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Fractional Order Glucose Insulin System Using Fractional Back-Stepping Sliding Mode Control

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Abstract

In this paper, based on a fractional order Bergman minimal model, a robust strategy for regulation of blood glucose in type 1 diabetic patients is presented. Glucose/insulin concentration in the patient body is controlled through the injection under the patients skin by the pump. Many various controllers for this system have been proposed in the literature. However, most of them have consider the system as an integer order system. Moreover, the majority of the presented methods suffer from an important disadvantage that is long settling time of the control system. Thus, the contribution of this paper in comparison with previous related works is presenting a fractional back-stepping sliding mode control that considerably reduces the required time for glucose to reach its desired level. Due to the sliding mode design, the proposed controller is robust against external disturbances. Due to the back-stepping design, convergence of each state variable of the system to its desired value can be guaranteed separately. Simulation results verify the satisfactory performance of the proposed controller.

Keywords: Fractional order control, sliding mode control, back-stepping design, blood glucose regulation, Fractional Bergman minimal model, Lyapunov fractional.
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1. Introduction

Diabetes is discussed as a serious condition in which the bodys production and use of insulin are impaired, causing glucose concentration level to increase in the bloodstream. Insulin is a hormone

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generated by specic cells, called beta cells, in the pancreas. In order to transfer blood glucose into cells, insulin is required. Two types of diabetes have been recognized. In type I diabetes mellitus (T1DM), the b-cells in the pancreas that are responsible for producing insulin are destroyed by the immune system of the patients. Thus, the current solution for treatment is the delivery of exogenous insulin to maintain the glucose levels close to normal [22-30].

Based on continuous glucose monitoring (CGM) systems and insulin pumps technologies, a controller that automatically monitors and regulates the blood glucose level can be designed. In other words, it can play the role of an articial pancreas system to replace the conventional treatment strategies in T1DM. In recent decades, various approaches have been presented in the literature for intelligent control of blood glucose. In this paper, the 3rd order minimal model of Bergman [1] is adopted. Various approaches have been presented to design a feedback controller for blood glucose regulation, such as fuzzy logic control [2-5], recurrent neural networks [6], model predictive control (MPC) [7], high order sliding mode control [8], optimal control [9] and back-stepping sliding mode control [10].

Fractional calculus can be considered as a generalized version of classical differentiation and integration to arbitrary (noninteger) order. Recently, fractional calculus has been the focus of many active researches in several fields in engineering. For example, in control engineering, this approach coming from applied mathematics has resulted in the new field of fractionalorder modeling control. One important superiority of fractional differentiation and integration in comparison with their integerorder counterparts is providing an extra degree of freedom for the designer to improve the performance of the control system. As a result, fractional calculus has attracted increasing interests and there has been a rapid growth in the number of applications where fractional calculus has been used such as secure communication and chaos synchronization [11], viscoelastic systems [12, 13], magnetic levitation system [14], power systems [15] and many other systems. Biological systems such as glucose-insulin system were no exception and various fractional order controllers for this system have been presented in the literature [16-20].

This paper presents a fractional order controller for fractional model of glucose-insulin system using back-stepping sliding mode design. Although various controllers for this system have been presented in the literature, most of them suffer from an important disadvantage that is the long time required for glucose to reach the desired level. For example, glucose settling time in [10] is about 350 minutes. Also, the back-stepping sliding mode controller presented in [16] requires 400 minutes to reduce the glucose level to the desired value. For another example, this time for the $H\mathcal{H}\infty$ controller presented in [18] is also about 400 minutes which is too long. Therefore, designing a more powerful controller with shorter glucose settling time is an important contribution of this paper. Moreover, according to [18], considering a fractional order model for this system results in more satisfactory responses. Thus, in this paper, the fractional order controller is designed for fractional order model of the system.

This paper is organized as follows. Section 2, describes the glucose-insulin model. Section 3 develops the proposed controller and presents the stability analysis. Section 4 illustrates simulation results and comparisons. Finally, section 5 concludes the paper

2. Glucose-insulin dynamics

Many models for describing glucose-insulin process has been presented. Bergmans minimal model has been proposed in 1980 by Richard Bergman. The main advantage of the Bergman minimal model is its simplicity. According to [18], it is the common model that is usually referenced in the literature.

Bergman Minimal Model (BeM) is described as [18]:

$$ {}_{t_0}^{C}D_t^{\alpha} x_1 = -p_1[x_1 - G_b] - x_1x_2 + \delta_1 + D(t) \tag{2.1} $$

$$ {}_{t_0}^{C}D_t^{\alpha} x_2 = -p_2x_2 + p_3[x_3 - I_b] + \Delta_2 \tag{2.2} $$

$$ {}_{t_0}^{C}D_t^{\alpha} x_3 = -n[x_3 - I_b] + \gamma t[x_1 - G_b]^{+}\Delta_3 + u(t) \tag{2.3} $$

in which ${}_{t_0}^{C}D_t^{\alpha}$ is the αth-order Caputo fractional derivative, $x_1(t)$, $x_2(t)$ and $x_3(t)$ are plasma glucose concentration, the insulin inuence on glucose concentration reduction, and insulin concentration in plasma respectively, $u(t) \in R$ is injected insulin rate in (mU/min). All of the parameters have been completely explained in [18]. In this paper, it has been assumed that the parameters in (2.1)-(2.3) are nominal parameters that may be different from their actual values. Thus, the terms Δ_1, Δ_2 and Δ_3 are uncertainties originated from these mismatches. It is assumed that these uncertainties are bounded as $|\Delta_1| \leq k_1$, $|\Delta_2| \leq d_2$ and $|\Delta_3| \leq d_3$ where k_1, d_2 and d_3 are known positive constants. This disturbance can be modeled by a decaying exponential function of the following form [16]

$$ D(t) = A\ exp(-Bt) \quad B > 0 \tag{2.4} $$

The pump can be modeled as a rst order linear system

$$ {}_{t_0}^{C}D_t^{\alpha} u(t) = \frac{1}{a}(w(t) - u(t)) + \Delta_4 \tag{2.5} $$

where $w(t)$ is insulin rate command in pump as input, and the parameter a is pump time constant. Also, Δ_4 is the uncertainty originated from the mismatch between the actual and nominal a.

3. The proposed controller and stability analysis

Define the tracking error of glucose as [20-23]

$$ e_1 = x_1(t) - x_{1d}(t) \tag{3.1} $$

where $x_{1d}(t)$ is the desired blood glucose Also, consider the following sliding surface

$$ s_1 = e_1 + \lambda_1 D^{-\alpha} e_1 \tag{3.2} $$

in which λ_1 is a design parameter. Taking the derivative of (3.2) results

$$ D^{\alpha}s_1 = D^{\alpha}e_1 + \lambda e_1 = D^{\alpha}x_1 - D^{\alpha}x_{1d} + \lambda_1 e_1 \tag{3.3} $$

Substitution of $D^{\alpha}x_1$ from (2.1) into (3.3) and solving $D^{\alpha}s_1 = 0$ results in

$$ x_{2eq} = (x_1)^{-1}(-p_1x_1 + p_1G_b - D^{\alpha}G_d + \lambda_1x_1 - \lambda_1G_d + d_1sign(s_1)) \tag{3.4} $$

in which $d_1sign(s_1)$ has been added to the control law to compensate for the external disturbance $D(t)$ and the uncertainty $\Delta_1 = \delta_1 + D(t)$. In other words, we have, $d_1 > |\Delta_1|$. Now, applying the control law (3.4) into the (2.1) results in

$$ D^{\alpha}x_1 = D^{\alpha}x_{1d} - \lambda_1e_1 + D + \Delta_1 - d_1sign(s_1) \tag{3.5} $$

which can be rewritten as

$$D^{\alpha}s_1 = D + \Delta_1 - d_1 sign(s_1) \tag{3.6}$$

In order to verify that the control law (3.4) guarantees the stability, consider the following positive definite function:

$$V_1 = \frac{1}{2}s_1^2 \tag{3.7}$$

According to Lemma 1 in [21], taking the derivative of (3.7) leads to

$$D^{\alpha}V_1 \leq s_1 D^{\alpha}s_1 \tag{3.8}$$

Substitution of (3.6) into (3.8) results in

$$D^{\alpha}V_1 \leq s_1(D + \Delta_1 - d_1 sign(s_1)) \tag{3.9}$$

It is obvious that

$$D^{\alpha}V_1 \leq |s_1||D + \Delta_1| - s_1 d_1 sign(s_1) \tag{3.10}$$

In other words, we have

$$D^{\alpha}V_1 \leq |s_1||\Delta_1| - d_1|s_1| = |s_1|(|\Delta_1| - d_1) \tag{3.11}$$

Since $d_1 \geq |\Delta_1|$, it can be concluded that

$$D^{\alpha}V_1 \leq -\tilde{d}_1|s_1| \tag{3.12}$$

in which $\tilde{d}_1 = d_1 - |\Delta_1| \geq 0$. Now, consider (2.2). Define the tracking error as

$$e_2 = x_2 - x_{2d} \tag{3.13}$$

where x_{2d} is the desired value of x_2. Also, consider the following sliding surface

$$s_2 = e_2 + \lambda_2 D^{-\alpha}e_2 \tag{3.14}$$

in which λ_2 is a design parameter. Taking the derivative of (3.14) results in

$$D^{\alpha}s_2 = D^{\alpha}e_2 + \lambda_2 e_2 = D^{\alpha}x_2 - D^{\alpha}x_{2d} + \lambda_2 e_2 \tag{3.15}$$

Substitution of $D^{\alpha}x_2$ from (2.2) into (3.15) and solving $D^{\alpha}s_2 = 0$ results in

$$x_{3eq} = \frac{p_2 x_2 + p_3 I_b + D^{\alpha}x_{2d} - \lambda_2 e_2 + d_2\ sign(s_2)}{p_3} \tag{3.16}$$

in which $d_2 sign(s_2)$ has been considered for compensation of the lumped uncertainty Δ_2.

Now, applying the control law (3.16) into the (2.2) results in

$$D^{\alpha}x_2 = D^{\alpha}x_{2d} - \lambda_2 e_2 + \Delta_2 - d_2\ sign(s_2) \tag{3.17}$$

In other words

$$D^{\alpha}s_2 = \Delta_2 - d_2\ sign(s_2) \tag{3.18}$$

consider the following positive definite function

$$V_2 = \frac{1}{2}s_2^2 \tag{3.19}$$

According to Lemma 1 in [21], taking the time derivative of (3.19) results in

$$D^{\alpha}V_2 \leq s_2 D^{\alpha}s_2 \tag{3.20}$$

Substitution of (3.18) into (3.20) leads to

$$D^{\alpha}V_2 \leq s_2(\Delta_2 - d_2\ sign(s_2)) \tag{3.21}$$

It follows from (3.21) that

$$D^{\alpha}V_2 \leq |s_2||\Delta_2| - s_2 d_2\ sign(s_2) \tag{3.22}$$

which can be rewritten as

$$D^{\alpha}V_2 \leq |s_2||\Delta_2| - d_2|s_2| = |s_2|(|\Delta_2| - d_2) \tag{3.23}$$

Since $d_2 > |\Delta_2|$, it can be concluded that

$$D^{\alpha}V_2 \leq -\tilde{d}_2|s_2| \tag{3.24}$$

in which $\tilde{d}_2 = d_2 - |\Delta_2| \geq 0$. Now, consider (2.3). Define the tracking error as

$$e_3 = x_3 - x_{3d} \tag{3.25}$$

where x_{3d} is the desired value of x_3. Also, consider the following sliding surface

$$s_3 = e_3 + \lambda_3 D^{-\alpha}e_3 \tag{3.26}$$

in which λ_3 is a design parameter. Taking the derivative of (3.26) results in

$$D^{\alpha}s_3 = D^{\alpha}e_3 + \lambda_3 e_3 = D^{\alpha}x_3 - D^{\alpha}x_{3d} + \lambda_3 e_3 \tag{3.27}$$

Substitution of $\dot{x}_3$ from (2.3) into (3.27) and solving $\dot{s}_3 = 0$ results in

$$u_d = n[x_3 - I_b] + D^{\alpha}x_{3d} - \lambda_3 e_3 + d_3\ sign(s_3) \tag{3.28}$$

Similar to the procedure given in (3.1) to (3.12), it can be shown that

$$D^{\alpha}V_3 \leq |s_3|(|\Delta_3| - d_3) \leq 0 \tag{3.29}$$

in which

$$V_3 = \frac{1}{2}s_3^2 \tag{3.30}$$

Table 1: The model parameters

Bergman minimal model	
$P_1(min)^{-1}$	0
$P_2(min)^{-1}$	0.0123
$P_2(min)^{-1}$	8.2×10^{-8}
$n(min^{-1})$	0.2659
I_b	7
G_b	70
$X_1(0)$	200
$X_3(0)$	50

Also, the same procedure will lead to

$$w_d = u + aD^{\alpha}u_d - a\lambda_4 e_4 - ad_4 \; sign(s_4) \tag{3.31}$$

In fact, it can be simply shown that this control law will result in

$$D^{\alpha}V_4 \leq |s_4|(|\Delta_4| - d_4) \leq 0 \tag{3.32}$$

in which

$$V_4 = \frac{1}{2}s_4^2 \tag{3.33}$$

$$s_4 = e_4 + \lambda_4 D^{-\alpha}e_4 \tag{3.34}$$

$$e_4 = u - u_d \tag{3.35}$$

Now the following theorem is presented.

Theorem 3.1. *Consider the dynamic system (1). If the control laws* (3.4), (3.16), (3.28) *and* (3.31) *are applied to this system, then the closed-loop signals are bounded and the tracking errors* e_1*,* e_2*,* e_3 *and* e_4 *asymptotically converge to zero.*
Proof . *Define the Lyapunov function candidate as*

$$V = \sum_{i=1}^{4} V_i = \sum_{i=1}^{4} \frac{1}{2}s_i^2 \tag{3.36}$$

in which V_1*,* V_2*,* V_3 *and* V_4 *are defined in* (3.7), (3.19), (3.30) *and* (3.33)*, respectively. Based on* (3.11), (3.23), (3.29) *and* (3.32) *it can be concluded that*

$$D^{\alpha}V \leq -\sum_{i=1}^{4} |s_i|\tilde{d}_i \tag{3.37}$$

According to Theorem1 in [21], (3.37) *implies the asymptotic stability of the system.* □

4. SIMULATION RESULTS

Consider the model described in [20]. Its parameters are given in Table 1. The parameter of the controller have been set to $\lambda_1 = 0.08$, $\lambda_2 = 7.6$, $\lambda_3 = 0.1$, $\lambda_4 = 0.5$. The fractional order has been set to $\alpha = 0.88$. In order to investigate the controller robustness against parametric uncertainties, we have applied 10% uncertaintiy to the parameters presented in Table 1 and used them in the controller design. To be more presice, the parameters p_2, p_3 and n have been multiplied by 1.1. The blood glucose level is presented in Fig. 1. As shown in this figure, the controller can reduce the blood glucose concentration from the initial value of 200 (mg/dl) to the approximate value of 80 (mg/dl) which is our desired level within 150 minutes. The external disturbance is selected as [20]

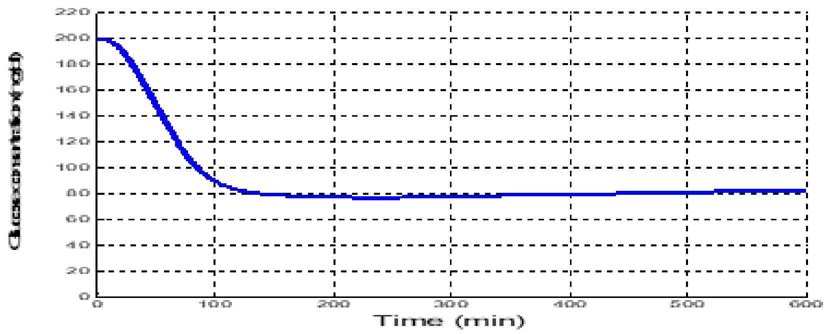

Figure 1: Blood glucose concentration with $\alpha = 0.88$

$$D(t) = 10\sin(\omega t) \tag{4.1}$$

in which $\omega = \frac{2\pi}{T}$ and $T = 6h$. In comparison with [20], the settling time of the proposed method is improved. The settling time in this paper is less than 150 minutes while in [20], the settling time is about 400 minutes. Thus, the proposed fractional controller is superior than the controller presented in [20, 26]. Insulin concentration has been plotted in Fig. 2. As shown in this figure, this signal is bounded and converges to $7(\mu U/dl)$ in the steady state.

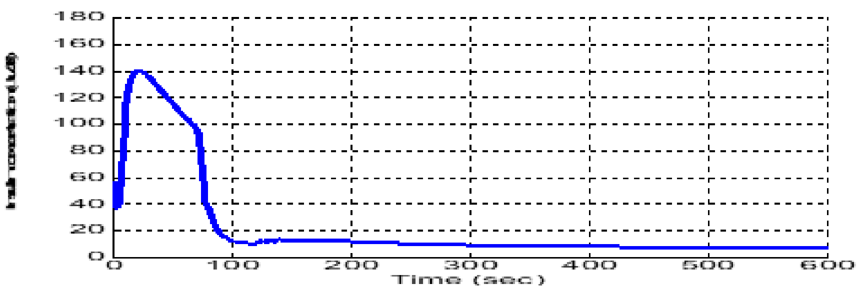

Figure 2: Insulin concentration using the proposed controller

The rate of injected insulin through pump is illustrated in Fig. 3. As shown in this figure, this signal is in acceptable range without any chattering. In order to show the superiority of fractional order control in comparison with integer order control, the parameter α can be set to 1. All of the other controller parameters are the same. In this situation, blood glucose concentration is presented in

Fig. 4. As shown in this figure, the performance of the integer order controller is not satisfactory and the blood glucose concentration reduces considerably which is dangerous. Therefore, fractional controller outperforms the classical integer controller. Also, the proposed fractional controller can reduce the settling time of glucose concentration in patient body considerably which the important superiority of this controller in comparison with previous related works.

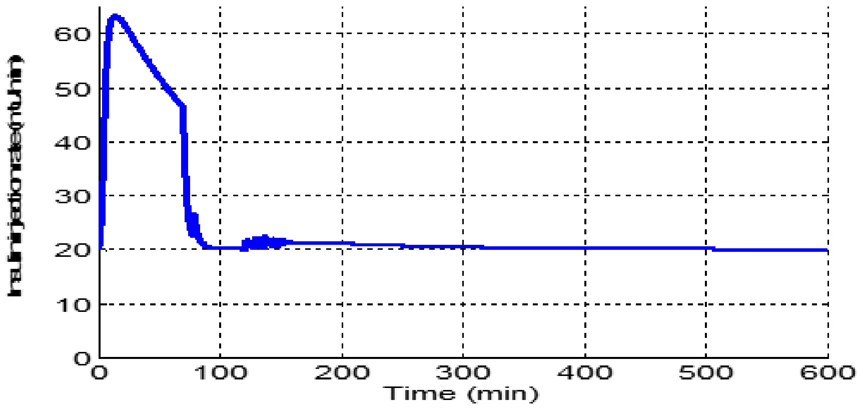

Figure 3: Insulin injection rate using the proposed method

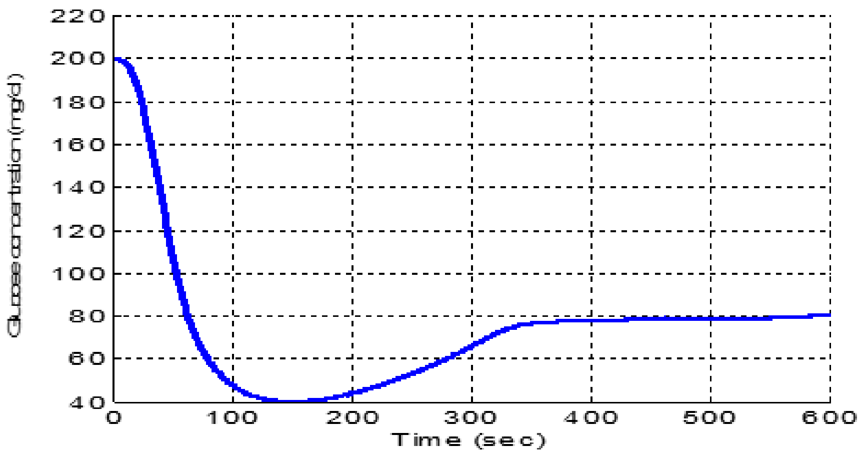

Figure 4: Blood glucose concentration with $\alpha = 1$

5. CONCLUSION

In this paper, a fractional order controller for blood glucose regulation in type I diabetes patients has been presented. Uncertainties have been estimated and compensated using the Fourier series expansion which is less computational in comparison with other uncertainty estimators. The sliding mode control strategy has been adopted to make the controller robust against external disturbances. Simulation results verify the satisfactory performance of the proposed controller in comparison with a previous related work. In fact, the settling time of glucose concentration has been improved considerably in the proposed method. Moreover, the fractional controller outperforms the integer controller with the same parameters.

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