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Continuous time portfolio optimization

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Abstract

This paper presents dynamic portfolio model based on the Merton's optimal investment-consumption model, which combines dynamic synthetic put option using risk-free and risky assets. This paper is extended version of methodological paper published by Yuan Yao in 2012. Because of the long history of the development of foreign financial market, with a variety of financial derivatives, the study on theory or empirical analysis of portfolio insurance focused on how best can portfolio strategies be used in minimizing risk and market volatility. In this paper, stock and risk-free assets are used to replicate options and to establish a new dynamic model to analyze the implementation of the dynamic process of investors' actions using dynamic replication strategy. Our results show that investors' optimal strategies of portfolio are not dependent on their wealth, but are dependent on market risk and this new methodology is broaden in compare to paper of Yuan Yao (2012).

Keywords: Portfolio; Investment Strategy; Dynamic Optimization.
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1. Introduction and preliminaries

In the past two decades, alongside with globalization and development of international finance, financial innovation are employed to avoid financial risks and improve their competitiveness. In the early 1980s, a new hedging tool based on financial derivatives and portfolio strategy became popular in financial markets. Investors used new types of hedging tools, which may affect the entire financial system stability. There are two cases, one is the chain reaction of the collapse of Barings Bank in 1995, the other is the outbreak of the sub-prime crisis in 2007 that led to the global proliferation

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and amplification of financial risk, whose root cause is the credit risk of excessive use of financial derivatives.

The problem of optimal investment-consumption has been an area of active research in the last few decades. Samuelson [20] considered a discrete-time consumption-investment model with the objective of maximizing the overall expected utility of consumption. Using dynamic stochastic programming approach, he succeeded in obtaining the optimal decision for the consumption-investment model. Merton [13] extended the model of Samuelson [20] to a continuous-time framework and used stochastic optimal control methodology to obtain the optimal portfolio strategy [15]. Rubinstein and Leland [18] put forward the concept of portfolio insurance. Grossman and vila [11] gave a specific definition of hedging and shifting the risk by financial derivative instruments such as options, futures or simulated options. The appraisal of portfolio was more affirmative, and the research focused on how best can minimizing risk of stock market and how to utilize hedge. Literatures (Rubinstein and Leland [18]; Cox and Leland [8]; Cox and Huang [7]) proposed the model of portfolio which is only deformation of optimal control model of Merton [14]; with an ultimate goal of ensuring that the end wealth is no less than a preset value. After the US stock market crashed in October 1987, questions surrounding portfolio strategy emerged, that is whether the implementation of such strategy would decrease the market volatility and how to effectively use the strategy. The results showed that the implementation of portfolio insurance using options influenced the increased in market liquidity, transaction costs and subsequent market volatility increase (Harris [12]; Grossman [10]; Campbell et al., [5]). Studies showed the increasing market volatility was not included by using of options to minimizing risks, but was included by the uncertainty of the model itself [4]. Some researches found the relations among portfolio, benefits and risk of investors in high-risk market conditions. Some analyzed the problem of the minimum cost of portfolio strategies, as well as the application of lattice theory in portfolio (Aliprantis et al., [1]; Polyrakis [17]). Vanden et al., [21] analyzed portfolio insurance strategies by diversifying market volatility and structural changes. Zimmerer and Meyer [24] studied the optimization of the CPPI strategy. Nicole and Asma [16] and Annaert et al., [2] proved the portfolio insurance is a stochastic dominance strategy with limitation using random process theory. Balder et al., [3] studied the CPPI strategy in the conditions of discrete-time. Chi et al,. [6] studied the problem of portfolio insurance using VaR-based futures markets. Zhai and Yan [23] analyzed the dynamic insurance with the constraints of EaR.

Development of foreign financial market and a variety of financial derivatives stimulate the study on theory and empirical analysis of portfolio which focused on how to use portfolio strategies to avoid risk and how to affect market volatility. This paper, shows the use of stock and risk-free assets to achieve optimized portfolio and establishes a new dynamic model to analyze the implementation of the dynamic process and investors actions to market risk and stock returns. First, this paper establishes a dynamic portfolio model in continuous-framework model by using the technology of dynamic replication options with the lower limit in the final level of wealth. Secondly, the investor's personal inter-phase dynamic portfolio decision-making problem is changed into a static utility maximization problem. Finally, this paper attains the best optimal portfolio strategy in the level of participants optimal asset and the similarities and differences between the optimal investment portfolio model and the optimal investment strategy of optimal investment and consumption of the Merton model [14] are compared.

2. Model of Portfolio

In this paper, with the assumptions of principle of no arbitrage and a complete market, based on optimal consumption model and portfolio strategy in a continuous Merton model [14], a model of dynamic portfolio is created using dynamic replication of put option.

2.1. Complete Market Hypothesis

The complete market model assumes under the condition of continuous-time is as follows [19].

1) Investment exists in a time period $[0, T]$, $T < \infty$, investors will consume and invest in any (consecutive) time points.

2) Uncertainty of Market is described by a complete probability space $\{\Omega, \mathcal{F}, P\}$, which Ω is sample space, $\mathcal{F}$ is σ-field of subsets on Ω and P is probability measure.

3) Information structure generated by the Brownian motion on the definition of the above probability space, is

$$\mathcal{F} = \{\mathcal{F}_t\} = \sigma\{W(u)|0 \leq u \leq t\}$$

where $\mathcal{F}_0$=Ω, $\mathcal{F}_T$=F.

4) Suppose there are two financial assets in the market, one is a risk-free asset, $\frac{dB(t)}{B(t)} = r(t)dt, B(0) = \overline{B}$, the other is risk asset with price $S(t)$ following the Brownian motion.

$$\frac{dS(t)}{S(t)} = \mu(t)dt + \sigma(t)dW(t), S(0) = \overline{S}$$

where the risk-free interest rate $r(t)$, expected rate of return of risk assets $\mu(t)$; and the transient volatility is adapted to $\mathcal{F}_t$ and uniformly bounded in $(t, \omega) \in [0, T] \times \Omega$; $dW(t)$ is a standard Wiener process, the process of discounted price is $d\tilde{S} = \dfrac{S(t)}{B(t)}$.

5) The decision-making variables which investors control are two:

1. the consumption for each moment, that is, the consumption process $C(t)$,
2. the trading process or portfolio process $\theta(t)$ (representing the number of assets);

the number of risk-free assets is $\theta_0(t)$, the number of risk assets is $\theta_1(t)$, and $\theta(t)$ is a predictable two-dimensional random variable, where $\int_0^T |\theta_i(t)|^2 dt < \infty, i = 0, 1$.

6) The initial wealth of investor is $X(0) = x = \theta_0(0)\overline{B} + \theta_1(0)\overline{S}$. θ is the trading strategy with initial wealth $X(0)$, where $x > 0$, followed by the process of wealth of investor $X(t) = \theta_0(t)B(t) + \theta_1(t)S(t)$. where $\int_0^T C(t)dt < \infty$, the process of wealth generated by the trading strategy and consumption is:

$$X(t) = X(0) + \int_0^t \theta_0(s)dB(s) + \int_0^t \theta_1(s)dS(s) - \int_0^t C(s)ds \ , \ \forall t \in [0, T] \tag{2.1}$$

The above equation demonstrates that the total wealth of the investor at time t equals to the sum of wealth and payoff of risk assets and risk-free assets, minus the total accumulating consumption. The proportions of investment in risk-free assets and risk assets are $w_0(t), w_1(t)$, when the investment strategy is defined by using the relative proportion of the assets invested in the total personal wealth.

$$w_0(t) = \frac{\theta_0(t)B(t)}{X(t)} \ , \ w_1(t) = \frac{\theta_1(t)S(t)}{X(t)} \tag{2.2}$$

where $w_0(t), w_1(t)$ is the portfolio process $\theta_i(t), i = 1, 2$, and $\int_0^T w_i^2(t)dt < \infty, i = 0, 1$ is a self-financing strategy, according to (2.1), the simple process of wealth is achieved:

$$dX(t) = \theta_0(t)dB(t) + \theta_1(t)dS(t) - C(t)dt \tag{2.3}$$

$$\begin{cases} dX(t) = (r(t)X(t) - C(t))dt + X(t)w_1(t)(\mu(t) - r(t)dt + \sigma(t)dW(t)) \\ X(0) = x \end{cases} \tag{2.4}$$

Given a positive initial wealth x, a self-financing strategy $[\theta(t), C(t)]$ or $[w(t), C(t)]$ produce a non negative wealth process $X(t) \geq 0, \forall t \in [0, T]$, which is an admissible strategy. The entire set of feasible strategies (w, C) based on x is denoted $A(x)$.

7) The market price of risk in complete market $K(t) = \sigma^{-1}[\mu(t) - r(t)]$ has a unique solution which is bounded. There are Radon-Nikodym derivative

$$\xi(t) = exp(-\int_0^t K(s)dW(s) - \frac{1}{2}\int_0^t \|K(s)\|^2 ds)$$

and the only measure of risk-neutral (martingale probability measure Q) according to Camron-Martin-Girsanov theorem [9], where $\xi(t) = \frac{dQ}{dP}|F_t$, and the risky asset under the martingale probability measure Q is

$$\frac{dS(t)}{S(t)} = r(t)dt + \sigma(t)d\tilde{W}(t) \tag{2.5}$$

After discounting, it becomes

$$\frac{d\tilde{S}(t)}{\tilde{S}(t)} = \sigma(t)d\tilde{W}(t) \tag{2.6}$$

and hence the price of risky asset $\tilde{S}$ converges to a martingale Q.

8) The state price density process

$$H(t) = \gamma(t)\xi(t) = exp(-\int_0^t K(s)dW(s) - \int_0^t (r(s) + \frac{\|K(s)\|^2}{2})ds) \tag{2.7}$$

is positive, continuous and progressively predictable, and is the unique solution of the following stochastic differential equation.

$$\begin{cases} dH(t) = -H(t)(r(t)dt + K(t)dW(t)) \\ H(0) = 1 \end{cases} \tag{2.8}$$

where $\gamma(t) = exp(-\int_0^t r(s)ds)$ is the discount factor.

9) There are a variety of options matching with risky asset in the market.

Theorem 1. Given the initial wealth $x \geq 0$ and an admissible self-financing strategy $\{\theta(t), C(t)\} \in A(x)$, the corresponding process of wealth meets

$$E(\int_0^t H(s)C(s)ds + H(t)X(t)) \leq x, \quad \forall t \in [0, T]$$

M is a non-negative random variable and adapted to F_t, $C(t)$ is a consumption process, $t \in [0, T]$, where

$$x = E(\int_0^t H(s)C(s)ds + H(t)M) < \infty$$

Thus there is a portfolio process $\theta(t), t \in [0, T]$, where $\{\theta(t), C(t)\} \in A(x)$, and the corresponding wealth process satisfies.

2.2. Portfolio Market Model

The optimal portfolio model is one that to maximizes the total expected utility and the final wealth, that is,

$$\max_{A'} E(\int_0^t U(C(s))ds + U(X(T))) \tag{2.9}$$

Thus, an optimal strategy is solved,

$$[\theta(t), C(t)] \in A^{'}(x),\ A^{'}(x) = ((\theta, C) \in A(x),\ E(\int_0^t U(C(s))^{-}ds + U(X(T)))^{-} < \infty) \tag{2.10}$$

where $a^{-} = \max(-a, 0)$, which demonstrates a weak and integrable condition.

The variables of dynamic trading strategies $\theta(t)$ include the number of risk-free assets, risky assets and the put option whose underlying is the risky assets with investment in portfolio.

In order to simplify the process of analysis, the consumption $C(t)$ during the period of investment is zero, and the variables of trading strategy controlled is $\theta(t)$. Because of dynamic replication strategy, the premium of options is zero. Based on these descriptions, the portfolio model is expressed as

$$\max_{(\theta,0)\in A'(x)} E(U(X(T))) \tag{2.11}$$

According to (2.1), the process of wealth generated by the portfolio trading strategy is

$$X(t) = X(0) + \int_0^t \theta_0(s)dB(s) + \int_0^t \theta_1(s)dS(s) + \int_0^t \theta_2(s)dP(s), \qquad \forall t \in [0, T] \tag{2.12}$$

Investors hold one risky asset and one put option P (the exercise price is K which is equal to the initial value of underlying assets), since they invest in a portfolio. Then $\theta_1(t) = \theta_2(t)$, and (2.12) becomes

$$X(t) = X(0) + \int_0^t \theta_0(s)dB(s) + \int_0^t \theta_1(s)dS(s) + \int_0^t \theta_1(s)dP(s), \qquad \forall t \in [0, T] \tag{2.13}$$

That is the total wealth of investors at the time t equal to the initial wealth plus the payoff of the investment in risk-free assets and risky assets as well as put option. If $\{\theta(t), 0\}$ is self-financing, the change of wealth are from the change of prices and the risk-free rate, the process of wealth is expressed as

$$dX(t) = \theta_0(t)dB(t) + \theta_1(t)dS(t) + \theta_1(t)dP(t) \tag{2.14}$$

Under the condition of no-arbitrage, we can replicate a put option by investing in the long position of risky assets $\Delta = \frac{\partial P}{\partial S}$ and a short position of risk-free assets; (2.14), then becomes

$$dX(t) = (\theta_0(t) - \theta_1(t))dB(t) + \theta_1(t)(1 + \frac{\partial P}{\partial S})dS(t) \tag{2.15}$$

ordering

$$\nu_0(t) = \theta_0(t) - \theta_1(t), \nu_1(t) = \theta_1(t)(1 + \frac{\partial P}{\partial S}) \tag{2.16}$$

then, (2.15) becomes

$$dX(t) = \nu_0(t)dB(t) + \nu_1(t)dS(t) \tag{2.17}$$

$\{\nu(t), 0\}$ is the set of investment strategy which includes investments in risk-free assets and risky assets, according to dynamic replication of options under the condition of no arbitrary equilibrium.

Comparing (2.17) and (2.3), the model of portfolio is similar to the general model of investment and consumption, because they have the same objective function which is maximizing the utility function of the wealth of investors. However the investment strategies $\{\theta(t), 0\}$ which include investment in risk-free assets, risky assets as well as the put option, is now transformed to $\{\nu(t), 0\}$ which only include investment in risk-free assets and risky assets. So the model of portfolio is

$$\max_{(\nu,0)\in A'(x)} E(U(X(T))) \tag{2.18}$$

where the process of wealth is (2.17). Similarly, according to Theorem 1, there is $\{\nu(t), 0\} \in A(x)$, then $X(T) = M$. The most important difference between our model and the models of literatures [18, 7, 14] is to add a restriction to final level of wealth.

3. Optimization of Portfolio Model

In the continuous-time conditions, the optimization of portfolio model consist of two steps: the first step is that the decision-making problem of the investor's personal inter-phase dynamic portfolio is changed into a static utility maximization problem, then optimize the wealth level of the investor; the second step is that the corresponding optimal strategy is obtained with martingale representation theorem.

3.1. Deformation of the Merton Model

Portfolio model (2.18) is similar to the investment-consumption model in micro-finance, and the investment strategy is changed from $\{\theta(t), 0\}$ of the investment-consumption model into $\{\nu(t), 0\}$ of the portfolio model. Then

$$\nu_0(t) = \theta_0(t) - \theta_1(t),$$
$$\nu_1(t) = \theta_1(t)(1 + \frac{\partial P}{\partial S})$$

According to optimization theory, we define

$$M(x) = \{M \geq 0, \quad F-predictable, \quad E(H(T)M)) \leq x, \quad E(U(M^-)) < \infty\}$$

is predictable.

$M(x)$ is the set of all final wealth which is generated by a feasible trading strategy with the initial wealth $y \in [0, x]$, and $E(U(M^-)) < \infty$.To optimize the portfolio model (2.18) and get the optimal final wealth $X(t)$, we can maximize all the random variables of $M(x)$, which is achieved with the martingale probability measure Q:

$$\max \quad \max_{M\in M(x)} \quad E(U(M)) \tag{3.1}$$

$$s.t. \qquad E^Q(\gamma(T)M) = E(\gamma(T)\xi(T)M) = x$$

As a result, the portfolio model (2.18) becomes the static optimization of model (3.1).

3.2. Optimization of New Model

If $\overline{M}$ is the optimal solution to the equation (3.1), $\{\nu(t), 0\} \in A(x)$ with $X(T) = \overline{M}$ must be achieved. To find a trading strategy with $X(T) = \overline{M}$, we construct the Lagrange function:

$$\begin{cases} L(M, \lambda) = E(U(M) - \lambda(H(T)M - x)), \\ \lambda > 0, \end{cases} \tag{3.2}$$

We calculate derivatives of M, λ in equation (3.2).

$$L_M(M, \lambda) = E(U^{'}(M) - \lambda(H(T)) = 0 \Rightarrow U^{'}(M) = \lambda(H(T)) \tag{3.3}$$

$$L_\lambda(M, \lambda) = E(H(T)M) - x = 0 \Rightarrow E(H(T)M) = x \tag{3.4}$$

Because the utility function is strictly monotonic decreasing, there is an inverse function $I = (U^{'})^{-1}$ which makes the optimal level of final wealth is

$$M = I[\lambda H(T)] \tag{3.5}$$

To substitute equation (3.5) into (3.4).

$$E(H(T)I(\lambda H(T))) = x = K(\lambda) \tag{3.6}$$

Ordering

$$\Gamma(x) = K^{-1}(x) \tag{3.7}$$

According to equation (3.5), the optimal solution is

$$\overline{M} = I(\Gamma(x)H(T)) \tag{3.8}$$

Because of

$$H(T) = \gamma(T)\xi(T) = exp(-\int_0^T K(t)dW(t) - \int_0^T (r(t) + \tfrac{\|K(t)\|^2}{2})dt)$$

the optimal solution is

$$\overline{M} = I(\Gamma(x)exp(-\int_0^T K(t)dW(t) - \int_0^T (r(t) + \tfrac{\|K(t)\|^2}{2})dt))$$

3.3. Analysis of the Optimal Portfolio Strategy

To find the solution of optimal portfolio strategy, from (2.2) the value of risk-free assets and risky assets which the investors invest in portfolio is

$$w_0(t) = 1 - w_1(t), w_1(t) = \frac{\nu_1(t)S(t)}{X(t)} \tag{3.9}$$

where $\{\nu(t), 0\}$ is the investment strategy set investing in risk-free assets and risky assets using dynamic replicating options method under no-arbitrage equilibrium analysis. As the wealth process of the self-financing portfolio process $\nu(t)$ is expressed as in equation (2.4), using the state price function, there is

$$H(t)X^{x,\overline{M},0}(t) = E(\int_t^T H(s)0ds + H(T)\overline{M}|F_t) \geq 0 \tag{3.10}$$

The integral 0 indicated the consumption process. The optimal portfolio process is $\{w(t), 0\}$. Using Ito's theorem, there is

$$x \leq x + \int_0^T H(t)X^{x,\overline{M},0}(t)(w_1(t)\sigma(t) - K(t))dW(t), \tag{3.11}$$

As the investor participates in portfolio, the smallest earning of the investor at the end is x,

$$x = x + \int_0^T H(t)X^{x,\overline{M},0}(t)(w_1(t)\sigma(t) - K(t))dW(t)$$

and thus,

$$H(t)X^{x,\overline{M},0}(t)(w_1(t)\sigma(t) - K(t))dW(t) = 0, \forall t \in [0,T] \tag{3.12}$$

Because $H(t)X^{x,\overline{M},0}(t)$ is always positive, so

$$w_1(t) = \frac{K(t)}{\sigma(t)} = \frac{\mu(t) - r(t)}{\sigma^2(t)} \tag{3.13}$$

After dynamic replicating option, we can obtain the investment strategy set for the model (3.1); the investment strategy set $\{\nu(t), 0\}$ for risk-free assets and risky assets is

$$\nu_1(t) = \frac{X(t)w_1}{S(t)}, \nu_0(t) = \frac{X(t)(1 - w_1)}{B(t)} \tag{3.14}$$

If the solution of the portfolio model is obtained through dynamic replication option, then the investors proportion of investment in risky assets, upon investment, is similar to that of optimal investment consumption model.
Since $H(t)$ is the solution of

$$\begin{cases} dH(t) = -H(t)(r(t)dt + K(t)dW(t)), \\ H(0) = 1 \end{cases} \tag{3.15}$$

and satisfies equation (3.10), the process of consumption $C(t)$ is always 0,

$$H^{-1}(t) = exp(\int_0^t K(s)dW(s) + \int_0^t (r(s) + \frac{\|K(s)\|^2}{2})ds) \tag{3.16}$$

is the process of wealth accompanied by the optimal trading strategy $\{w(t), 0\}$.
Comparing $\{\theta(t), 0\}$ with $\{\nu(t), 0\}$, we can obtain the trading strategy $\{\theta(t), 0\}$ of the portfolio problem (10) which includes investments in risk-free assets, risky assets as well as options underlying risk assets.

$$\theta_1(t) = \frac{X(t)w_1}{S(t)(1 + \frac{\partial P}{\partial S})} \tag{3.17}$$

$$\theta_0(t) = \frac{X(t)(1 - w_1)}{B(t)} + \frac{X(t)w_1}{S(t)(1 + \frac{\partial P}{\partial S})} \tag{3.18}$$

Comparing (3.17), (3.18) with (3.14), there are similarities and differences between the optimal portfolio model and the optimum investment strategy of the optimum investment consumption model. First, the former set of optimal strategies is divided into three parts: investment in risk-free assets, risky assets and options; the latter is divided into the two parts: investments in risk-free assets and risky assets. Second, the share of investment in risk-free assets $\theta_0(t)$ in the optimal investment strategy of the portfolio model includes two parts. One is the investment in risk-free assets (the first part of the right of equation (3.18)), the other is the investment in risky assets which replicates options (the latter part of the right of equation (3.18)). The sum is less than share of investment in risk-free assets of the optimal investment and consumption model. The share of investment in risky assets is more than that of the optimal investment and consumption model by comparing the denominator of equation (3.17) with that of equation (3.9), which the denominator of $\theta_1(t)$ is less than denominator of $\nu_1(t)$. Because Δ is always less than 1, which is the same to the result of replicating one option by investing in the long of risky assets Δ and the short of risk-free assets in the analysis of no-arbitrage equilibrium. In order to implement the portfolio strategy the investor must invest more Δ in the risky assets.

Finally, if the risk-free interest rate $r(t)$, the expected return rate of risky assets $\mu(t)$ and the instantaneous volatility rate $\sigma(t)$ are constant, the trading strategy $\{\theta(t), 0\}$ of portfolio model (2.11), after substituting (3.17) and (3.18) becomes

$$\theta_1(t) = \frac{X(t)}{S(t)(1+\frac{\partial P}{\partial S})}\frac{\mu - r}{\sigma^2} \tag{3.19}$$

$$\theta_0(t) = \frac{X(t)}{B(t)}(1 - \frac{\mu - r}{\sigma^2}) + \frac{X(t)w_1}{S(t)(1+\frac{\partial P}{\partial S})}\frac{\mu - r}{\sigma^2} \tag{3.20}$$

Therefore, to achieve a positive initial wealth x, the investment in risky assets and options are both $(1+\dfrac{\partial P}{\partial S})^{-1}\dfrac{\mu - r}{\sigma^2}$. Notice that this is independent to the level of wealth, which is decided by the risk premium (the ratio of the difference between the average return rate of risky assets and the return rate of risk-free assets to volatility rate) and the amount of Δ invested in risky assets to replicate portfolio strategy.

4. Conclusion

The new portfolio model of this paper is similar to the investment consumption model, that is the maximization of the investor's utility function. The advantage is to get rid of restrictions on the final wealth. This paper developed the portfolio strategy by replicating put option with risk-free assets and risky assets in the entire investment process. Through optimization of new dynamic portfolio model, the share of investment in risk-free assets of the optimal investment strategy is less than the share for the general model.

The proportion investing in risky assets and options of portfolio which is determined by the risk premium and Δ is independent of the level of wealth. It indicates the demand of investors for portfolio is independent of the wealth, but dependent on the market risk. In the other words, the higher of the market risk, the greater the demand for portfolio. The conclusions of this paper reflect the function of avoiding the risk of the portfolio strategy.

The implementation of portfolio strategy is a complicated systems engineering. It is used to avoid and manage the market risk and to face potential risks because of the special character of the strategy itself. This paper forms a basis for further research on portfolio as based on dynamic portfolio model and its optimal strategy, apart from providing some guidance for theoretical and practical strategy in financial market.

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