, \frac{l}{rc}, \frac{pd}{B}, d_{50}/h_1$	Cd	0.65	0.80	0.256	0.197	0.43	0.62	0.144	0.108
Model 5	$Fr, p/h_1$	Cd	0.83	0.91	0.166	0.140	0.72	0.85	0.175	0.102

Regarding the prediction error, the GEP models generally produced lower RMSE and MAE values than the corresponding SVR models. In particular, GEP Model 2 achieved the highest R^2 (0.92) and the lowest RMSE (0.061) among the developed models. Therefore, based on the reported R^2 and error indices, GEP Model 2 provided the best overall predictive performance.

AUTHOR VERIFICATION REQUIRED: Recheck the SVM Model 2 MAE value reported in Table 3 (currently 0.026) against the original computational output before submission, as requested by the reviewer.

4.4 Study Limitations

The applicability of the developed GEP and SVR models is bounded by the experimental conditions used in this study. The data were obtained from a single laboratory flume configuration with a 2.5 m bend radius, a 120° weir location, a Type C PKW geometry, and an erodible bed with $d_{50} = 0.6$ mm. Consequently, the reported variable-importance patterns and predictive performance should not be extrapolated to substantially different bend geometries, weir configurations, sediment conditions, or hydraulic ranges without independent validation.

5 Conclusion

This study compared Gene Expression Programming (GEP) and Support Vector Regression (SVR) for estimating the discharge coefficient of a lateral Piano Key Weir in a curved channel with an erodible bed. Across the five dimensionless-input configurations, GEP generally produced lower prediction errors than SVR. The best overall performance was obtained with GEP Model 2, which excluded d_{50}/h_1 and achieved $R^2 = 0.92$ and $RMSE = 0.061$.

The largest deterioration in predictive performance occurred when P/h_1 was omitted, while the reduced Model 5 based only on Fr and P/h_1 retained satisfactory accuracy. These findings indicate that relative weir height and Froude number are influential predictors within the present experimental range, but the ranking should not be generalized beyond the tested configuration without validation. Within this defined domain, GEP provides a practical data-driven approach for estimating the discharge coefficient when a compact predictive relationship is required.

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7 Conflict of Interest

The authors declare that there are no conflicts of interest regarding the publication of this study.

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