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Solvability of infinite system of nonlinear singular integral equations in the $C(I \times I, c)$ space and modified semi-analytic method to find a closed-form of solution

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Abstract

In this article, we discuss about solvability of infinite systems of singular integral equations with two variables in the Banach sequence space $C(I \times I, c)$ by applying measure of noncompactness and Meir-Keeler condensing operators. By presenting an example, we have illustrated our results. For validity of the results we introduce a modified semi-analytic method in the case of two variables to make an iteration algorithm to find a closed-form of solution for the above problem. The numerical results show that the produced sequence for approximating the solution of example is in the c space with a high accuracy.

Keywords: Measure of noncompactness; Infinite systems of singular integral equations; Meir-Keeler condensing operators; Fixed point theorem; Modified homotopy perturbation.
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1. Introduction

In nonlinear analysis the theory of infinite systems of differential or integral equations plays a very important role. This theory has many applications in the theory of branching process, the theory of neural nets and etc.

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The solvability of in infinite systems of equations have been discussed by many authors in Banach spaces (we refer [2, 3, 14, 16, 17, 20, 30, 36]).

In the year 1930, Kuratowski [24] first introduced and studied the measure of noncompactness. For different types of measure of noncompactness we refer [11] for the reader. Measures of noncompactness are useful tools which are widely used in fixed point theory, differential equations, functional equations, integral and integro-differential equations, and optimization etc (see [12, 27]). Many authors have been solved the various infinite system of equations by applying the measure of noncompactness (see [4, 5, 7, 8, 9, 10, 18, 19, 28, 29]).

Throughout the article we consider $I=[0,T],\ T>0$. Suppose E_1 is a real Banach space with the norm $\|\ .\ \|$. Let $B(x_0,d_1)$ be a closed ball in E_1 centered at x_0 and with radius d_1. If X_1 is a nonempty subset of E_1 then by $\bar{X}_1$ and ConvX_1 we denote the closure and convex closure of X_1. Moreover, let $\mathcal{M}_{E_1}$ denote the family of all nonempty and bounded subsets of E_1 and $\mathcal{N}_{E_1}$ its subfamily consisting of all relatively compact sets. The following axiomatic definition of a measure of noncompactness was introduced in [11].

Definition 1.1. A function $\mu_1:\mathcal{M}_{E_1}\to\mathbb{R}_+$ is called a measure of noncompactness if it satisfies the following conditions:

(i) the family $\ker\mu_1=\{X_1\in\mathcal{M}_{E_1}:\mu_1(X_1)=0\}$ is nonempty and $\ker\mu_1\subset\mathcal{N}_{E_1}$.
(ii) $X_1\subset Y_1 \implies \mu_1(X_1)\le\mu_1(Y_1)$.
(iii) $\mu_1\left(\bar{X}_1\right)=\mu_1(X_1)$.
(iv) $\mu_1(ConvX_1)=\mu_1(X_1)$.
(v) $\mu_1(\lambda X_1+(1-\lambda)Y_1)\le\lambda\mu_1(X_1)+(1-\lambda)\mu_1(Y_1)$ for $\lambda\in[0,1]$.
(vi) if $X_n^1\in\mathcal{M}_{E_1},\ X_n^1=\bar{X}_n^1,\ X_{n+1}^1\subset X_n^1$ for $n=1,2,3,\ldots$ and $\lim\limits_{n\to\infty}\mu_1(X_n^1)=0$, then $\bigcap\limits_{n=1}^{\infty}X_n^1$ is nonempty.

The family $\ker\mu_1$ is said to be the *kernel of measure of noncompactness* μ_1.
A measure μ_1 is said to be the sublinear if it satisfies the following conditions:

(1) $\mu_1(\lambda X_1)=|\lambda|\,\mu_1(X_1)$ for $\lambda\in\mathbb{R}$.
(2) $\mu_1(X_1+Y_1)\le\mu_1(X_1)+\mu_1(Y_1)$.

A sublinear measure of noncompactness μ_1 satisfying the condition:

$$\mu_1(X_1\cup Y_1)=\max\{\mu_1(X_1),\mu_1(Y_1)\}$$

and such that $\ker\mu_1=\mathcal{N}_{E_1}$ is said to be regular.
For a nonempty and bounded subset S of a metric space X_1, the Kuratowski measure of noncompactness is defined as

$$\alpha(S)=\inf\left\{\delta>0:S\subset\bigcup_{i=1}^{n}S_i,\ \text{diam}(S_i)\le\delta \text{ for } 1\le i\le n, n\in\mathbb{N}\right\},$$

where diam(S_i) denotes the diameter of the set S_i, that is,

$$\text{diam}(S_i)=\sup\{d(x,y):x,y\in S_i\}.$$

The Hausdorff measure of noncompactness for a bounded set S is defined by

$$\chi(S)=\inf\{\epsilon>0:S \text{ has finite } \epsilon-\text{net in } X_1\}.$$

Definition 1.2. *[6]* Let G_1 and G_2 be two Banach spaces and let μ_1 and μ_2 be arbitrary measures of noncompactness on G_1 and G_2, respectively. An operator f from G_1 to G_2 is called a (μ_1, μ_2)-condensing operator if it is continuous and $\mu_2 (f(D)) < \mu_1(D)$ for every set $D \subset G_1$ with compact closure.

Remark 1.3. If $G_1 = G_2$ and $\mu_1 = \mu_2 = \mu$, then f is called a μ-condensing operator.

The contractive maps and the compact maps are condensing if we take as measures of noncompactness the diameter of a set and the indicator function of a family of non-relatively compact sets, respectively (see [6]). In 1969, Meir and Keeler[26] proved the following interesting fixed point theorem, which is a generalization of the Banach contraction principle.

Definition 1.4. [26] Let (X, d) be a metric space. Then a mapping T on X is said to be a Meir-Keeler contraction if for any $\epsilon > 0$, there exists $\delta > 0$ such that

$$\epsilon \leq d(x, y) < \epsilon + \delta \implies d(Tx, Ty) < \epsilon,\ \forall x, y \in X.$$

Theorem 1.5. [26] Let (X, d) be a complete metric space. If $T : X \to X$ is a Meir-Keeler contraction, then T has a unique fixed point.

In [4], the following results are given, which are very useful in our study.

Definition 1.6. *[4] Let C be a nonempty subset of a Banach space E and let μ be an arbitrary measure of noncompactness on E. We say that an operator $T : C \to C$ is a Meir-Keeler condensing operator if for any $\epsilon > 0$, there exists $\delta > 0$ such that*

$$\epsilon \leq \mu(X) < \epsilon + \delta \implies \mu(T(X)) < \epsilon$$

for any bounded subset X of C.

Theorem 1.7. *[4] Let C be a nonempty, bounded, closed and convex subset of a Banach space E and let μ be an arbitrary measure of noncompactness on E. If $T : C \to C$ is a continuous and Meir-Keeler condensing operator, then T has at least one fixed point and the set of all fixed points of T in C is compact.*

2. Measure of noncompactness in sequence spaces

In this article, we establish the existence of solution of infinite systems of integral equations in two variables in the sequence space $C(I \times I, c)$ by using Meir-Keeler condensing operators. We explain the results with the help of simple example.

In the Banach space $(c, \| . \|_c)$ the Hausdorff measure of noncompactness can't be expressed in simple rule but we have an equivalent measure of noncompactness in c can be formulated as follows (see [11]):

$$\mu_c(\bar{V}) = \lim_{n\to\infty}\left[\sup_{z\in\bar{V}}\left(\sup_{k\geq n} \mid z_k - \lim_{m\to\infty} z_m \mid\right)\right], \tag{2.1}$$

where $z = (z_i)_{i=1}^{\infty} \in c$ and $\bar{V} \in \mathcal{M}_c$.

Let us denote by $C(I \times I, c)$ the space of all continuous functions on $I \times I$ with values in c. Then $C(I \times I, c)$ is also a Banach space with norm $\| x(t, s) \|_{C(I\times I,c)} = \sup \{\| x(t, s) \|_c : t, s \in I\}$ where $x(t, s) \in C(I \times I, c)$.

For any non-empty bounded subset $\hat{E}$ of $C(I\times I,c)$ and $t,s\in I$, let $\hat{E}(t,s)=\left\{x(t,s):x\in\hat{E}\right\}$. Now, using (2.1), we conclude that the measure of noncompactness for $\hat{E}\subset C(I\times I,c)$ can be defined by

$$\mu_{C(I\times I,c)}(\hat{E})=\sup\left\{\mu_c(\hat{E}(t,s)):t,s\in I\right\}.$$

In this article, the existence of solution of the following infinite systems of nonlinear singular integral equations with two variables will be studied

$$z_i(t,s)=H_i\left(t,s,z(t,s)\right)+F_i\left(t,s,z(t,s),\int\limits_0^s\int\limits_0^t\frac{u_i\left(t,s,v,w,z(v,w)\right)}{(t-v)^\alpha(s-w)^\beta}dvdw\right),\tag{2.2}$$

where $z(t,s)=(z_i(t,s))_{i=1}^\infty\in E$, $(t,s)\in I\times I$ and $z_i(t,s)\in C(I\times I,\mathbb{R})$ for all $i\in\mathbb{N}$ and $\alpha,\beta\in(0,1)$. $C(I\times I,\mathbb{R})$ denotes the Banach space of all real continuous functions on $I\times I$ with norm $\| z \|=\sup\{| z(t,s) |: t,s\in I\}$ and E is some Banach sequence space $(E,\| \,.\, \|)$. Assume that

(i) $F_i: I\times I\times C(I\times I,c)\times\mathbb{R}\to\mathbb{R}\,(i\in\mathbb{N})$ are continuous functions. Also there exists continuous functions $\hat{A}_i:I\times I\to\mathbb{R}_+$ and $B_i:I\times I\to\mathbb{R}_+$ such that for all $i\in\mathbb{N}$,

$$F_i\left(t,s,z(t,s),l(t,s)\right)=\hat{A}_i(t,s)z_i(t,s)+B_i(t,s)l(t,s)$$

where $z(t,s)=(z_i(t,s))_{i=1}^\infty\in C(I\times I,c), z_i(t,s)\in C(I\times I,\mathbb{R})$ for all $i\in\mathbb{N}$ and $l:I\times I\to\mathbb{R}$.

(ii) $u_i: I\times I\times I\times I\times C(I\times I,c)\to\mathbb{R}$ $(i\in\mathbb{N})$ are continuous. Moreover,

$$U_i=\sup\left\{|u_i\left(t,s,v,w,z(v,w)\right)|:t,s,v,w\in I,z(v,w)\in C(I\times I,c)\right\}<\infty.$$

Also we assume that $\sup\limits_i U_i=U$ and $\lim\limits_{i\to\infty}U_i=0$.

(iii) $H_i: I\times I\times C(I\times I,c)\to\mathbb{R}$ $(i\in\mathbb{N})$ are continuous and there exist continuous functions $\hat{Q}_i:I\times I\to\mathbb{R}$ $(i\in\mathbb{N})$ such that

$$H_i(t,s,z(t,s))=\hat{Q}_i(t,s)z_i(t,s)$$

with the condition that $\left|\hat{Q}_i(t,s)\right|\le Q_i$ for all $t,s\in I$, where Q_i is a constant. Also $\sup_i Q_i=Q$, $\lim\limits_{i\to\infty}Q_i=0$ and for all $i\in\mathbb{N}$.

(iv) Define an operator S on $I\times I\times C(I\times I,c)$ to $C(I\times I,c)$ as follows

$$(t,s,z(t,s))\to\left(Sz\right)(t,s),$$

where

$$\begin{aligned}\left(Sz\right)(t,s)&=\left((S_1z)(t,s),(S_2z)(t,s),(S_3z)(t,s),\ldots\right),\\ (S_iz)(t,s)&=H_i\left(t,s,z(t,s)\right)+F_i\left(t,s,z(t,s),I_i(z)\right)\\ \text{and}\quad I_i(z)&=\int\limits_0^s\int\limits_0^t\frac{u_i\left(t,s,v,w,z(v,w)\right)}{(t-v)^\alpha(s-w)^\beta}dvdw,\ i\in\mathbb{N}.\end{aligned}$$

(v) Let $\sup\limits_{t,s\in I}\left|\hat{A}_i(t,s)\right|=A_i$ and as $i\to\infty, A_i\to 0$. Also $\sup\limits_i A_i=A<\infty$.

(vi) Let us define $\bar{B}_i : I \times I \to \mathbb{R}_+$ by

$$\bar{B}_i(t,s) = t^{1-\alpha}s^{1-\beta}B_i(t,s) \text{ and } \hat{B} = \sup\left\{\bar{B}_i : t,s \in I, i \in \mathbb{N}\right\} < \infty.$$

(vii) We also assume that $0 < A + Q < 1$.

Theorem 2.1. Under the hypothesis $(i) - (vii)$, infinite system (2.2) has at least one solution $z(t,s) = (z_i(t,s))_{i=1}^{\infty} \in C(I \times I, c)$ for all $t,s \in I$ and $z_i(t,s) \in C(I \times I, \mathbb{R})$ for all $i \in \mathbb{N}$.

Proof . By using (2.2) and (i)-(vii), then for all arbitrary fixed $t,s \in I$, we have

$$\begin{aligned}
&\| z(t,s) \|_c \\
&= \sup_{i\geq 1}\left| H_i\left(t,s,z(t,s)\right) + F_i\left(t,s,z(t,s), \int_0^s\int_0^t \frac{u_i\left(t,s,v,w,z(v,w)\right)}{(t-v)^{\alpha}(s-w)^{\beta}}dvdw\right)\right| \\
&= \sup_{i\geq 1}\left| \hat{Q}_i(t,s)z_i(t,s) + \hat{A}_i(t,s)z_i(t,s) + B_i(t,s)\int_0^s\int_0^t \frac{u_i\left(t,s,v,w,z(v,w)\right)}{(t-v)^{\alpha}(s-w)^{\beta}}dvdw\right| \\
&\leq \sup_{i\geq 1}\left[\left|\hat{Q}_i(t,s)\right| |z_i(t,s)| + \left|\hat{A}_i(t,s)\right| |z_i(t,s)| + |B_i(t,s)| \left|\int_0^s\int_0^t \frac{u_i\left(t,s,v,w,z(v,w)\right)}{(t-v)^{\alpha}(s-w)^{\beta}}dvdw\right|\right] \\
&\leq \sup_{i\geq 1}\left[(Q+A)\,|z_i(t,s)| + B_i(t,s)\left|\int_0^s\int_0^t \frac{u_i\left(t,s,v,w,z(v,w)\right)}{(t-v)^{\alpha}(s-w)^{\beta}}dvdw\right|\right] \\
&\leq (Q+A) \| z(t,s) \|_c + \sup_{i\geq 1}\left\{\frac{Ut^{1-\alpha}s^{1-\beta}B_i(t,s)}{(1-\alpha)(1-\beta)}\right\} \\
&\leq (Q+A) \| z(t,s) \|_c + \frac{U\hat{B}}{(1-\alpha)(1-\beta)}
\end{aligned}$$

i.e. $(1-Q-A) \| z(t,s) \|_c \leq \frac{U\hat{B}}{(1-\alpha)(1-\beta)}$

i.e. $\| z(t,s) \|_c \leq \frac{U\hat{B}}{(1-Q-A)(1-\alpha)(1-\beta)} = r < \infty$(say) which gives $\| z(t,s) \|_{C(I\times I,c)} \leq r$.

Therefore $z(t,s) \in C(I \times I, c)$.

Suppose $B_1 = B_1\left(z^0(t,s), r\right)$ be the closed ball with center at $z^0(t,s) = (z_i^0(t,s))$ where $z_i^0(t,s) = 0$ for all $i \in \mathbb{N}, t,s \in I$ and radius r, thus B_1 is a non-empty, bounded, closed and convex subset of $C(I \times I, c)$. Also let $S = (S_i)$ be an operator which is defined as follows, for all $t,s \in I$

$$\left(Sz\right)(t,s) = \left\{\left(S_i z\right)(t,s)\right\} = \left\{H_i(t,s,z(t,s)) + F_i\left(t,s,z(t,s),I_i(z)\right)\right\},$$

where $z(t,s) = (z_i(t,s))_{i=1}^{\infty} \in B_1$ and $z_i(t,s) \in C(I \times I, \mathbb{R})$, for all $i \in \mathbb{N}$.

Now, we have to show that for fixed $t,s \in I$, that $\left(Sz\right)(t,s)$ is a Cauchy sequence.

Let us consider fixed $z(t,s) \in B_1$ and $t, s \in I$. For arbitrary $m, n \in \mathbb{N}$ we have

$$
\begin{aligned}
&\left|\left(S_n z\right)(t,s) - \left(S_m z\right)(t,s)\right| \\
&= \left|H_n(t,s,z(t,s)) + F_n\left(t,s,z(t,s),I_n(z)\right) - H_m(t,s,z(t,s)) - F_m\left(t,s,z(t,s),I_m(z)\right)\right| \\
&= \left|\hat{Q}_n(t,s)z_n(t,s) + \hat{A}_n(t,s)z_n(t,s) + B_n(t,s)I_n(z) - \hat{Q}_m(t,s)z_m(t,s) - \hat{A}_m(t,s)z_m(t,s) - B_m(t,s)I_m(z)\right| \\
&\le Q_n\left|z_n(t,s)\right| + A_n\left|z_n(t,s)\right| + B_n(t,s)\left|I_n(z)\right| + Q_m\left|z_m(t,s)\right| + A_m\left|z_m(t,s)\right| + B_m(t,s)\left|I_m(z)\right| \\
&\le \left(Q_n + A_n\right)\left|z_n(t,s)\right| + B_n(t,s)\frac{U_n t^{1-\alpha}s^{1-\beta}}{(1-\alpha)(1-\beta)} + \left(Q_m + A_m\right)\left|z_m(t,s)\right| + B_m(t,s)\frac{U_m t^{1-\alpha}s^{1-\beta}}{(1-\alpha)(1-\beta)} \\
&\le \left(Q_n + A_n\right)\left|z_n(t,s)\right| + \frac{U_n\hat{B}}{(1-\alpha)(1-\beta)} + \left(Q_n + A_n\right)\left|z_n(t,s)\right| + \frac{U_n\hat{B}}{(1-\alpha)(1-\beta)}
\end{aligned}
$$

As $m, n \to \infty$ we have $\left|\left(S_n z\right)(t,s) - \left(S_m z\right)(t,s)\right| \to 0$. Thus $\left(Sz\right)(t,s)$ is a real Cauchy sequence hence it is convergent i.e. $\left(Sz\right)(t,s) \in C(I \times I, c)$.

Also $\| \left(Sz\right)(t,s) - z^0(t,s) \|_{C(I\times I,c)} \le r$ so S is self mapping on B_1.

Let us consider a real number $\epsilon > 0$ and arbitrary $z(t,s) = \left(z_i(t,s)\right)_{i=1}^{\infty}$, $\bar{z}(t,s) = \left(\bar{z}_i(t,s)\right)_{i=1}^{\infty} \in B_1$ and $z_i(t,s), \bar{z}_i(t,s) \in C(I \times I, \mathbb{R})$ such that $\| z - \bar{z} \|_{C(I\times I,c)} < \frac{\epsilon}{2(A+Q)}$.

For all $i \in \mathbb{N}$ and arbitrary fixed $t, s \in I$ we have

$$
\begin{aligned}
&\left|\left(S_i z\right)(t,s) - \left(S_i \bar{z}\right)(t,s)\right| \\
&= \left|H_i(t,s,z(t,s)) + F_i\left(t,s,z(t,s),I_i(z(t,s))\right) - H_i(t,s,\bar{z}(t,s)) - F_i\left(t,s,\bar{z}(t,s),I_i(\bar{z}(t,s))\right)\right| \\
&= \left|\hat{Q}_i(t,s)z_i(t,s) + \hat{A}_i(t,s)z_i(t,s) + B_i(t,s)I_i(z) - \hat{Q}_i(t,s)\bar{z}_i(t,s) - \hat{A}_i(t,s)\bar{z}_i(t,s) - B_i(t,s)I_i(\bar{z})\right| \\
&\le (A+Q) \| z - \bar{z} \|_c + B_i(t,s)\int_0^s\int_0^t \frac{\left|u_i\left(t,s,v,w,z(v,w)\right) - u_i\left(t,s,v,w,\bar{z}(v,w)\right)\right|}{(t-v)^{\alpha}(s-w)^{\beta}}dvdw \\
&< \frac{\epsilon}{2} + B_i(t,s)\int_0^s\int_0^t \frac{\left|u_i\left(t,s,v,w,z(v,w)\right) - u_i\left(t,s,v,w,\bar{z}(v,w)\right)\right|}{(t-v)^{\alpha}(s-w)^{\beta}}dvdw.
\end{aligned}
$$

Let

$$
W = \sup_i \left\{\left|u_i\left(t,s,v,w,z(v,w)\right) - u_i\left(t,s,v,w,\bar{z}(v,w)\right)\right| : t,s,v,w \in I, z(v,w), \bar{z}(v,w) \in B_1\right\}.
$$

Then $\left|\left(S_i z\right)(t,s) - \left(S_i \bar{z}\right)(t,s)\right| < \frac{\epsilon}{2} + \frac{W t^{1-\alpha}s^{1-\beta}B_i(t,s)}{(1-\alpha)(1-\beta)} \le \frac{\epsilon}{2} + \frac{W\hat{B}}{(1-\alpha)(1-\beta)}$.
Since u_i is uniformly continuous on compact set $I \times I \times I \times I \times B_1$ we have $W \to 0$ as $\epsilon \to 0$, therefore for all $i \in \mathbb{N}$, we have $\left|\left(S_i z\right)(t,s) - \left(S_i \bar{z}\right)(t,s)\right| \to 0$ as $\| z(t,s) - \bar{z}(t,s) \|_{C(I\times I,c)} \to 0$. Since t, s is arbitrarily chosen therefore S is continuous on $B_1 \subset C(I \times I, c)$ for all $t, s \in I$.

Now we shall prove that S is a Meir-Keeler condensing operator.

We have for arbitrarily fixed $t, s \in I$,

$$\begin{aligned}
&\left|H_k(t,s,z(t,s)) - \lim_{m\to\infty} H_m(t,s,z(t,s))\right| \\
&= \left|\hat{Q}_k(t,s)z_k(t,s) - \lim_{m\to\infty} \hat{Q}_m(t,s)z_m(t,s)\right| \\
&= \left|\hat{Q}_k(t,s)\left(z_k(t,s) - \lim_{m\to\infty} z_m(t,s)\right) + \lim_{m\to\infty}\left(\hat{Q}_k(t,s) - \hat{Q}_m(t,s)\right) z_m(t,s)\right| \\
&\leq \left|\hat{Q}_k(t,s)\right| \left|z_k(t,s) - \lim_{m\to\infty} z_m(t,s)\right| + \left|\lim_{m\to\infty}\left(\hat{Q}_k(t,s) - \hat{Q}_m(t,s)\right) z_m(t,s)\right| \\
&\leq Q\left|z_k(t,s) - \lim_{m\to\infty} z_m(t,s)\right| + \left|\hat{Q}_k(t,s)\lim_{m\to\infty} z_m(t,s)\right|
\end{aligned}$$

and

$$\begin{aligned}
&\left|F_k(t,s,z(t,s),I_k(z)) - \lim_{m\to\infty} F_m(t,s,z(t,s),I_m(z))\right| \\
&= \left|\hat{A}_k(t,s)z_k(t,s) + B_k(t,s)I_k(z) - \lim_{m\to\infty}\left(\hat{A}_m(t,s)z_m(t,s) + B_m(t,s)I_m(z)\right)\right| \\
&\leq A\left|z_k(t,s) - \lim_{m\to\infty} z_m(t,s)\right| + \left|\hat{A}_k(t,s)\lim_{m\to\infty} z_m(t,s)\right| + B_k(t,s)\left|I_k(z)\right| + \lim_{m\to\infty} B_m(t,s)\left|I_m(z)\right| \\
&\leq A\left|z_k(t,s) - \lim_{m\to\infty} z_m(t,s)\right| + \left|\hat{A}_k(t,s)\lim_{m\to\infty} z_m(t,s)\right| + \frac{\hat{B}U_k}{(1-\alpha)(1-\beta)} + \lim_{m\to\infty}\frac{\hat{B}U_m}{(1-\alpha)(1-\beta)} \\
&= A\left|z_k(t,s) - \lim_{m\to\infty} z_m(t,s)\right| + \left|\hat{A}_k(t,s)\lim_{m\to\infty} z_m(t,s)\right| + \frac{\hat{B}U_k}{(1-\alpha)(1-\beta)}.
\end{aligned}$$

Therefore

$$\begin{aligned}
&\mu_c\left(S\left(B_1\right)\right) \\
&= \lim_{n\to\infty}\left[\sup_{z(t,s)\in B_1}\left\{\sup_{k\geq n}\left|H_k(t,s,z) + F_k(t,s,z,I_k(z)) - \lim_{m\to\infty}\left\{H_m(t,s,z) + F_m(t,s,z,I_m(z))\right\}\right|\right\}\right] \\
&\leq (A+Q)\mu_c(B_1)
\end{aligned}$$

i.e.

$$\mu_{C(I\times I,c)}\left(S\left(B_1\right)\right) \leq (A+Q)\mu_{C(I\times I,c)}(B_1).$$

We observe that $\mu_{C(I\times I,c)}\left(S\left(B_1\right)\right) \leq (A+Q)\mu_{C(I\times I,c)}(B_1) < \epsilon \Rightarrow \mu_c\left(B_1\right) < \frac{\epsilon}{A+Q}$.
If we choose $\delta = \frac{\epsilon(1-A-Q)}{A+Q}$ we get $\epsilon \leq \mu_{C(I\times I,c)}\left(B_1\right) < \epsilon + \delta$. Thus S is a Meir-Keeler condensing operator on $B_1 \subset C(I\times I,c)$. So S satisfies all the conditions of Theorem 1.7 which implies S has at least one fixed point in B_1. Therefore the system (2.2) has a solution in $C(I\times I,c)$. □

3. Applications

Example 3.1. Consider the following infinite system of singular integral equations

$$z_i(t,s) = \frac{1}{4i+t^2s^2}\sum_{j=1}^{3i}\left(\frac{z_i(t,s)}{j^2}\right) + \sum_{j=1}^{i}\left(\frac{z_i(t,s)}{4j^2i^2}\right) + \frac{1}{e^{ts}}\int_0^s\int_0^t \frac{\cos\left(\sum_{j=1}^{5i} z_j(v,w)\right)}{(t^2s^2+i^2)(t-v)^{\frac{1}{2}}(s-w)^{\frac{1}{2}}}dvdw, \quad (3.1)$$

where $i \in \mathbb{N}$ and $I = [0, 7]$. Here

$$H_i(t, s, z(t, s)) = \frac{1}{4i + t^2 s^2} \sum_{j=1}^{3i} \left(\frac{z_i(t, s)}{j^2} \right)$$

$$F_i(t, s, z(t, s), I_i(z)) = \sum_{j=1}^{i} \left(\frac{z_i(t, s)}{4i^2 j^2} \right) + \frac{1}{e^{ts}} I_i(z),$$

$$I_i(z) = \int_0^s \int_0^t \frac{\cos\left(\sum_{j=1}^{5i} z_j(v, w) \right)}{(t^2 s^2 + i^2)(t - v)^{\frac{1}{2}} (s - w)^{\frac{1}{2}}} dv dw,$$

$$u_i(t, s, v, w, z(v, w)) = \frac{\cos\left(\sum_{j=1}^{5i} z_j(v, w) \right)}{t^2 s^2 + i^2}$$

and $\alpha = \beta = \frac{1}{2}$.
Now if $z(t, s) = (z_i(t, s)) \in C(I \times I, c)$ and $z_i(t, s) \in C(I \times I, \mathbb{R})$ for all $i \in \mathbb{N}$ then,

$$\begin{aligned} &F_i(t, s, z(t, s), I_i(z)) \\ &= \frac{z_i(t, s)}{4i^2} \sum_{j=1}^{i} \frac{1}{j^2} + \frac{1}{e^{ts}} I_i(z). \end{aligned}$$

Here $\hat{A}_i(t, s) = \frac{1}{4i^2} \sum\limits_{j=1}^{i} \frac{1}{j^2}$, $B_i(t, s) = \frac{1}{e^{ts}}$. B_i is both continuous and bounded function for all $t, s \in I$ and $i \in \mathbb{N}$.

Also, $A_i = \frac{\pi^2}{24i^2}$, $A = \frac{\pi^2}{24}$ and $\lim\limits_{i \to \infty} A_i = 0$.
Also,

$$H_i(t, s, z(t, s)) = \frac{z_i(t, s)}{4i + t^2 s^2} \sum_{j=1}^{3i} \frac{1}{j^2}.$$

So we have $\hat{Q}_i(t, s) = \frac{1}{4i + t^2 s^2} \sum_{j=1}^{3i} \frac{1}{j^2}$, $Q_i = \frac{\pi^2}{24i}$, $Q = \frac{\pi^2}{24}$, $\lim\limits_{i \to \infty} Q_i = 0$ and $0 < A + Q < 1$. We can easily see that $u_i(t, s, v, w, z(v, w))$ are continuous for all $i \in \mathbb{N}$. We also have $U_i = \frac{1}{i^2}$,$U = 1$ and $\lim\limits_{i \to \infty} U_i = 0$.
Again we have $\bar{B}_i(t, s) = \frac{\sqrt{ts}}{e^{ts}}$ and $\hat{B} = \frac{1}{\sqrt{2e}}$.
It is obvious that H_i and F_i are continuous functions. So all the assumptions from (i)-(vii) are satisfied. Hence by theorem 2.1 we conclude that the system 3.1 has a solution in $C(I \times I, c)$.

4. Coupled semi-analytic method to find solution of infinite system of nonlinear singular integral equations

In the section 3 we proved existence of solution for infinite system of nonlinear singular integral equations of two variables (see example 3.1 as an application of Theorem 2.1). Now, we obtain an approximation of solution for the above problem by a coupled technique that is created by modified homotopy perturbation method with infinite functions of two variables and Adomian decomposition method. Applications of modified homotopy perturbation method to solve nonlinear integral equations, nonlinear singular integral equations and nonlinear differential equations can be seen in [21, 31, 32, 34], respectively. Adomian in [1] introduced a decomposition method for solving frontier problem of physics and this technique is used in [35] to solve Fredholm integro-differential equations system. In [22] Hazarika et al. was applied a modified homotopy perturbation and Adomian decomposition method to solve infinite system of nonlinear integral equations in the case of one variable. Also to solve singular integral equation can be seen in [25, 33, 34, 37]. But in this article we introduce a modified homotopy perturbation method in terms of infinite number of functions with two variables and for simplification of nonlinear terms we use Adomian decomposition method in the suitable form. Consider nonlinear problem with infinite functions of two variables in the general form

$$\begin{cases} A(z_1(t,s), z_2(t,s), \cdots, z_i(t,s), \cdots) - f(t,s,i) = 0, \\ (t,s) \in \Omega = [0,T] \times [0,T], \quad i \in \mathbb{N} \end{cases} \tag{4.1}$$

where A is a general nonlinear operator and f's are known analytic functions. Similar to [31, 32], we divide the general operator A to two nonlinear operators N_1 and N_2. Of course N_1 or N_2 can be linear operator in special case. Also every one of f's are converted to f_1 and f_2 functions in other word we have

$$\begin{cases} N_1(z_1(t,s), \cdots, z_i(t,s), \cdots) - f_1(t,s,i) \\ \qquad +N_2(z_1(t,s), \cdots, z_i(t,s), \cdots) - f_2(t,s,i) = 0, \; i \in \mathbb{N}, \end{cases}$$

By assumption $\widehat{\eta}(t,s) = (\eta_1(t,s), \eta_2(t,s), \cdots)$, we introduce a modified homotopy perturbation for infinite functions of two variables as follows

$$\begin{cases} H(\widehat{\eta}(t,s), p) & = N_1(\eta_1(t,s), \cdots, \eta_i(t,s), \cdots) - f_1(t,s,i) \\ & +p(N_2(\eta_1(t,s), \cdots, \eta_i(t,s), \cdots) - f_2(t,s,i)) = 0, \; p \in [0,1], \end{cases} \tag{4.2}$$

where p is an embedding parameter and η_i's are approximation of z_i's for $i \in \mathbb{N}$. By variations of $p = 0$ to $p = 1$ it's concluded that $N_1(\eta_1(t,s), \cdots, \eta_i(t,s), \cdots) = f_1(t,s,i)$ to $A(\eta_1(t,s), \eta_2(t,s), \cdots, \eta_i(t,s), \cdots) - f(t,s,i) = 0$. In fact by choosing of $p = 1$ in (4.2) we can get the solution of (4.1) and also we have

$$\begin{cases} z_i(t,s) \approx \eta_i(t,s) = \sum_{k=0}^{\infty} p^k \eta_{i,k}(t,s), i \in \mathbb{N}. \\ z_i(t,s) = \lim\limits_{p \to 1} \eta_i(t,s) \end{cases} \tag{4.3}$$

For $(t,s) \in [0,7] \times [0,7]$, we define N_1 and N_2 operators and f's functions to solve of (3.1) to this form;

$$\begin{aligned} N_1(z_1(t,s), \cdots, z_i(t,s), \cdots) &= z_i(t,s), \\ N_2(z_1(t,s), \cdots, z_i(t,s), \cdots) &= -\frac{1}{4i + t^2 s^2} \sum_{j=1}^{3i} \left(\frac{z_i(t,s)}{j^2} \right) - \frac{1}{4} \sum_{j=1}^{i} \left(\frac{z_i(t,s)}{j^2 i^2} \right) \\ & \quad - \frac{1}{e^{ts}} \int_0^s \int_0^t \frac{\cos\left(\sum_{j=1}^{5i} z_j(v,w) \right)}{(i^2 + t^2 s^2)(t-v)^{\frac{1}{2}} (s-w)^{\frac{1}{2}}} dv dw, \end{aligned} \tag{4.4}$$

$$f(t,s,i) = f_1(t,s,i) + f_2(t,s,i).$$

By substituting (4.4) and (4.3) in the homotopy perturbation (4.2), we have

$$\begin{aligned}&\Big(\sum_{k=0}^{\infty}p^k\eta_{i,k}(t,s)-f_1(t,s,i)\Big)+p\Big(-\frac{1}{4i+t^2s^2}\sum_{j=1}^{3i}(\frac{\sum_{k=0}^{\infty}p^k\eta_{i,k}(t,s)}{j^2})\\&-\frac{1}{4}\sum_{j=1}^{i}(\frac{\sum_{k=0}^{\infty}p^k\eta_{i,k}(t,s)}{j^2i^2})-\frac{1}{e^{ts}}\int_0^s\int_0^t\frac{\cos\left(\sum_{j=1}^{5i}\sum_{k=0}^{\infty}p^k\eta_{j,k}(v,w)\right)}{(t^2s^2+i^2)(t-v)^{\frac{1}{2}}(s-w)^{\frac{1}{2}}}dvdw-f_2(t,s,i)\Big)=0,\end{aligned} \tag{4.5}$$

In (4.5), we apply Adomian decomposition method to convert nonlinear terms to smaller separable nonlinear terms

$$\begin{aligned}\sum_{j=1}^{3i}(\frac{\sum_{k=0}^{\infty}p^k\eta_{i,k}(t,s)}{j^2})&=\sum_{k=0}^{\infty}p^kB_{i,k}(t,s),\\\sum_{j=1}^{i}(\frac{\sum_{k=0}^{\infty}p^k\eta_{i,k}(t,s)}{j^2i^2})&=\sum_{k=0}^{\infty}p^k\widehat{B}_{i,k}(t,s),\\\cos\left(\sum_{j=1}^{5i}\sum_{k=0}^{\infty}p^k\eta_{j,k}(v,w)\right)&=\sum_{k=0}^{\infty}p^k\widehat{\widehat{B}}_{i,k}(t,s),\end{aligned} \tag{4.6}$$

where Adomian polynomials are given by

$$\begin{aligned}B_{i,k}(t,s)&=\frac{1}{k!}\Big(\frac{d^k}{dp^k}\sum_{j=1}^{3i}(\frac{\sum_{k=0}^{\infty}p^k\eta_{i,k}(t,s)}{j^2})\Big)_{p=0,}\\\widehat{B}_{i,k}(t,s)&=\frac{1}{k!}\Big(\frac{d^k}{dp^k}\sum_{j=1}^{i}(\frac{\sum_{k=0}^{\infty}p^k\eta_{i,k}(t,s)}{j^2i^2})\Big)_{p=0,}\\\widehat{\widehat{B}}_{i,k}(t,s)&=\frac{1}{k!}\Big(\frac{d^k}{dp^k}\cos\left(\sum_{j=1}^{5i}\sum_{k=0}^{\infty}p^k\eta_{j,k}(v,w)\right)\Big)_{p=0.}\end{aligned} \tag{4.7}$$

Placing (4.6) into (4.5), it concludes that

$$\begin{aligned}&\Big(\sum_{k=0}^{\infty}p^k\eta_{i,k}(t,s)-f_1(t,s,i)\Big)+p\Big(-\frac{1}{4i+t^2s^2}\sum_{k=0}^{\infty}p^kB_{i,k}(t,s)\\&-\frac{1}{4}\sum_{k=0}^{\infty}p^k\widehat{B}_{i,k}(t,s)-\frac{1}{e^{ts}}\int_0^s\int_0^t\frac{\sum_{k=0}^{\infty}p^k\widehat{\widehat{B}}_{i,k}(v,w)dvdw}{(t^2s^2+i^2)(t-v)^{\frac{1}{2}}(s-w)^{\frac{1}{2}}}-f_2(t,s,i)\Big)=0,\end{aligned} \tag{4.8}$$

By rearranging of (4.8) in terms of p powers we can get

$$\begin{aligned}&p^0:(\eta_{i,0}(t,s)-f_1(t,s,i)),\\&p^1:(\eta_{i,1}(t,s)-\frac{B_{i,0}(t,s)}{4i+t^2s^2}-\frac{1}{4}\widehat{B}_{i,0}(t,s)-\frac{1}{e^{ts}}\int_0^s\int_0^t\frac{\widehat{\widehat{B}}_{i,0}(v,w)dvdw}{(t^2s^2+i^2)(t-v)^{\frac{1}{2}}(s-w)^{\frac{1}{2}}}-f_2(t,s,i),\\&p^n:(\eta_{i,n}(t,s)-\frac{B_{i,n-1}(t,s)}{4i+t^2s^2}-\frac{1}{4}\widehat{B}_{i,n-1}(t,s)-\frac{1}{e^{ts}}\int_0^s\int_0^t\frac{\widehat{\widehat{B}}_{i,n-1}(v,w)dvdw}{(t^2s^2+i^2)(t-v)^{\frac{1}{2}}(s-w)^{\frac{1}{2}}},n\geq 2.\end{aligned}$$

According to modified homotopy perturbation (4.2) the coefficients of p powers must be equal to zero and we can give an iterative algorithm to solve (3.1).

Algorithm:

$$\begin{aligned} &\eta_{i,0}(t,s) = f_1(t,s,i),\\ &\eta_{i,1}(t,s) = f_2(t,s,i) + \frac{B_{i,0}(t,s)}{4i+t^2s^2} + \frac{1}{4}\widehat{B}_{i,0}(t,s) + \frac{1}{e^{ts}}\int_0^s\int_0^t \frac{\widehat{\widehat{B}}_{i,0}(v,w)dvdw}{(t^2s^2+i^2)(t-v)^{\frac{1}{2}}(s-w)^{\frac{1}{2}}},\\ &\eta_{i,n}(t,s) = \frac{B_{i,n-1}(t,s)}{4i+t^2s^2} + \frac{1}{4}\widehat{B}_{i,n-1}(t,s) + \frac{1}{e^{ts}}\int_0^s\int_0^t \frac{\widehat{\widehat{B}}_{i,n-1}(v,w)dvdw}{(t^2s^2+i^2)(t-v)^{\frac{1}{2}}(s-w)^{\frac{1}{2}}}, n\geq 2. \end{aligned} \tag{4.9}$$

Convergence of the above algorithm can be proved similar to [23]. Now, we compute terms of sequence $\{z_1(t,s), z_2(t,s), ..\}$ and then we introduce closed form of solution for the infinite system of non-linear singular integral equations (3.1) by the above algorithm. To this end at first we calculate Adomian polynomials in the case of $k = 0$ as follows,

$$\begin{aligned} &B_{i,0}(t,s) = \sum_{j=1}^{3i}(\frac{\eta_{i,0}(t,s)}{j^2}), \quad \widehat{B}_{i,0}(t,s) = \sum_{j=1}^{i}(\frac{\eta_{i,0}(t,s)}{j^2i^2})\\ &\widehat{\widehat{B}}_{i,0}(t,s) = \cos\left(\sum_{j=1}^{5i}\eta_{j,0}(v,w)\right). \end{aligned} \tag{4.10}$$

Since in (3.1), $f(t,s,i) = 0$ then $f_1(t,s,i) = f_2(t,s,i) = 0$ and also in the algorithm (4.9) we have

$$\begin{aligned} \eta_{i,0}(t,s) &= f_1(t,s,i) = 0,\\ \eta_{i,1}(t,s) &= f_2(t,s,i) + \frac{B_{i,0}(t,s)}{4i+t^2s^2} + \frac{1}{4}\widehat{B}_{i,0}(t,s) + \frac{1}{e^{ts}}\int_0^s\int_0^t \frac{\widehat{\widehat{B}}_{i,0}(v,w)dvdw}{(t^2s^2+i^2)(t-v)^{\frac{1}{2}}(s-w)^{\frac{1}{2}}}\\ &= \frac{4e^{-st}\sqrt{st}}{(i^2+s^2t^2)}. \end{aligned}$$

We use from (4.3) to approximate of the some elements of sequence $(z_i(t,s))_{i=1}^{\infty}$ by a few terms of the above approximations

$$z_1(t,s) \simeq \sum_{k=0}^{1}\eta_{1,k}(t,s) = \frac{4e^{-st}\sqrt{st}}{(1+s^2t^2)}, \tag{4.11}$$

and similarly

$$\begin{aligned} z_2(t,s) &= \frac{4e^{-st}\sqrt{st}}{(2^2+s^2t^2)},\\ z_{10}(t,s) &= \frac{4e^{-st}\sqrt{st}}{(10^2+s^2t^2)},\\ z_{100}(t,s) &= \frac{4e^{-st}\sqrt{st}}{(100^2+s^2t^2)}. \end{aligned} \tag{4.12}$$

Therefore we can give solution of (3.1) by a closed form,

$$z_i(t,s) \simeq \sum_{k=0}^{1}\eta_{i,k}(t,s) = \frac{4e^{-st}\sqrt{st}}{(i^2+s^2t^2)}. \tag{4.13}$$

We choose a subinterval of $[0,7] \times [0,7]$ to compute absolute errors. So by substituting (4.13) in Eq.(3.1) absolute errors in some points for some elements of solution such as $z_1(t,s), z_{10}(t,s)$ and $z_{100}(t,s)$ are given in the Table 1-3.

Table 1: Absolute errors for $z_1(t,s)$ in some points

$(t,s) \in [5,7] \times [5,7]$	5.0	5.5	6.0	6.5	7.0
5.0	1.1×10^{-13}	7.9×10^{-15}	5.7×10^{-16}	4.1×10^{-17}	3.0×10^{-18}
5.5	7.9×10^{-15}	4.4×10^{-16}	2.4×10^{-17}	1.3×10^{-18}	7.9×10^{-20}
6.0	5.7×10^{-16}	2.4×10^{-17}	1.0×10^{-18}	4.7×10^{-20}	2.1×10^{-21}
6.5	4.1×10^{-17}	8.3×10^{-18}	4.7×10^{-20}	1.6×10^{-21}	5.6×10^{-23}
7.0	3.0×10^{-18}	7.9×10^{-20}	2.1×10^{-21}	5.6×10^{-23}	1.5×10^{-24}

Table 2: Absolute errors for $z_{10}(t,s)$ in some points

$(t,s) \in [5,7] \times [5,7]$	5.0	5.5	6.0	6.5	7.0
5.0	6.3×10^{-14}	3.8×10^{-15}	2.2×10^{-16}	1.4×10^{-17}	9.6×10^{-19}
5.5	3.8×10^{-15}	1.7×10^{-16}	8.6×10^{-18}	4.2×10^{-19}	2.1×10^{-20}
6.0	2.3×10^{-16}	8.6×10^{-18}	3.2×10^{-19}	1.2×10^{-20}	4.8×10^{-22}
6.5	1.4×10^{-17}	4.2×10^{-19}	1.2×10^{-20}	3.7×10^{-22}	1.1×10^{-23}
7.0	9.6×10^{-19}	2.1×10^{-20}	4.8×10^{-22}	1.1×10^{-23}	2.6×10^{-25}

Table 3: Absolute errors for $z_{100}(t,s)$ in some points

$(t,s) \in [5,7] \times [5,7]$	5.0	5.5	6.0	6.5	7.0
5.0	4.1×10^{-13}	2.9×10^{-14}	2.0×10^{-15}	1.4×10^{-16}	1.0×10^{-18}
5.5	2.9×10^{-14}	1.6×10^{-15}	8.8×10^{-17}	4.9×10^{-18}	2.7×10^{-20}
6.0	2.0×10^{-15}	8.8×10^{-17}	3.7×10^{-18}	1.6×10^{-19}	7.1×10^{-21}
6.5	1.4×10^{-16}	4.9×10^{-18}	1.6×10^{-19}	5.5×10^{-21}	1.8×10^{-23}
7.0	1.0×10^{-17}	2.7×10^{-19}	7.1×10^{-21}	1.8×10^{-22}	4.9×10^{-24}

As we showed in Table 1-3 the proposed method has a acceptable accuracy.

5. Conclusion

In this article, we proved existence of solution for infinite system of nonlinear singular integral equations with two variables. Efficiency of our results was confirmed by an example. Also we constructed an iteration algorithm to get solution of the above equations system with a high accuracy.

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