

On the fixed points of Kannan contraction mappings on fuzzy metric spaces

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Abstract

In this paper, comparison functions are employed to introduce and investigate new classes of contractive mappings in fuzzy and classical metric spaces. After defining φ -large contractions and establishing fixed-point results for them in fuzzy and classical settings, the study focuses on generalizations of Khan-type contractions. In this regard, φ -large Khan contractions are first introduced in fuzzy metric spaces and the existence of fixed points is proved for such mappings. Their counterparts in classical metric spaces are then defined, and by using the results obtained in fuzzy metric spaces, the existence of fixed points in the classical case is also established. Furthermore, large Khan contractions and extended φ -large Khan contractions are studied as generalizations of the aforementioned mappings, and sufficient conditions ensuring the existence of fixed points in both fuzzy and classical metric spaces are provided. Finally, a more general form of large Khan contractions is introduced in fuzzy metric spaces, and the existence of fixed points for functions in this broader class is proved. The results of this study not only expand the theory of fixed points in fuzzy metric spaces, but also provide a more general framework for analyzing stability and convergence in related problems.

Keywords: Large contraction, Kannan contraction, Fuzzy metric space, Fixed point

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1 Introduction

Fixed-point theorems constitute a cornerstone in the study of equilibrium analysis, with wide-range applications in dynamical systems and economic models. They also serve as essential tools for the investigation of solutions to numerous problems in mathematical physics, game theory, and dynamic programming (see [6, 25]). In addition, these theorems have been effectively applied to boundary value problems, such as first-order nonlinear ordinary differential equations and matrix equations, in order to establish the existence of solutions under specific conditions [22]. One of the earliest and most influential results in this field is Banach's fixed-point theorem (1922). Later, in 1936, Nemytskii [21] extended the scope of contraction mappings by formulating fixed-point results under more general conditions. Since then, various important generalizations and refinements have been proposed, including those of Bailey [1], Suzuki [38], Popescu [26], Hardy–Rogers [17], and Karapınar [19]. Edelstein [20] further enriched the theory by employing the concept of w -limits to generalize both Banach's contraction principle and Nemytskii's theorems.

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The study of fuzzy metric spaces began with the seminal contribution of Kramosil and Michalek [4], who extended the notion of probabilistic metric spaces to incorporate fuzzy sets. This pioneering work was followed by several alternative formulations of fuzzy metric spaces proposed by other researchers [5, 18, 3]. In 1994, George and Veeramani [7] refined the original definition by introducing a Hausdorff topology on fuzzy metric spaces, which significantly advanced the field. Since then, a considerable body of literature has been devoted to exploring their topological properties (see [8, 12]). More recently, Gregori, Miana, and Miravet [10] introduced the concept of extended fuzzy metrics, developing a topology derived from these metrics and analyzing their structural properties.

Within the framework of fuzzy metric spaces, the theory of fixed points for contraction-type mappings is closely aligned with the corresponding theory in probabilistic metric spaces of the Menger type. These two domains share a strong conceptual connection, as both address fixed-point theory for analogous classes of mappings (see [14, 23, 24, 27, 34, 39]). In 1988, Grabiec extended the fixed-point theorems of Banach and Edelstein to the setting of fuzzy metric spaces defined by Kramosil and Michalek. Since then, many researchers have contributed to the development of fixed-point results for various contractions in fuzzy metric spaces [9, 13, 15, 16, 34, 35, 36, 37, 40].

In this context, various types of contractions have been studied, including large contractions, φ -large contractions, and Khan-type contractions, each introduced to generalize the existing contractive conditions and establish the existence of fixed points under weaker or more flexible assumptions. Building on these developments, this study introduces new classes of contractions by combining the concepts of φ -large contractions and Khan-type contractions in both fuzzy and classical metric spaces. In particular, comparison functions are employed to define φ -large Khan contractions in fuzzy metric spaces and to establish the existence of fixed points for such mappings. Then, their counterparts in classical metric spaces are investigated, and by relying on the results obtained in the fuzzy setting, the existence of fixed points in the classical case is also established. Moreover, large Khan contractions and extended φ -large Khan contractions are examined, and sufficient conditions ensuring the existence of fixed points in both fuzzy and classical metric spaces are provided. Finally, a more general form of large Khan contractions is introduced in fuzzy metric spaces, and it is shown that this generalized class also guarantees the existence of fixed points.

The results of this study not only unify and extend several well-known theorems in the field of fixed-point theory, but also contribute to a deeper understanding of contractive mappings in fuzzy settings. In addition, the generalized framework developed in this paper provides new perspectives for the study of stability and convergence in applications where uncertainty plays a central role. Moreover, the study of the problem in the nonlinear setting is a challenging problem (see [28, 29, 30, 31, 32]) and there is a broad set of applications.

2 Preliminaries

This section introduces the foundational concepts necessary for understanding the remainder of the paper. In particular, we recall the definition of a fuzzy metric space from [7, 9, 11].

Definition 2.1. A fuzzy metric space is defined as a triple $(\mathcal{X}, \mathcal{M}, *)$, where $\mathcal{X}$ represents a non-empty set, $*$ denotes a continuous t-norm, and $\mathcal{M}$ is a function from $\mathcal{X} \times \mathcal{X} \times (0, \infty)$ to the interval $[0, 1]$ that satisfies the following conditions for all $x, y, z \in \mathcal{X}$ and $s, t > 0$.

- (FM1) $\mathcal{M}(x, y, t) > 0$.
- (FM2) $\mathcal{M}(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$.
- (FM3) $\mathcal{M}(x, y, t) = \mathcal{M}(y, x, t)$.
- (FM4) $\mathcal{M}(x, y, \cdot) : (0, \infty) \rightarrow [0, 1]$ is continuous.
- (FM5) $\mathcal{M}(x, z, t + s) \geq \mathcal{M}(x, y, t) * \mathcal{M}(y, z, s)$.

The following additional condition must be included in certain theorems.

- (FM6) $\lim_{t \rightarrow \infty} \mathcal{M}(x, y, t) = 1$ for all $x, y \in \mathcal{X}$.

Definition 2.2. Let $(\mathcal{X}, \mathcal{M}, *)$ be a fuzzy metric space.

- (i) A sequence x_n in $\mathcal{X}$ is called *Cauchy* whenever for any $\epsilon \in (0, 1)$ and any $t > 0$, one can find a natural number N such that for all $i > j \geq N$,

$$\mathcal{M}(x_i, x_j, t) > 1 - \epsilon.$$

- (ii) The sequence x_n is called *convergent* in $\mathcal{X}$ if there exists some $x \in \mathcal{X}$ satisfying

$$\lim_{i \rightarrow \infty} \mathcal{M}(x_i, x, t) = 1, \text{ for every } t > 0.$$

Lemma 2.3. Let $(\mathcal{X}, \mathcal{M}, *)$ be a fuzzy metric space. For arbitrary points x and y in $\mathcal{X}$, the function $\mathcal{M}(x, y, t)$ is a non-decreasing function of t .

We now turn to n -dimensional comparison functions, adopting the definition from [33].

Definition 2.4. [33] Let $\Phi : ((0, \infty))^n \rightarrow (0, \infty)$ be a function.

1. A function Φ is called an n -dimensional comparison function if it satisfies the following conditions.

- (i) Φ is monotone increasing; that is, if $0 \leq t_i \leq s_i$ for all $i = 1, \dots, n$, then

$$\Phi(t_1, \dots, t_n) \leq \Phi(s_1, \dots, s_n).$$

- (ii) For every $t > 0$, the sequence $\{\Psi^n(t)\}_{n=0}^\infty$ converges to zero, where the function $\Psi : (0, \infty) \rightarrow (0, \infty)$ is defined by $\Psi(t) = \Phi(t, \dots, t)$.

2. A function Φ is called an n -dimensional (c)-comparison function if it satisfies the following conditions.

- (i) Φ is monotone increasing; that is, if $0 \leq t_i \leq s_i$ for all $i = 1, \dots, n$, then

$$\Phi(t_1, \dots, t_n) \leq \Phi(s_1, \dots, s_n).$$

- (ii) The series $\sum_{k=0}^\infty \Psi^k(t)$ converges for every $t \in (0, \infty)$, where the function $\Psi : (0, \infty) \rightarrow (0, \infty)$ is defined by $\Psi(t) = \Phi(t, \dots, t)$.

3 Large-Kannan Contraction Mappings on Fuzzy Metric Spaces

In this section, we use the concept of comparison functions to introduce a new class of contraction mappings, namely, Φ -large contraction mappings. This definition is proposed as a generalization of large contraction mappings within the framework of fuzzy metric spaces. The aim of this extension is to broaden the class of contraction mappings that exhibit convergence behavior in fuzzy settings.

Definition 3.1. Let $(\mathcal{X}, \mathcal{M}, \min)$ be a fuzzy metric space. A map $\mathcal{F} : \mathcal{X} \rightarrow \mathcal{X}$ is said to be a Φ -large contraction if for every $x, y \in \mathcal{X}$, the following statements are true.

- (i) If $x \neq y$, then for every $\alpha \in (0, 1)$ there exist $0 < t_\alpha \leq s_\alpha < s'_\alpha$ such that

$$\mathcal{M}(x, y, t) < \alpha, \text{ for all } 0 < t \leq s'_\alpha, \text{ and } \mathcal{M}(\mathcal{F}x, \mathcal{F}y, t_\alpha) \geq \alpha.$$

- (ii) For every $\epsilon > 0$, a (1-dimensional) comparison function Φ exists such that for all $\alpha \in (0, 1)$, $\mathcal{M}(\mathcal{F}x, \mathcal{F}y, \Phi(t)) \geq \alpha$, where $\mathcal{M}(y, x, t) \geq \alpha$, and $\mathcal{M}(x, y, r) < \alpha$ for all $r < \epsilon$.

Lemma 3.2. Let $(\mathcal{X}, \mathcal{M}, \min)$ be a fuzzy metric space, and $\mathcal{F} : \mathcal{X} \rightarrow \mathcal{X}$ be a function for which statement (i) of Definition 3.1 is true. Moreover, let $\{x_n\}$ be a sequence that converges to $x \in \mathcal{X}$. Then, the sequence $\{\mathcal{F}x_n\}$ converges to $\mathcal{F}x$.

Proof . Let $t > 0$ and $\alpha \in (0, 1)$. Since the sequence $\{x_n\}$ converges to $x \in \mathcal{X}$, there exists $N \in \mathbb{N}$ such that $\mathcal{M}(x_n, x, t) \geq \alpha$ for all $n \geq N$.

Now, for every $n \geq N$, the following statements are true.

- (i) If $x_n = x$, then $\mathcal{F}x_n = \mathcal{F}x$. So, $\mathcal{M}(\mathcal{F}x_n, \mathcal{F}x, t) \geq \alpha$.
- (ii) If $x_n \neq x$, then by (i) there exist $0 < t_\alpha \leq s_\alpha < s'_\alpha$ such that $\mathcal{M}(x_n, x, t) < \alpha$ for all $0 < t \leq s'_\alpha$, and $\mathcal{M}(\mathcal{F}x_n, \mathcal{F}x, t_\alpha) \geq \alpha$. Hence, $0 < t_\alpha \leq s_\alpha < s'_\alpha < t$. Therefore,

$$\mathcal{M}(\mathcal{F}x_n, \mathcal{F}x, t) \geq \mathcal{M}(\mathcal{F}x_n, \mathcal{F}x, t_\alpha) \geq \alpha.$$

Thus, $\mathcal{M}(\mathcal{F}x_n, \mathcal{F}x, t) \geq \alpha$ for every $n \geq N$. This implies that the sequence $\{\mathcal{F}x_n\}$ converges to $\mathcal{F}x$. $\square$

Having introduced the notion of Φ -large contraction mappings, we proceed to examine the existence of their fixed points. Under certain sufficient conditions, we show that each Φ -large contraction mapping on a fuzzy metric space admits a unique fixed point. This result generalizes some well-known fixed-point theorems to the context of fuzzy metric spaces.

Theorem 3.3. Let $(\mathcal{X}, \mathcal{M}, \min)$ be a complete fuzzy metric space, and $\mathcal{F} : \mathcal{X} \rightarrow \mathcal{X}$ be a Φ -large contraction. Also, suppose there exist $x \in \mathcal{X}$ and $L > 0$ such that for every $\alpha \in (0, 1)$, $\mathcal{M}(x, \mathcal{F}^n x, L) \geq \alpha$ for all $n \geq 1$. Then, $\mathcal{F}$ has a unique fixed point in $\mathcal{X}$.

Proof . Assume that the sequence $\{\mathcal{F}^n x\}$ is not Cauchy. Then, there exist $\alpha > 0$ and $t_0 > 0$ such that for every $k \in \mathbb{N}$, one finds $m_k > n_k \geq k$ for which $\mathcal{M}(x_{n_k}, x_{m_k}, t_0) < \alpha$ holds.

Since $\mathcal{F}$ is a Φ -large contraction, by (i), for $x_{n_k} \neq x_{m_k}$ there exist $0 < t_\alpha \leq s_\alpha < s'_\alpha$ such that $\mathcal{M}(x_{n_k-1}, x_{m_k-1}, t) < \alpha$ for all $0 < t \leq s'_\alpha$, and $\mathcal{M}(x_{n_k}, x_{m_k}, t_\alpha) \geq \alpha$. Since $\mathcal{M}(x_{n_k}, x_{m_k}, t_0) < \alpha$, it follows that $t_0 < t_\alpha \leq s_\alpha < s'_\alpha$. So, $\mathcal{M}(x_{n_k-1}, x_{m_k-1}, t_0) < \alpha$. Similarly, $\mathcal{M}(x_{n_k-i}, x_{m_k-i}, t_0) < \alpha$ for all $i \leq n_k$. Since $\mathcal{F}$ is a large contraction, by (ii), for $t_0 > 0$ there exists a comparison function Φ such that for all $\alpha \in (0, 1)$, $\mathcal{M}(\mathcal{F}x, \mathcal{F}y, \Phi(t)) \geq \alpha$, where $\mathcal{M}(y, x, t) \geq \alpha$, and $\mathcal{M}(x, y, r) < \alpha$ for all $r < t_0$. Since $\mathcal{M}(x_{n_k}, x_{m_k}, r) < \alpha$ for all $r < t_0$, and $\mathcal{M}(x, x_{m_k-n_k}, L) \geq \alpha$, it follows that

$$\mathcal{M}(x_1, x_{m_k-n_k+1}, \Phi(L)) \geq \alpha.$$

Similarly, $\mathcal{M}(x_{n_k}, x_{m_k}, \Phi^{n_k}(L)) \geq \alpha$. As Φ is a comparison function, $\Phi^{n_k}(L) \leq t_0$ for large n_k . So,

$$\mathcal{M}(x_1, x_{m_k-n_k+1}, t_0) \geq \mathcal{M}(x_{n_k}, x_{m_k}, \Phi^{n_k}(L)) \geq \alpha.$$

This is a contradiction. Hence, the sequence $\{\mathcal{F}^n x\}$ is Cauchy. Since the sequence $\{\mathcal{F}^n x\}$ is Cauchy, and $\mathcal{X}$ is a complete fuzzy metric space, there exists $y \in \mathcal{X}$ such that the sequence $\{\mathcal{F}^n x\}$ converges to y . Hence, by Lemma 3.2, $y = \lim_{n \rightarrow \infty} \mathcal{F}^n x = \lim_{n \rightarrow \infty} \mathcal{F}^{n+1} x = \mathcal{F}y$.

Next, we demonstrate the existence and uniqueness of a fixed point for $\mathcal{F}$ in $\mathcal{X}$. Let $y, z \in \mathcal{X}$ be fixed points of $\mathcal{F}$. If $y \neq z$, then by the assumption that $\mathcal{F}$ is a large contraction, for every $\alpha \in (0, 1)$ there exist $0 < t_\alpha \leq s_\alpha < s'_\alpha$ such that

$$\mathcal{M}(z, y, t) < \alpha, \text{ for all } 0 < t \leq s'_\alpha, \text{ and } \mathcal{M}(z, y, t) = \mathcal{M}(\mathcal{F}z, \mathcal{F}y, t_\alpha) \geq \alpha.$$

This is a contradiction. $\square$

In what follows, we extend the concept of Φ -large contraction mappings from fuzzy metric spaces to classical metric spaces. Using the results obtained in the fuzzy case, we derive conditions under which any Φ -large contraction mapping considered in the classical metric framework also possesses a fixed point. This establishes a bridge between fixed-point theory in fuzzy and classical settings.

Definition 3.4. Let $(\mathcal{X}, d)$ be a metric space. A map $\mathcal{F} : \mathcal{X} \rightarrow \mathcal{X}$ is said to be a Φ -large contraction, if

- (i) $d(\mathcal{F}x, \mathcal{F}y) < d(x, y)$ for all $x, y \in \mathcal{X}$ with $x \neq y$, and
- (ii) for every $\epsilon > 0$, there exists a (1-dimensional) comparison function Φ such that

$$d(\mathcal{F}x, \mathcal{F}y) \leq \Phi(d(x, y)), \text{ for all } x, y \in \mathcal{X} \text{ with } d(x, y) \geq \epsilon.$$

Proposition 3.5. Let $(\mathcal{X}, d)$ be a complete metric space, and $\mathcal{F} : \mathcal{X} \rightarrow \mathcal{X}$ be a Φ -large contraction. Also, suppose there exist $x \in \mathcal{X}$ and $L > 0$ such that $d(x, \mathcal{F}^n x) \leq L$ for all $n \geq 1$. Then, $\mathcal{F}$ has a unique fixed point in $\mathcal{X}$.

Proof . We define a fuzzy metric $\mathcal{M}$ on $\mathcal{X}$ by

$$\mathcal{M}(x, y, t) = \begin{cases} 0, & t < d(x, y), \\ 1, & d(x, y) \leq t, \end{cases}$$

where $x, y \in \mathcal{X}$ and $t > 0$. Now, we show that $\mathcal{F}$ is a Φ -large contraction on the resulting fuzzy metric space $\mathcal{M}$.

- (i) If $x \neq y$, then for every $\alpha \in (0, 1)$ there exist $0 < t_\alpha = d(\mathcal{F}x, \mathcal{F}y) \leq s_\alpha < s'_\alpha < d(x, y)$ such that $\mathcal{M}(x, y, t) = 0 < \alpha$ for all $0 < t \leq s'_\alpha$, and $\mathcal{M}(\mathcal{F}x, \mathcal{F}y, t_\alpha) = 1 \geq \alpha$.
- (ii) Let $\epsilon > 0$. By condition (ii) of Definition 3.4, there exists a comparison function Φ such that $d(\mathcal{F}x, \mathcal{F}y) \leq \Phi(d(x, y))$ for all $x, y \in \mathcal{X}$ with $d(x, y) \geq \epsilon$. Suppose that $\alpha \in (0, 1)$, $\mathcal{M}(y, x, t) \geq \alpha$, and $\mathcal{M}(x, y, r) < \alpha$ for all $r < \epsilon$. Then, $d(x, y) \leq t$ and $d(x, y) \geq \epsilon$. Thus,

$$d(\mathcal{F}x, \mathcal{F}y) \leq \Phi(d(x, y)) \leq \Phi(t)$$

and $d(x, y) \geq \epsilon$. Therefore, $\mathcal{M}(\mathcal{F}x, \mathcal{F}y, \Phi(t)) = 1 \geq \alpha$.

This implies that $\mathcal{F}$ is a Φ -large contraction on the defined fuzzy metric space $\mathcal{M}$. By Theorem 3.3, $\mathcal{F}$ has a unique fixed point in $\mathcal{X}$. $\square$

Here, we introduce a special type of Φ -large contraction mappings, called large contraction mappings, in the framework of fuzzy metric spaces. This definition aims to generalize classical fixed-point results to fuzzy settings. We then prove that, under appropriate conditions, any such mapping has a unique fixed point in the context of fuzzy metric spaces.

Definition 3.6. Let $(\mathcal{X}, \mathcal{M}, \min)$ be a fuzzy metric space. A map $\mathcal{F} : \mathcal{X} \rightarrow \mathcal{X}$ is said to be a large contraction if for every $x, y \in \mathcal{X}$, the following statements are true.

- (i) If $x \neq y$, then for every $\alpha \in (0, 1)$ there exist $0 < t_\alpha \leq s_\alpha < s'_\alpha$ such that

$$\mathcal{M}(x, y, t) < \alpha, \text{ for all } 0 < t \leq s'_\alpha, \text{ and } \mathcal{M}(\mathcal{F}x, \mathcal{F}y, t_\alpha) \geq \alpha.$$

- (ii) For every $\epsilon > 0$, there exists $\delta < 1$ such that for all $\alpha \in (0, 1)$, $\mathcal{M}(\mathcal{F}x, \mathcal{F}y, \delta t) \geq \alpha$, where $\mathcal{M}(y, x, t) \geq \alpha$, and $\mathcal{M}(x, y, r) < \alpha$ for all $r < \epsilon$.

Theorem 3.7. Let $(\mathcal{X}, \mathcal{M}, \min)$ be a complete fuzzy metric space, and $\mathcal{F} : \mathcal{X} \rightarrow \mathcal{X}$ be a large contraction. Also, assume that $x \in \mathcal{M}$ and $L > 0$ exist such that for every $\alpha \in (0, 1)$, $\mathcal{M}(x, \mathcal{F}^n x, L) \geq \alpha$ for all $n \geq 1$. Then, $\mathcal{F}$ has a unique fixed point in $\mathcal{X}$.

Proof . Let $\delta < 1$. We define $\Phi : (0, \infty) \rightarrow (0, \infty)$ by $\Phi(t) = \delta t$ for all $t \in (0, \infty)$. By Theorem 3.3, $\mathcal{F}$ has a unique fixed point in $\mathcal{X}$. $\square$

Example 3.8. Suppose that $\mathcal{F} : [0, 1] \rightarrow [0, 1]$ is defined by $\mathcal{F}(x) = x - x^4/4$.

- (i) We have

$$\begin{aligned} |\mathcal{F}(x) - \mathcal{F}(y)| &\leq |x - x^4/4 - y + y^4/4| \\ &\leq |x - y| + |x^4/4 - y^4/4| \\ &< |x - y|, \text{ for all } x \neq y. \end{aligned}$$

- (ii) By Example 5 in [2],

$$|\mathcal{F}(x) - \mathcal{F}(y)| \leq |x - y| (1 - |x - y|^3/8).$$

Hence, $|\mathcal{F}(x) - \mathcal{F}(y)| \leq |x - y| (1 - \epsilon^3/8)$ with $|x - y| \geq \epsilon$. Put $\delta_\epsilon = 1 - \epsilon^3/8$. Then, $|\mathcal{F}(x) - \mathcal{F}(y)| \leq \delta_\epsilon |x - y|$ with $|x - y| \geq \epsilon$.

Thus, by Definition 3.4, $\mathcal{F}$ is a Φ -large contraction, where $\Phi_\epsilon(t) = \delta_\epsilon t$.

Now, we use comparison functions to introduce a new class of mappings, namely, Φ -large-Kannan contraction mappings, thus providing a generalization of Khan contraction mappings to fuzzy metric spaces. We then establish sufficient conditions ensuring that any such mapping admits a fixed point in the context of fuzzy metric spaces.

Definition 3.9. Let $(\mathcal{X}, \mathcal{M}, \min)$ be a fuzzy metric space. A map $\mathcal{F} : \mathcal{X} \rightarrow \mathcal{X}$ is said to be a Φ -large-Kannan contraction if for every $x, y \in \mathcal{X}$, the following statements are true.

- (i) If $x \neq y$, then for every $\alpha \in (0, 1)$ there exist $0 < t_\alpha \leq s_\alpha < s'_\alpha$ such that

$$\mathcal{M}(x, y, t) < \alpha, \text{ for all } 0 < t \leq s'_\alpha, \text{ and } \mathcal{M}(\mathcal{F}x, \mathcal{F}y, t_\alpha) \geq \alpha.$$

- (ii) For every $\epsilon > 0$, there exists a (2-dimensional) comparison function Φ such that for all $\alpha \in (0, 1)$, $\mathcal{M}(\mathcal{F}x, \mathcal{F}y, \Phi(t, s)) \geq \alpha$, where

$$\mathcal{M}(\mathcal{F}x, x, t) \geq \alpha, \mathcal{M}(y, \mathcal{F}y, s) \geq \alpha, \text{ and } \mathcal{M}(x, y, r) < \alpha, \text{ for all } r < \epsilon.$$

Theorem 3.10. Let $(\mathcal{X}, \mathcal{M}, \min)$ be a compact fuzzy metric space, and $\mathcal{F} : \mathcal{X} \rightarrow \mathcal{X}$ be a Φ -large-Kannan contraction. Also, assume that the function Ψ , defined by $\Psi(t) = \Phi(t, \dots, t)$ for every $t \in \mathbb{R}^+$, is continuous. Then, $\mathcal{F}$ has a unique fixed point in $\mathcal{X}$.

Proof . Let $x_0 \in \mathcal{X}$. If there exists $m \in \mathbb{N}$ such that $\mathcal{F}^m(x_0) = \mathcal{F}^{m+1}(x_0)$, then $\mathcal{F}^m x_0$ is a fixed point of $\mathcal{F}$. Now, assume that $\mathcal{F}^n x_0 \neq \mathcal{F}^{n+1} x_0$ for every $n \geq 1$. Suppose that $\alpha \in (0, 1)$, and

$$t_{m,n} = \inf\{t > 0 : \mathcal{M}(\mathcal{F}^m x_0, \mathcal{F}^n x_0, t) \geq \alpha\}, \text{ for all } m, n \in \mathbb{N} \cup \{0\}.$$

By (FM4), $\mathcal{M}(\mathcal{F}^m x_0, \mathcal{F}^n x_0, t_{m,n}) \geq \alpha$ for all $m, n \in \mathbb{N} \cup \{0\}$. Since $\mathcal{F}$ is a Φ -large-Kannan contraction, there exist $0 < t_\alpha \leq s_\alpha < s'_\alpha$ such that

$$\mathcal{M}(x_0, \mathcal{F}x_0, t) < \alpha, \text{ for all } 0 < t \leq s'_\alpha, \text{ and } \mathcal{M}(\mathcal{F}x_0, \mathcal{F}^2 x_0, t_\alpha) \geq \alpha.$$

Hence, $t_{1,2} \leq t_\alpha \leq s_\alpha < s'_\alpha \leq t_{0,1}$. Similarly, we obtain

$$0 \leq t_{n,n+1} \leq \dots \leq t_{2,3} \leq t_{1,2} \leq t_{0,1}.$$

Thus, the sequence $\{t_{n,n+1}\}$ is decreasing. Assume that $\lim_{n \rightarrow \infty} t_{n,n+1} = \epsilon$. Then, $t_{n,n+1} \geq \epsilon$. If $\epsilon > 0$, then $\mathcal{M}(\mathcal{F}^n x_0, \mathcal{F}^{n+1} x_0, r) < \alpha$ for every $r < \epsilon$. On the other hand, since $\mathcal{F}$ is a Φ -large-Kannan contraction, there exists a comparison function such that $\mathcal{M}(\mathcal{F}^{n+1} x_0, \mathcal{F}^{n+2} x_0, \Phi(t_{n,n+1}, t_{n+1,n+2})) \geq \alpha$, where $\mathcal{M}(\mathcal{F}^n x_0, \mathcal{F}^{n+1} x_0, t_{n,n+1}) \geq \alpha$, and $\mathcal{M}(\mathcal{F}^{n+1} x_0, \mathcal{F}^{n+2} x_0, t_{n+1,n+2}) \geq \alpha$. Then, $t_{n+1,n+2} \leq \Phi(t_{n,n+1}, t_{n+1,n+2})$. Since the sequence $\{t_{n,n+1}\}$ is decreasing, $t_{n+1,n+2} \leq \Phi(t_{n,n+1}, t_{n,n+1}) = \Psi(t_{n,n+1})$. Hence, $t_{n+1,n+2} \leq \Psi(t_{n,n+1})$. Similarly, we obtain

$$\begin{aligned} t_{n+1,n+2} &\leq \Psi(t_{n,n+1}) \\ &\leq \Psi^2(t_{n-1,n}) \\ &\vdots \\ &\leq \Psi^{n+1}(t_{0,1}). \end{aligned}$$

Since the sequence $\{\Psi^n(t)\}_{n=0}^\infty$ converges to zero for all $t \in \mathbb{R}^+$, it follows that $\lim_{n \rightarrow \infty} t_{n,n+1} = 0$. This is a contradiction.

Now, we show that the sequence $\{\mathcal{F}^n x_0\}$ is Cauchy. Otherwise, there exist subsequences $\{n_k\}$ and $\{m_k\}$, $\epsilon > 0$ and $\alpha \in (0, 1)$ such that $\mathcal{M}(\mathcal{F}^{m_k} x_0, \mathcal{F}^{n_k} x_0, \epsilon) < \alpha$ for all $k \in \mathbb{N}$. So, $\epsilon \leq t_{n_k, m_k}$ for all $k \in \mathbb{N}$. Since $\mathcal{F}$ is a Φ -large-Kannan contraction, $\mathcal{M}(\mathcal{F}^{m_k-1} x_0, \mathcal{F}^{n_k-1} x_0, r) < \alpha$ for all $r < \epsilon$, $\mathcal{M}(\mathcal{F}^{n_k} x_0, \mathcal{F}^{n_k-1} x_0, t_{n_k, n_k-1}) \geq \alpha$, and $\mathcal{M}(\mathcal{F}^{m_k} x_0, \mathcal{F}^{m_k-1} x_0, t_{m_k, m_k-1}) \geq \alpha$, it follows that $\mathcal{M}(\mathcal{F}^{m_k} x_0, \mathcal{F}^{n_k} x_0, \Phi(t_{n_k, n_k-1}, t_{m_k, m_k-1})) \geq \alpha$. Therefore, $t_{n_k, m_k} \leq \Phi(t_{n_k, n_k-1}, t_{m_k, m_k-1})$. Since the sequence $\{t_{n,n+1}\}$ is decreasing,

$$t_{n_k, m_k} \leq \Phi(t_{m_k, m_k-1}, t_{m_k, m_k-1}) = \Psi(t_{m_k, m_k-1}).$$

The assumption that Ψ is continuous implies $t_{n_k, m_k} \rightarrow 0$ as $k \rightarrow \infty$. This is a contradiction, which shows that the sequence $\{\mathcal{F}^n x_0\}$ is Cauchy. Since the sequence $\{\mathcal{F}^n x\}$ is Cauchy, and $\mathcal{X}$ is a complete fuzzy metric space, there exists $y \in \mathcal{X}$ such that the sequence $\{\mathcal{F}^n x_0\}$ converges to y . Hence, by Lemma 3.2, $y = \lim_{n \rightarrow \infty} \mathcal{F}^n x_0 = \lim_{n \rightarrow \infty} \mathcal{F}^{n+1} x_0 = \mathcal{F}y$.

Next, we demonstrate the existence and uniqueness of a fixed point for $\mathcal{F}$ in $\mathcal{X}$. Let $y, z \in \mathcal{X}$ be fixed points of $\mathcal{F}$, and assume that $y \neq z$. Since $\mathcal{F}$ is a large contraction, and $y \neq z$, for every $\alpha \in (0, 1)$ there exist $0 < t_\alpha \leq s_\alpha < s'_\alpha$ such that $\mathcal{M}(z, y, t) < \alpha$ for all $0 < t \leq s'_\alpha$, and $\mathcal{M}(z, y, t) = \mathcal{M}(\mathcal{F}z, \mathcal{F}y, t_\alpha) \geq \alpha$. This is a contradiction. $\square$

Next, we define Φ -large-Kannan contraction mappings in the setting of classical metric spaces. By applying the results obtained for fuzzy metric spaces and under certain conditions, we show that such mappings also possess fixed points in classical metric spaces. This creates a direct connection between fuzzy and classical frameworks in fixed-point theory.

Definition 3.11. Let $(\mathcal{X}, d)$ be a metric space. A map $\mathcal{F} : \mathcal{X} \rightarrow \mathcal{X}$ is said to be a Φ -large-Kannan contraction, if

- (i) $d(\mathcal{F}x, \mathcal{F}y) < d(x, y)$ for all $x, y \in \mathcal{X}$ with $x \neq y$, and
- (ii) for every $\epsilon > 0$ there exists a (2-dimensional) comparison function Φ such that

$$d(\mathcal{F}x, \mathcal{F}y) \leq \Phi(d(x, \mathcal{F}x), d(y, \mathcal{F}y)), \text{ for all } x, y \in \mathcal{X} \text{ with } d(x, y) \geq \epsilon.$$

Proposition 3.12. Let $(\mathcal{X}, d)$ be a complete metric space, and $\mathcal{F} : \mathcal{X} \rightarrow \mathcal{X}$ be a Φ -large-Kannan contraction. Moreover, assume that the function Ψ , defined by $\Psi(t) = \Phi(t, \dots, t)$ for every $t \in \mathbb{R}^+$, is continuous. Then, $\mathcal{F}$ has a unique fixed point in $\mathcal{X}$.

Proof . We define a fuzzy metric $\mathcal{M}$ on $\mathcal{X}$ by

$$\mathcal{M}(x, y, t) = \begin{cases} 0, & t < d(x, y), \\ 1, & d(x, y) \leq t, \end{cases}$$

where $x, y \in \mathcal{X}$ and $t > 0$. We now demonstrate that $\mathcal{F}$ constitutes a Φ -large-Kannan contraction on the fuzzy metric space $\mathcal{M}$ as defined.

- (i) If $x \neq y$, then for every $\alpha \in (0, 1)$ there exist $0 < t_\alpha = d(\mathcal{F}x, \mathcal{F}y) \leq s_\alpha < s'_\alpha < d(x, y)$ such that $\mathcal{M}(x, y, t) = 0 < \alpha$ for all $0 < t \leq s'_\alpha$, and $\mathcal{M}(\mathcal{F}x, \mathcal{F}y, t_\alpha) = 1 \geq \alpha$.
- (ii) Let $\epsilon > 0$. By condition (ii) of Definition 3.11, there exists a comparison function Φ such that $d(\mathcal{F}x, \mathcal{F}y) \leq \Phi(d(x, \mathcal{F}x), d(y, \mathcal{F}y))$ for all $x, y \in \mathcal{X}$ with $d(x, y) \geq \epsilon$. Suppose that $\alpha \in (0, 1)$, $\mathcal{M}(\mathcal{F}x, x, t) \geq \alpha$, $\mathcal{M}(y, \mathcal{F}y, s) \geq \alpha$, and $\mathcal{M}(x, y, r) < \alpha$ for all $r < \epsilon$. Then, $d(x, \mathcal{F}x) \leq t$, $d(y, \mathcal{F}y) \leq s$, and $d(x, y) \geq \epsilon$. Thus, $d(\mathcal{F}x, \mathcal{F}y) \leq \Phi(d(x, \mathcal{F}x), d(y, \mathcal{F}y)) \leq \Phi(t, s)$, and $d(x, y) \geq \epsilon$. Therefore, $\mathcal{M}(\mathcal{F}x, \mathcal{F}y, \Phi(t, s)) = 1 \geq \alpha$.

This implies that $\mathcal{F}$ is a Φ -large-Kannan contraction on the defined fuzzy metric space $\mathcal{M}$. By Theorem 3.10, $\mathcal{F}$ has a unique fixed point in $\mathcal{X}$. $\square$

Here, we introduce a special type of Φ -large-Kannan contraction mappings, namely, large-Kannan contraction mappings in fuzzy metric spaces. This concept extends Khan's classical contractions to the fuzzy context. Furthermore, we show that under certain conditions, these mappings have fixed points in fuzzy metric spaces.

Definition 3.13. Let $(\mathcal{X}, \mathcal{M}, \min)$ be a fuzzy metric space. A map $\mathcal{F} : \mathcal{X} \rightarrow \mathcal{X}$ is said to be a large-Kannan contraction if for every $x, y \in \mathcal{X}$, the following statements are true.

- (i) If $x \neq y$, then for every $\alpha \in (0, 1)$ there exist $0 < t_\alpha \leq s_\alpha < s'_\alpha$ such that

$$\mathcal{M}(x, y, t) < \alpha, \text{ for all } 0 < t \leq s'_\alpha, \text{ and } \mathcal{M}(\mathcal{F}x, \mathcal{F}y, t_\alpha) \geq \alpha.$$

- (ii) For every $\epsilon > 0$, there exists $\delta < 1/2$ such that for all $\alpha \in (0, 1)$, $\mathcal{M}(\mathcal{F}x, \mathcal{F}y, \delta(t + s)) \geq \alpha$, where $\mathcal{M}(\mathcal{F}x, x, t) \geq \alpha$, $\mathcal{M}(y, \mathcal{F}y, s) \geq \alpha$, and $\mathcal{M}(x, y, r) < \alpha$ for all $r < \epsilon$.

Theorem 3.14. Let $(\mathcal{X}, \mathcal{M}, \min)$ be a compact fuzzy metric space. Moreover, let $\mathcal{F} : \mathcal{X} \rightarrow \mathcal{X}$ be a large-Kannan contraction. Then, $\mathcal{F}$ has a unique fixed point in $\mathcal{X}$.

Proof . Let $\delta < 1/2$. We define $\Phi : (0, \infty)^2 \rightarrow (0, \infty)$ by $\Phi(t, s) = \delta(t + s)$ for all $t \in (0, \infty)$. By Theorem 3.10, $\mathcal{F}$ has a unique fixed point in $\mathcal{X}$. $\square$

Example 3.15. Let $D = \{x \in \mathbb{R} : |x| < 1/2\sqrt{3}\}$, and consider $\mathcal{F} : D \rightarrow D$ defined by

$$\mathcal{F}(x) = -x^3.$$

Now, we show that $\mathcal{F}$ is a Φ -large contraction on D . Suppose that $x, y \in D$, and $\epsilon > 0$. Then, $|x|, |y| < 1/2\sqrt{3}$. Hence, $|x| + |y| < 1/\sqrt{3}$. Therefore, $(|x| + |y|)^2 < 1/3$. Thus,

$$\begin{aligned} |x^2 + y^2 + xy| + (1/2)|x - y|^2 &\leq (3/2)x^2 + (3/2)y^2 + |xy| - xy \\ &\leq (3/2)x^2 + (3/2)y^2 + 2|xy| \\ &\leq (3/2)x^2 + (3/2)y^2 + 3|xy| \\ &\leq (3/2)(|x| + |y|)^2 \\ &\leq 1/2. \end{aligned}$$

Now, the following statements are true.

- (i) If $x \geq 0$, then $|x| = x \leq x + x^3 = x - (-x^3) = |x - \mathcal{F}(x)|$.
- (ii) If $x \leq 0$, then $|x| = -x \leq -x - x^3 = -x + (-x^3) = |x - \mathcal{F}(x)|$.

It follows that

$$\begin{aligned}
 |\mathcal{F}(x) - \mathcal{F}(y)| &= |x^3 - y^3| \\
 &= |x - y||x^2 + y^2 + xy| \\
 &\leq (|x| + |y|)|x^2 + y^2 + xy| \\
 &\leq (|x - \mathcal{F}(x)| + |y - \mathcal{F}(y)|)|x^2 + y^2 + xy| \\
 &\leq (|x - \mathcal{F}(x)| + |y - \mathcal{F}(y)|)((1 - |x - y^2|)/2).
 \end{aligned}$$

So, if $x, y \in D$ and $|x - y| \geq \epsilon$, then

$$|\mathcal{F}(x) - \mathcal{F}(y)| \leq (|x - \mathcal{F}(x)| + |y - \mathcal{F}(y)|)((1 - \epsilon^2)/2).$$

Hence, $\mathcal{F}$ is a Φ -large contraction on D , where

$$\Phi_\epsilon(t, s) = \delta_\epsilon(t + s) = ((1 - \epsilon^2)/2)(t + s), \text{ for all } t \in (0, \infty).$$

Definition 3.16. A map $\Phi : ((0, \infty))^n \rightarrow (0, \infty)$ is said to be an n -dimensional (s)-comparison function whenever the following conditions are satisfied.

- (i) If $0 \leq t_i \leq s_i$ for all $i = 1, \dots, n$, then $\Phi(t_1, \dots, t_n) \leq \Phi(s_1, \dots, s_n)$.
- (ii) $\sum 2^n \Psi^n(t)$ converges for every $t \in (0, \infty)$.
- (iii) $\Psi(t + s) \leq \Psi(t) + \Psi(s)$.

Here, $\Psi(t) = \Phi(t, \dots, t)$, for all $t \in (0, \infty)$.

Now, we propose a new and broader class of mappings, called extended Φ -large-Khan contraction mappings, as a generalization of Φ -large-Khan contractions in fuzzy metric spaces. These mappings are introduced to cover a wider range of nonlinear mappings. We show that, under certain conditions, such mappings possess fixed points in fuzzy metric spaces.

Definition 3.17. Let $(\mathcal{X}, \mathcal{M}, \min)$ be a fuzzy metric space. A map $\mathcal{F} : \mathcal{X} \rightarrow \mathcal{X}$ is said to be a extended Φ -large-Kannan contraction if for every $x, y \in \mathcal{X}$, the following statements are true.

- (i) If $x \neq y$, then for every $\alpha \in (0, 1)$ there exist $0 < t_\alpha \leq s_\alpha < s'_\alpha$ such that

$$\mathcal{M}(x, y, t) < \alpha, \text{ for all } 0 < t \leq s'_\alpha, \text{ and } \mathcal{M}(\mathcal{F}x, \mathcal{F}y, t_\alpha) \geq \alpha.$$

- (ii) For every $\epsilon > 0$, there exists a (5-dimensional) (s)-comparison function Φ such that for all $\alpha \in (0, 1)$,

$$\mathcal{M}(\mathcal{F}x, \mathcal{F}y, \Phi(t_1, t_2, t_3, t_4, t_5)) \geq \alpha,$$

where $\mathcal{M}(\mathcal{F}x, x, t_1) \geq \alpha$, $\mathcal{M}(y, \mathcal{F}y, t_2) \geq \alpha$, $\mathcal{M}(\mathcal{F}y, x, t_3) \geq \alpha$, $\mathcal{M}(\mathcal{F}x, y, t_4) \geq \alpha$, $\mathcal{M}(x, y, t_5) \geq \alpha$, and $\mathcal{M}(x, y, r) < \alpha$ for all $r < \epsilon$.

Theorem 3.18. Let $(\mathcal{X}, \mathcal{M}, \min)$ be a compact fuzzy metric space. Moreover, let $\mathcal{F} : \mathcal{X} \rightarrow \mathcal{X}$ be an extended Φ -large-Kannan contraction. Then, $\mathcal{F}$ has a unique fixed point in $\mathcal{X}$.

Proof . Assume that $x_0 \in \mathcal{X}$. If there exists an integer $m \in \mathbb{N}$ such that $\mathcal{F}^m(x_0) = \mathcal{F}^{m+1}(x_0)$, then $\mathcal{F}^m x_0$ is a fixed point of $\mathcal{F}$. Now, let $\mathcal{F}^n x_0 \neq \mathcal{F}^{n+1} x_0$ for all $n \geq 1$. Suppose that $\alpha \in (0, 1)$, and

$$t_{m,n} = \inf\{t > 0 : \mathcal{M}(\mathcal{F}^m x_0, \mathcal{F}^n x_0, t) \geq \alpha\}, \text{ for all } m, n \in \mathbb{N} \cup \{0\}.$$

By (FM4), $\mathcal{M}(\mathcal{F}^m x_0, \mathcal{F}^n x_0, t_{m,n}) \geq \alpha$ for all $m, n \in \mathbb{N} \cup \{0\}$. Since $\mathcal{F}$ is a extended Φ -large-Kannan contraction, there exist $0 < t_\alpha \leq s_\alpha < s'_\alpha$ such that $\mathcal{M}(x_0, \mathcal{F}x_0, t) < \alpha$ for all $0 < t \leq s'_\alpha$, and $\mathcal{M}(\mathcal{F}x_0, \mathcal{F}^2 x_0, t_\alpha) \geq \alpha$. Hence, $t_{1,2} \leq t_\alpha \leq s_\alpha < s'_\alpha \leq t_{0,1}$. Similarly, we obtain

$$0 \leq t_{n,n+1} \leq \dots \leq t_{2,3} \leq t_{1,2} \leq t_{0,1}.$$

Thus, the sequence $\{t_{n,n+1}\}$ is decreasing. Assume that $\lim_{n \rightarrow \infty} t_{n,n+1} = \epsilon$. Then, $t_{n,n+1} \geq \epsilon$. If $\epsilon > 0$, then $\mathcal{M}(\mathcal{F}^n x_0, \mathcal{F}^{n+1} x_0, r) < \alpha$ for all $r < \epsilon$.

On the other hand, since $\mathcal{F}$ is a extended Φ -large-Kannan contraction, there exists a comparison function such that

$$\mathcal{M}(\mathcal{F}^{n+1} x_0, \mathcal{F}^{n+2} x_0, \Phi(t_{n,n+1}, t_{n+1,n+2}, t_{n,n+1} + t_{n+1,n+2}, t_{n,n+1}, t_{n,n+1})) \geq \alpha,$$

where $\mathcal{M}(\mathcal{F}^n x_0, \mathcal{F}^{n+1} x_0, t_{n,n+1}) \geq \alpha$, $\mathcal{M}(\mathcal{F}^{n+1} x_0, \mathcal{F}^{n+2} x_0, t_{n+1,n+2}) \geq \alpha$, and

$$\begin{aligned} \mathcal{M}(\mathcal{F}^n x_0, \mathcal{F}^{n+2} x_0, t_{n,n+1} + t_{n+1,n+2}) &\geq \min \left\{ \begin{array}{l} \mathcal{M}(\mathcal{F}^n x_0, \mathcal{F}^{n+1} x_0, t_{n,n+1}), \\ \mathcal{M}(\mathcal{F}^{n+1} x_0, \mathcal{F}^{n+2} x_0, t_{n+1,n+2}) \end{array} \right\} \\ &\geq \alpha. \end{aligned}$$

Then, $t_{n+1,n+2} \leq \Phi(t_{n,n+1}, t_{n+1,n+2}, t_{n,n+1} + t_{n+1,n+2}, t_{n,n+1}, t_{n,n+1})$. Since the sequence $\{t_{n,n+1}\}$ is decreasing, $t_{n+1,n+2} \leq \Psi(2t_{n,n+1}) \leq 2\Psi(t_{n,n+1})$. Hence, $t_{n+1,n+2} \leq 2\Psi(t_{n,n+1})$. Similarly, we obtain

$$\begin{aligned} t_{n+1,n+2} &\leq 2\Psi(t_{n,n+1}) \\ &\leq 2^2\Psi^2(t_{n-1,n}) \\ &\vdots \\ &\leq 2^{n+1}\Psi^{n+1}(t_{0,1}). \end{aligned} \tag{3.1}$$

Since the sequence $\{2^n \Psi^n(t)\}_{n=0}^\infty$ converges to zero for every $t \in (0, \infty)$, it follows that

$$\lim_{n \rightarrow \infty} t_{n,n+1} = 0.$$

This is a contradiction. Now, we show that the sequence $\{\mathcal{F}^n x_0\}$ is Cauchy. Otherwise, there exist subsequences $\{n_k\}$ and $\{m_k\}$, $\epsilon > 0$, and $\alpha \in (0, 1)$ such that $\mathcal{M}(\mathcal{F}^{m_k} x_0, \mathcal{F}^{n_k} x_0, \epsilon) < \alpha$ for all $k \in \mathbb{N}$. So, $\epsilon \leq t_{n_k, m_k}$ for all $k \in \mathbb{N}$. Thus,

$$\begin{aligned} \mathcal{M}\left(\mathcal{F}^{j_k} x_0, \mathcal{F}^{r_k} x_0, \sum_{i=r_k}^{j_k-1} t_{i_k, i_k+1}\right) &\geq \min \left\{ \begin{array}{l} \mathcal{M}(\mathcal{F}^{r_k} x_0, \mathcal{F}^{r_k+1} x_0, t_{r_k, r_k+1}), \\ \vdots \\ \mathcal{M}(\mathcal{F}^{j_k-1} x_0, \mathcal{F}^{j_k} x_0, t_{j_k-1, j_k}) \end{array} \right\} \\ &\geq \alpha, \text{ for all } j_k > r_k. \end{aligned}$$

Since $\mathcal{F}$ is a Φ -large-Kannan contraction, and

$$\begin{aligned} \mathcal{M}(\mathcal{F}^{m_k-1} x_0, \mathcal{F}^{n_k-1} x_0, r) &< \alpha \text{ for all } r < \epsilon, \\ \mathcal{M}(\mathcal{F}^{n_k} x_0, \mathcal{F}^{n_k-1} x_0, t_{n_k, n_k-1}) &\geq \alpha, \\ \mathcal{M}(\mathcal{F}^{m_k} x_0, \mathcal{F}^{m_k-1} x_0, t_{m_k, m_k-1}) &\geq \alpha, \\ \mathcal{M}\left(\mathcal{F}^{m_k} x_0, \mathcal{F}^{n_k-1} x_0, \sum_{i=n_k-1}^{m_k-1} t_{i_k, i_k+1}\right) &\geq \alpha, \\ \mathcal{M}\left(\mathcal{F}^{n_k} x_0, \mathcal{F}^{m_k-1} x_0, \sum_{i=n_k}^{m_k-2} t_{i_k, i_k+1}\right) &\geq \alpha, \\ \mathcal{M}\left(\mathcal{F}^{m_k-1} x_0, \mathcal{F}^{n_k-1} x_0, \sum_{i=n_k-1}^{m_k-2} t_{i_k, i_k+1}\right) &\geq \alpha, \end{aligned}$$

it follows that

$$\mathcal{M}\left(\mathcal{F}^{m_k} x_0, \mathcal{F}^{n_k} x_0, \Psi\left(\sum_{i=n_k-1}^{m_k-1} t_{i_k, i_k+1}\right)\right) \geq \alpha.$$

Therefore,

$$t_{n_k, m_k} \leq \Psi \left(\sum_{i=n_k-1}^{m_k-1} t_{i_k, i_k+1} \right) \leq \sum_{i=n_k-1}^{m_k-1} \Psi(t_{i_k, i_k+1}).$$

By (3.1),

$$t_{n_k, m_k} \leq \sum_{i=n_k-1}^{m_k-1} \Psi(t_{i_k, i_k+1}) \leq \sum_{i=n_k-1}^{m_k-1} 2^{i_k} \Psi^{i_k+1}(t_{0,1}).$$

Since $\sum 2^n \Psi^n(t)$ converges for every $t \in (0, \infty)$, it follows that $t_{n_k, m_k} \rightarrow 0$ as $k \rightarrow \infty$. This contradiction implies that the sequence $\{\mathcal{F}^n x_0\}$ is Cauchy. Since the sequence $\{\mathcal{F}^n x_0\}$ is Cauchy, and $\mathcal{X}$ is a complete fuzzy metric space, there exists $y \in \mathcal{X}$ such that the sequence $\{\mathcal{F}^n x_0\}$ converges to y . Hence, by Lemma 3.2, $y = \lim_{n \rightarrow \infty} \mathcal{F}^n x_0 = \lim_{n \rightarrow \infty} \mathcal{F}^{n+1} x_0 = \mathcal{F}y$.

Next, we demonstrate the existence and uniqueness of a fixed point for $\mathcal{F}$ in $\mathcal{X}$. Let $y, z \in \mathcal{X}$ be fixed points of $\mathcal{F}$ and $y \neq z$. Since $\mathcal{F}$ is a large contraction and $y \neq z$, for every $\alpha \in (0, 1)$ there exist $0 < t_\alpha \leq s_\alpha < s'_\alpha$ such that $\mathcal{M}(z, y, t) < \alpha$ for all $0 < t \leq s'_\alpha$, and $\mathcal{M}(z, y, t) = \mathcal{M}(\mathcal{F}z, \mathcal{F}y, t_\alpha) \geq \alpha$. This is a contradiction. $\square$

Next, we define extended Φ -large-Kannan contraction mappings in classical metric spaces. By utilizing the corresponding fuzzy version, we establish sufficient conditions under which these mappings also admit fixed points in the classical setting. This provides a unified framework for fixed-point results in fuzzy and classical metric spaces.

Definition 3.19. Let $(\mathcal{X}, d)$ be a metric space. A map $\mathcal{F} : \mathcal{X} \rightarrow \mathcal{X}$ is said to be an extended Φ -large-Kannan contraction, if the following statements are true.

- (i) $d(\mathcal{F}x, \mathcal{F}y) < d(x, y)$ for all $x, y \in \mathcal{X}$ with $x \neq y$.
- (ii) For every $\epsilon > 0$, there exists a (5-dimensional) (s)-comparison function Φ such that

$$d(\mathcal{F}x, \mathcal{F}y) \leq \Phi(d(x, \mathcal{F}x), d(y, \mathcal{F}y), d(x, \mathcal{F}y), d(y, \mathcal{F}x), d(x, y)) \text{ for all } x, y \in \mathcal{X},$$

with $d(x, y) \geq \epsilon$.

Proposition 3.20. Let $(\mathcal{X}, d)$ be a complete metric space, and $\mathcal{F} : \mathcal{X} \rightarrow \mathcal{X}$ be an extended Φ -large-Kannan contraction. Then, $\mathcal{F}$ has a unique fixed point in $\mathcal{X}$.

Proof . We define a fuzzy metric $\mathcal{M}$ on $\mathcal{X}$ by

$$\mathcal{M}(x, y, t) = \begin{cases} 0, & t < d(x, y), \\ 1, & d(x, y) \leq t, \end{cases}$$

where $x, y \in \mathcal{X}$ and $t > 0$. We now demonstrate that $\mathcal{F}$ constitutes a extended Φ -large-Kannan contraction on the fuzzy metric space $\mathcal{M}$ as defined.

- (i) If $x \neq y$, then for every $\alpha \in (0, 1)$ there exist $0 < t_\alpha = d(\mathcal{F}x, \mathcal{F}y) \leq s_\alpha < s'_\alpha < d(x, y)$ such that $\mathcal{M}(x, y, t) = 0 < \alpha$ for all $0 < t \leq s'_\alpha$, and $\mathcal{M}(\mathcal{F}x, \mathcal{F}y, t_\alpha) = 1 \geq \alpha$.
- (ii) Let $\epsilon > 0$. By condition (ii) of Definition 3.11, there exists a comparison function Φ such that

$$d(\mathcal{F}x, \mathcal{F}y) \leq \Phi(d(x, \mathcal{F}x), d(y, \mathcal{F}y), d(x, \mathcal{F}y), d(y, \mathcal{F}x), d(x, y)), \text{ for all } x, y \in \mathcal{X},$$

with $d(x, y) \geq \epsilon$. Suppose that $\alpha \in (0, 1)$,

$$\begin{aligned} \mathcal{M}(\mathcal{F}x, x, t_1) &\geq \alpha, \\ \mathcal{M}(y, \mathcal{F}y, t_2) &\geq \alpha, \\ \mathcal{M}(\mathcal{F}y, x, t_3) &\geq \alpha, \\ \mathcal{M}(\mathcal{F}x, y, t_4) &\geq \alpha, \\ \mathcal{M}(x, y, t_5) &\geq \alpha, \\ \mathcal{M}(x, y, r) &< \alpha, \text{ for all } r < \epsilon. \end{aligned}$$

Then, $d(\mathcal{F}x, x) \leq t_1$, $d(y, \mathcal{F}y) \leq t_2$, $d(\mathcal{F}y, x) \leq t_3$, $d(\mathcal{F}x, y) \leq t_4$, $d(x, y) \leq t_5$ and $d(x, y) \geq \epsilon$. Thus,

$$\begin{aligned} d(\mathcal{F}x, \mathcal{F}y) &\leq \Phi(d(x, \mathcal{F}x), d(y, \mathcal{F}y), d(x, \mathcal{F}y), d(y, \mathcal{F}x), d(x, y)) \\ &\leq \Phi(t_1, t_2, t_3, t_4, t_5), \end{aligned}$$

and $d(x, y) \geq \epsilon$. Therefore, $\mathcal{M}(\mathcal{F}x, \mathcal{F}y, \Phi(t_1, t_2, t_3, t_4, t_5)) = 1 \geq \alpha$.

This implies that $\mathcal{F}$ is an extended Φ -large-Kannan contraction on the defined fuzzy metric space $\mathcal{M}$. By Theorem 3.18, $\mathcal{F}$ has a unique fixed point in $\mathcal{X}$. $\square$

Finally, to unify and extend the previous results, we introduce a more general form of large-Khan contraction mappings in fuzzy metric spaces. This generalized definition encompasses the previously defined mappings as special cases. We then prove that such a mapping has a unique fixed point in fuzzy metric spaces.

Definition 3.21. A map $\Phi : ((0, \infty))^n \rightarrow (0, \infty)$ is said to be an n -dimensional weak comparison function whenever the following conditions are satisfied.

- (i) If $0 \leq t_i \leq s_i$ for all $i = 1, \dots, 5$, then $\Phi(t_1, \dots, t_n) \leq \Phi(s_1, \dots, s_n)$.
- (ii) $\Phi(t_1, \dots, t_n) \leq t_1 + \dots + t_n$ for all $(t_1, \dots, t_n) \in (0, \infty)^n$.

Definition 3.22. let $\sum$ denote the class of real-valued functions satisfying the following condition:

$$\sum = \left\{ f : (0, 1) \rightarrow [0, 1/2) : \lim_{n \rightarrow \infty} t_n = 0, \text{ when } \lim_{n \rightarrow \infty} f(t_n) = 1/2 \right\}.$$

Theorem 3.23. Let $(\mathcal{X}, \mathcal{M}, \min)$ be a complete fuzzy metric space. Moreover, let $\mathcal{F} : \mathcal{X} \rightarrow \mathcal{X}$ be a self-mapping for which the following statements are true.

- (i) If $x \neq y$, then for every $\alpha \in (0, 1)$ there exist $0 < t_\alpha \leq s_\alpha < s'_\alpha$ such that

$$\mathcal{M}(x, y, t) < \alpha, \text{ for all } 0 < t \leq s'_\alpha, \text{ and } \mathcal{M}(\mathcal{F}x, \mathcal{F}y, t_\alpha) \geq \alpha.$$

- (ii) For every $\epsilon > 0$, there exist a (2-dimensional) weak comparison function Φ and, $f_\epsilon \in \sum$ such that for all $\alpha \in (0, 1)$,

$$\mathcal{M}(\mathcal{F}x, \mathcal{F}y, f_\epsilon(u) \Phi(t, s)) \geq \alpha,$$

where $\mathcal{M}(y, x, u) \geq \alpha$, $\mathcal{M}(\mathcal{F}x, x, t) \geq \alpha$, $\mathcal{M}(y, \mathcal{F}y, s) \geq \alpha$, and $\mathcal{M}(x, y, r) < \alpha$ for all $r < \epsilon$.

Then, $\mathcal{F}$ has a unique fixed point x_0 in $\mathcal{X}$.

Proof . Let $x_0 \in \mathcal{X}$. If there exists an integer $m \in \mathbb{N}$ such that $\mathcal{F}^m(x_0) = \mathcal{F}^{m+1}(x_0)$, then $\mathcal{F}$ admits a fixed point. Henceforth, we assume that $\mathcal{F}^n(x_0) \neq \mathcal{F}^{n+1}(x_0)$ for all $n \in \mathbb{N}$, and define the iterative sequence $\{x_n\}$ by $x_n = \mathcal{F}^n(x_0)$ for all $n \in \mathbb{N}$. For a fixed $\alpha \in (0, 1)$, define

$$t_n = \inf \{ t > 0 : \mathcal{M}(x_n, x_{n+1}, t) \geq \alpha \}, \text{ for all } n \in \mathbb{N}.$$

By the definition of t_n , there exist positive numbers $0 < t_\alpha < s_\alpha < s'_\alpha$, such that $\mathcal{M}(x_{n-1}, x_n, t) < \alpha$ for all $0 < t \leq s'_\alpha$, and $\mathcal{M}(x_n, x_{n+1}, t_\alpha) \geq \alpha$. Consequently, $t_n \leq t_\alpha \leq s_\alpha < s'_\alpha \leq t_{n-1}$, which shows that the sequence $\{t_n\}$ is monotone decreasing. Since $t_n \geq 0$ for every n , it follows that $\{t_n\}$ converges. Therefore, there exists $\epsilon \geq 0$ such that $\lim_{n \rightarrow \infty} t_n = \epsilon$.

Assume that $\epsilon > 0$. Then, by the hypothesis, there exist a weak comparison function Φ and a function $f_\epsilon \in \sum$ such that $\mathcal{M}(x_n, x_{n+1}, f_\epsilon(u) \Phi(t, s)) \geq \alpha$, whenever $\mathcal{M}(x_n, x_{n-1}, u) \geq \alpha$, $\mathcal{M}(x_{n+1}, x_n, t) \geq \alpha$, $\mathcal{M}(x_{n-1}, x_n, s) \geq \alpha$, and $\mathcal{M}(x_n, x_{n-1}, r) < \alpha$ for every $r < \epsilon$. Since $\epsilon \leq t_n$ for every $n \in \mathbb{N}$, it follows that $\mathcal{M}(x_n, x_{n-1}, r) < \alpha$ for all $r < \epsilon$ and all $n \in \mathbb{N}$. Therefore, by the assumed contractive condition, $\mathcal{M}(x_n, x_{n+1}, f_\epsilon(u) \Phi(t, s)) \geq \alpha$, whenever $\mathcal{M}(x_n, x_{n-1}, u) \geq \alpha$, $\mathcal{M}(x_{n+1}, x_n, t) \geq \alpha$, $\mathcal{M}(x_{n-1}, x_n, s) \geq \alpha$. In particular, choosing $u = s = t_{n-1}$ and $t = t_n$, and using the facts that $\mathcal{M}(x_n, x_{n-1}, t_{n-1}) \geq \alpha$, $\mathcal{M}(x_{n+1}, x_n, t_n) \geq \alpha$, $\mathcal{M}(x_{n-1}, x_n, t_{n-1}) \geq \alpha$, we obtain

$$\mathcal{M}(x_n, x_{n+1}, f_\epsilon(t_{n-1}) \Phi(t_n, t_{n-1})) \geq \alpha.$$

By the definition of t_n , it follows that $t_n \leq f_\epsilon(t_{n-1}) \Phi(t_n, t_{n-1})$. Moreover, by the monotonicity of Φ ,

$$t_n \leq f_\epsilon(t_{n-1}) \Phi(t_n, t_{n-1}) \leq f_\epsilon(t_{n-1}) \Phi(t_{n-1}, t_{n-1}) \leq 2t_{n-1}.$$

Consequently, $t_n/2t_{n-1} \leq f_\epsilon(t_{n-1}) < 1/2$. Passing to the limit as $n \rightarrow \infty$, we deduce that $\lim_{n \rightarrow \infty} f_\epsilon(t_{n-1}) = 1/2$. By the defining property of f_ϵ , this yields $\lim_{n \rightarrow \infty} t_{n-1} = 0$, contradicting the assumption that $\lim_{n \rightarrow \infty} t_n = \epsilon > 0$. Hence, the only possible value of the limit is $\epsilon = 0$.

We next prove that the sequence $\{x_n\}$ is Cauchy in $\mathcal{X}$. Suppose, to the contrary, that $\{x_n\}$ is not a Cauchy sequence. Then there exist $\epsilon > 0$, $\alpha \in (0, 1)$, and two subsequences $\{x_{n_k}\}$ and $\{x_{m_k}\}$ with $m_k > n_k$ such that $\mathcal{M}(x_{n_k}, x_{m_k}, \epsilon) < \alpha$, for all $k \in \mathbb{N}$. In particular, $x_{n_k} \neq x_{m_k}$, and hence $x_{n_k-1} \neq x_{m_k-1}$. By condition (i), it follows that $\mathcal{M}(x_{n_k-1}, x_{m_k-1}, \epsilon) < \alpha$. Therefore, there exists a function $f_\epsilon \in \Sigma$ such that $\mathcal{M}(x_{n_k}, x_{m_k}, f_\epsilon(u) \Phi(t, s)) \geq \alpha$, whenever $\mathcal{M}(x_{m_k-1}, x_{n_k-1}, u) \geq \alpha$, $\mathcal{M}(x_{n_k}, x_{n_k-1}, t) \geq \alpha$, $\mathcal{M}(x_{m_k-1}, x_{m_k}, s) \geq \alpha$. Since $f_\epsilon(u) \leq 1/2$, it follows that

$$\mathcal{M}(x_{n_k}, x_{m_k}, (1/2) \Phi(t, s)) \geq \mathcal{M}(x_{n_k}, x_{m_k}, f_\epsilon(u) \Phi(t, s)) \geq \alpha,$$

whenever $\mathcal{M}(x_{m_k-1}, x_{n_k-1}, u) \geq \alpha$, $\mathcal{M}(x_{n_k}, x_{n_k-1}, t) \geq \alpha$, $\mathcal{M}(x_{m_k-1}, x_{m_k}, s) \geq \alpha$. By the definition of ϵ , this yields $\epsilon \leq (1/2) \Phi(t, s)$, provided that $\mathcal{M}(x_{n_k}, x_{n_k-1}, t) \geq \alpha$, $\mathcal{M}(x_{m_k-1}, x_{m_k}, s) \geq \alpha$. Taking $t = t_{n_k}$ and $s = t_{m_k}$, we deduce that $\epsilon \leq \frac{1}{2}$, $\Phi(t_{n_k}, t_{m_k}) \leq \frac{1}{2}$, $\Phi(t_{n_k}, t_{n_k}) \leq t_{n_k}$. Passing to the limit as $k \rightarrow \infty$ and using the fact that $t_{n_k} \rightarrow 0$, we arrive at $0 < \epsilon \leq 0$, which is impossible. This contradiction proves that $\{x_n\}$ is a Cauchy sequence in $\mathcal{X}$. Finally, since $\mathcal{X}$ is complete, there exists a point $z_0 \in \mathcal{X}$ such that $\lim_{n \rightarrow \infty} x_n = z_0$.

Now, we show that z_0 is a fixed point for $\mathcal{F}$. First, note that

$$\begin{aligned} \mathcal{M}(z_0, \mathcal{F}z_0, r + u) &\geq \min\{\mathcal{M}(z_0, x_{n+1}, r), \mathcal{M}(x_{n+1}, \mathcal{F}z_0, u)\} \\ &= \min\{\mathcal{M}(z_0, \mathcal{F}x_n, r), \mathcal{M}(\mathcal{F}x_n, \mathcal{F}z_0, u)\}. \end{aligned} \quad (3.2)$$

If $x_n \neq z_0$, then there exist $\epsilon > 0$ and f_ϵ such that for every $\alpha \in (0, 1)$,

$$\mathcal{M}(\mathcal{F}x_n, \mathcal{F}z_0, f_\epsilon(u) (t + s)) \geq \alpha,$$

where $\mathcal{M}(x_n, z_0, u) \geq \alpha$, $\mathcal{M}(\mathcal{F}x_n, x_n, t) \geq \alpha$, $\mathcal{M}(z_0, \mathcal{F}z_0, s) \geq \alpha$, and $\mathcal{M}(x_n, z_0, r) < \alpha$ for all $r < \epsilon$. Thus, for every $\alpha \in (0, 1)$, $\mathcal{M}(\mathcal{F}x_n, \mathcal{F}z_0, (1/2) \Phi(t, s)) \geq \mathcal{M}(\mathcal{F}x_n, \mathcal{F}z_0, f_\epsilon(u) \Phi(t, s)) \geq \alpha$, where $\mathcal{M}(x_n, z_0, u) \geq \alpha$, $\mathcal{M}(\mathcal{F}x_n, x_n, t) \geq \alpha$, $\mathcal{M}(z_0, \mathcal{F}z_0, s) \geq \alpha$, and $\mathcal{M}(x_n, z_0, r) < \alpha$ for all $r < \epsilon$. Therefore, for every $\alpha \in (0, 1)$, $\mathcal{M}(\mathcal{F}x_n, \mathcal{F}z_0, (1/2) \Phi(t, s)) \geq \alpha$, where $\mathcal{M}(\mathcal{F}x_n, x_n, t) \geq \alpha$, and $\mathcal{M}(z_0, \mathcal{F}z_0, s) \geq \alpha$. Hence, $\mathcal{M}(\mathcal{F}x_n, \mathcal{F}z_0, (1/2) \Phi(t, s)) \geq \min\{\mathcal{M}(\mathcal{F}x_n, x_n, t), \mathcal{M}(z_0, \mathcal{F}z_0, s)\}$. So, by (3.2),

$$\begin{aligned} \mathcal{M}(z_0, \mathcal{F}z_0, r + (1/2) \Phi(t, s)) &\geq \min \left\{ \begin{array}{l} \mathcal{M}(z_0, \mathcal{F}x_n, r), \\ \mathcal{M}(\mathcal{F}x_n, \mathcal{F}z_0, (1/2) \Phi(t, s)) \end{array} \right\} \\ &\geq \min \left\{ \begin{array}{l} \mathcal{M}(z_0, \mathcal{F}x_n, r), \\ \mathcal{M}(\mathcal{F}x_n, x_n, t), \\ \mathcal{M}(z_0, \mathcal{F}z_0, s) \end{array} \right\}. \end{aligned}$$

Let $t_\alpha = \inf\{t > 0 : \mathcal{M}(z_0, \mathcal{F}z_0, t) \geq \alpha\}$, $\alpha \in (0, 1)$, and $\xi > 0$. Since $\{x_n\}$ is Cauchy and $\lim x_n = z_0$, there exists $N \in \mathbb{N}$ such that $\min\{\mathcal{M}(z_0, \mathcal{F}x_n, \xi), \mathcal{M}(\mathcal{F}x_n, x_n, \xi)\} \geq \alpha$ for every $n \geq N$. Then,

$$\begin{aligned} \mathcal{M}\left(z_0, \mathcal{F}z_0, \xi + \frac{1}{2} \Phi(\xi, t_\alpha)\right) &\geq \min \left\{ \begin{array}{l} \mathcal{M}(z_0, \mathcal{F}x_n, \xi), \\ \mathcal{M}(\mathcal{F}x_n, x_n, \xi), \\ \mathcal{M}(z_0, \mathcal{F}z_0, t_\alpha) \end{array} \right\} \\ &\geq \alpha. \end{aligned}$$

Hence, $t_\alpha \leq \xi + (1/2) \Phi(\xi, t_\alpha) \leq \xi + (1/2) (\xi + t_\alpha)$. Thus, $(1/2) t_\alpha \leq (3/2) \xi$. As $\xi \rightarrow 0$, we obtain $t_\alpha = 0$. Therefore, $\mathcal{F}z_0 = z_0$.

Next, we demonstrate the existence and uniqueness of a fixed point for $\mathcal{F}$ in $\mathcal{X}$. Suppose that $\mathcal{F}$ has a fixed point $z_1 \in \mathcal{X}$ such that $z_0 \neq z_1$. Then, for every $\alpha \in (0, 1)$ there exist $0 < t_\alpha \leq s_\alpha < s'_\alpha$ such that $\mathcal{M}(z_0, z_1, t) < \alpha$ for all $0 < t \leq s'_\alpha$, and $\mathcal{M}(\mathcal{F}z_0, \mathcal{F}z_1, t_\alpha) \geq \alpha$. This contradiction implies that $z_0 = z_1$. $\square$

Theorem 3.24. Let $(\mathcal{X}, \mathcal{M}, \min)$ be a complete fuzzy metric space. Moreover, let $\mathcal{F} : \mathcal{X} \rightarrow \mathcal{X}$ be a self-mapping for which the following statements are true.

(i) If $x \neq y$, then for every $\alpha \in (0, 1)$ there exist $0 < t_\alpha \leq s_\alpha < s'_\alpha$ such that

$$\mathcal{M}(x, y, t) < \alpha, \text{ for all } 0 < t \leq s'_\alpha, \text{ and } \mathcal{M}(\mathcal{F}x, \mathcal{F}y, t_\alpha) \geq \alpha.$$

(ii) For every $\epsilon > 0$, there exists $f_\epsilon \in \Sigma$ such that for all $\alpha \in (0, 1)$, $\mathcal{M}(\mathcal{F}x, \mathcal{F}y, f_\epsilon(u) (t + s)) \geq \alpha$, where $\mathcal{M}(y, x, u) \geq \alpha$, $\mathcal{M}(\mathcal{F}x, x, t) \geq \alpha$, $\mathcal{M}(y, \mathcal{F}y, s) \geq \alpha$, and $\mathcal{M}(x, y, r) < \alpha$ for all $r < \epsilon$.

Then, $\mathcal{F}$ has a unique fixed point x_0 in $\mathcal{X}$.

Proof . We define $\Phi : (0, \infty)^2 \rightarrow (0, \infty)$ by $\Phi(t, s) = t + s$ for all $t, s \in (0, \infty)$. By Theorem 3.23, $\mathcal{F}$ has a unique fixed point in $\mathcal{X}$. $\square$

4 Conclusion

In this paper, we introduced and studied new classes of contractive mappings in both fuzzy and classical metric spaces. By employing comparison functions, we proposed generalizations of large and Khan contractions, including Φ -large contractions, Φ -large Khan contractions, and their extended forms. We established the existence of fixed points for all these classes under certain conditions, thereby not only broadening the scope of fixed-point theory in fuzzy metric spaces, but also strengthening the connection between classical and fuzzy frameworks. The significance of these results lies in providing a unified and comprehensive structure for the study of various types of contractive mappings, which can serve as a solid foundation for future research in mathematical analysis, integral equations, and dynamical systems. Hence, the present work offers valuable insights and potential directions for both theoretical advancements and practical applications within the field of fixed-point theory.

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