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Back-stepping sliding mode control design for glucose regulation in type 1 diabetic patients

S. Vakili[1], H.T. Shandiz[2*]

[1] *Faculty of Electrical Engineering, Shahrood University of Technology, Shahrood, Iran,*
[2] *Faculty of Electrical Engineering, Ferdowsi University, Mashhad, Iran*

Abstract

This article presents a robust scheme for regulating blood glucose in the patients suffering from type I diabetes based on BMM. A pump is used to inject glucose/insulin under the patient's skin to control its concentration. Previous studies have presented a variety of such controllers. But, they mainly suffered from long settling time. This study employs back-stepping sliding control to reduce the settling time. Since the controller presented in this study employs sliding mode control, it is robust against external disturbances. Since the back-stepping scheme is used in this design, it guarantees that the state variables of the system converge to the desired value. The results obtained from simulation verify performance of the presented scheme.

Keywords: Sliding mode control, back-stepping design, blood glucose regulation, Bergman minimal model.
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1. Introduction

One of the serious conditions, which impairs body's insulin secretion and usage and increases glucose level. Pancreas is responsible to generate the hormone, insulin which is required for transferring glucose into cells. Diabetes is classified into two types including type I mellitus (T1DM) in which the immune systems destroys the b-cells of pancreas which have to generate insulin. Currently, type I diabetes is treated through delivering exogenous insulin such that glucose levels is maintained at a normal level. A controller can be designed for automatic monitoring and regulation of the blood glucose level based on continuous glucose monitoring (CGM) systems. In other words, CGM emulates pancreas performance and can replace the current T1DM treatments. Recently, a number

*Corresponding author
Email address: saharsadatvakili@shahroodut.ac.ir, htoosian@ferdowsi.um.ac.ir (S. Vakili[1], H.T. Shandiz[2*])

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of studies have been conducted on intelligent blood glucose control. This study employs the 3rd order Bergman minimal model [1] A number of strategies have been proposed for feedback controller for regulating blood glucose among which fuzzy logic control [2, 3, 4, 5], recurrent neural networks [6], model predictive control (MPC) [7], high order sliding mode control [8], optimal control [9] and back-stepping sliding mode control [10] can be mentioned. In addition, some approaches have been presented for blood glucose regulation based on fractional order control [11, 12, 13, 14, 15].
In this study, the back-stepping sliding mode control is used to present a robust glucose-insulin controller. Most of the previous schemes require a long settling time (the time that takes the glucose level to reach a desired level). As an example, settling time of the schemes presented in [10] and [11] is 350 minutes. Thus, this paper tries to present a more robust design with shorter settling time.The proposed design integrates the tracking error in the sliding surface which decreases the tracking error and improves glucose settling time [18]-[33].
The rest of this paper is organized as follows. In section 2, the glucose-insulin model is described. The proposed controller and its stability analysis are presented in section 3. Simulation and comparison results are given in section 4. Finally, the paper is concluded in section 5.

2. Dynamics of Glucose-Insulin

A variety of models have been presented for describing glucose-insulin. In 1980, Doctor Richard Bergman presented the Bergman's minimal model which is simple. As mentioned in [1], Bergman's model is commonly used in the literature and it is described as follows [1]:

$$\dot{x}_1 = -p_1[x_1 - G_b] - x_1 x_2 + \delta_1 + D(t) \tag{2.1}$$

$$\dot{x}_2 = -p_2 x_2 + p_3[x_3 - I_b] + \Delta_2 \tag{2.2}$$

$$\dot{x}_3 = -n[x_3 - I_b] + \gamma t[x_1 - G_b]^+ + \Delta_3 + u(t) \tag{2.3}$$

where $x_1(t)$, $x_2(t)$ and $x_3(t)$ represent concentration of plasma glucose, impact of insulin influence on reducing concentration of glucose, and concentration of insulin in plasma respectively, $u(t) \in \mathbb{R}$ is the rate of injected insulin measured in (mU/min), is the basal pre-injection level of glucose (mg/dl), I_b is the basal pre-injection level of insulin (μU/ml), p_1 the insulin independent rate constant of glucose uptake in muscles and liver (1/min), p_2 the decrease rate of in tissue glucose uptake ability (1/min), p_3 the insulin-dependent increase in glucose uptake ability in tissue per-unit of insulin concentration above the basal level ((μU/ml)/min).
$\gamma t[B_1(t) - G_b]^+$ is the secretion of the pancreatic insulin after a meal in take at $t = 0$. The parameters in (2.1)-(2.3) are assumed to be nominal which might differ with their real values. Thus, the lumped uncertainties resulting from mismatches are represented by Δ_1, Δ_2 and Δ_3. These uncertainties are assumed to be bounded as $|\Delta_1| \leq k_1$, $|\Delta_2| \leq d_2$ and $|\Delta_3| \leq d_3$ in which k_1, d_2 and d_3 are known positive constants. Since the focus of this study is on insulin therapy as a treatment for type I diabetes mellitus, γ is assumed to be zero so that real dynamic of this disease is modelled and p is also assumed to be zero. The parameter n represents the first order decay rate of insulin in blood. A decaying exponential function can be used to model disturbance as follows [16]:

$$D(t) = A\exp(-Bt) \qquad B > 0 \tag{2.4}$$

A first order linear system can be used to model the pump:

$$\dot{u}(t) = \frac{1}{a}(w(t) - u(t)) + \Delta_4 \tag{2.5}$$

in which $w(t)$ is the command rate of insulin in the pump as input, and a is the time constant of the pump. Moreover, Δ_4 represent the lumped uncertainty resulting from disagreement of the actual and nominal values of a.

3. The Proposed Controller And Stability Analysis

The tracking error of glucose is defined as

$$e_1 = x_1(t) - x_{1d}(t) \tag{3.1}$$

in which $x_{1d}(t)$ shows the favorable level of blood glucose. The following sliding surface is considered:

$$s_1 = e_1 + \int_0^t \lambda_1 e_1 dt \tag{3.2}$$

where λ_1 is a design parameter. By differentiating (3.2), we have:

$$\dot{s}_1 = \dot{e}_1 + \lambda_1 e_1 = \dot{x}_1 - \dot{x}_{1d} + \lambda_1 e_1 \tag{3.3}$$

By substituting $\dot{x}_1$ from (2.1) into (3.3) and solving , we have:

$$x_{2eq} = (x_1)^{-1}(-p_1 x_1 + p_1 G_b - \dot{G}_d + \lambda_1 x_1 - \lambda_1 G_d + d_1 sign(s_1)) \tag{3.4}$$

where $d_1 sign(s_1)$ is added to the control law to compensate the lumped certainty $\Delta_1 = \delta_1 + D(t)$ and the external disturbance $D(t)$. In other words, we have, $d_1 > |\Delta_1|$. By applying the control law (3.4) to (2.1), we have:

$$\dot{x}_1 = \dot{x}_{1d} - \lambda_1 e_1 + D + \Delta_1 - d_1 sign(s_1) \tag{3.5}$$

it can be rewritten as follows

$$\dot{s}_1 = D + \Delta_1 - d_1 sign(s_1) \tag{3.6}$$

The following positive definite function is considered to verify that stability of the control law (3.4)

$$V_1 = \frac{1}{2}s_1^2 \tag{3.7}$$

By differentiating (3.7), we have:

$$\dot{V}_1 = \dot{s}_1 s_1 \tag{3.8}$$

By substituting (3.6) into (3.8), we have:

$$\dot{V}_1 = s_1(D + \Delta_1 - d_1 sign(s_1)) \tag{3.9}$$

It is clear that

$$\dot{V}_1 \leq |s_1||D + \Delta_1| - s_1 d_1 sign(s_1) \tag{3.10}$$

which can be considered as follows

$$\dot{V}_1 \leq |s_1||\Delta_1| - d_1|s_1| = |s_1|(|\Delta_1| - d_1) \tag{3.11}$$

The following is concluded since $d_1 \geq |\Delta_1|$

$$\dot{V}_1 \leq 0 \tag{3.12}$$

By considering (2.2), the tracking error is defined as follows:

$$e_2 = x_2 - x_{2d} \tag{3.13}$$

in which x_{2d} represents the desired value of x_2. Consider the sliding surface defined in the following

$$s_2 = e_2 + \int_0^t \lambda_2 e_2 dt \tag{3.14}$$

where λ_2 is a design parameter. By differentiating (3.14), we have:

$$\dot{s}_2 = \dot{e}_2 + \lambda_2 e_2 = \dot{x}_2 - \dot{x}_{2d} + \lambda_2 e_2 \tag{3.15}$$

By substituting $\dot{x}_2$ from (2.2) into (3.15) and solving $\dot{s}_2 = 0$, we have:

$$x_{3eq} = \frac{p_2 x_2 + p_3 I_b + \dot{x}_{2d} - \lambda_2 e_2 + d_2 sign(s_2)}{p_3} \tag{3.16}$$

where $d_2 sign(s_2)$ is used to compensate the lumped uncertainty Δ_2. By applying the control law (3.16) into (2.2), we have:

$$\dot{x}_2 = \dot{x}_{2d} - \lambda_2 e_2 + \Delta_2 - d_2 sign(s_2) \tag{3.17}$$

in other words

$$\dot{x}_2 = \Delta_2 - d_2 sign(s_2) \tag{3.18}$$

By considering the following positive definite function

$$V_2 = \frac{1}{2} s_2^2 \tag{3.19}$$

and differentiating (3.19), we have

$$\dot{V}_2 = \dot{s}_2 s_2 \tag{3.20}$$

By substituting (3.18) into (3.20), we have

$$\dot{V}_2 = s_2(\Delta_2 - d_2 sign(s_2)) \tag{3.21}$$

which results in the following

$$\dot{V}_2 \leq |s_2||\Delta_2| - s_2 d_2 sign(s_2)) \tag{3.22}$$

which can be rewritten as follows

$$\dot{V}_2 \leq |s_2||\Delta_2| - d_2|s_2| = |s_2|(|\Delta_2| - d_2) \tag{3.23}$$

The following is concluded since

$$\dot{V}_2 \leq 0 \tag{3.24}$$

By considering (2.3), the tracking error is defined as follows

$$e_3 = x_3 - x_{3d} \tag{3.25}$$

in which x_{3d} represents the desired value of x_3. In addition, consider the sliding surface defined as follows:

$$s_3 = e_3 + \int_0^t \lambda_3 e_3 dt \tag{3.26}$$

where λ_3 is a design parameter. By differentiating (3.26), we have:

$$\dot{s}_3 = \dot{e}_3 + \lambda_3 e_3 = \dot{x}_3 - \dot{x}_{3d} + \lambda_3 e_3 \tag{3.27}$$

By substituting $\dot{x}_3$ from (2.3) into (3.27) and solving $\dot{s}_3 = 0$, we have:

$$u_d = n[x_3 - I_b] + \dot{x}_{3d} - \lambda_3 e_3 + d_3 sign(s_3) \tag{3.28}$$

Just like the procedure represented in (3.1) to (3.12), it can be demonstrated that:

$$\dot{V}_3 \leq |s_3|(|\Delta_3| - d_3) \leq 0 \tag{3.29}$$

where

$$V_3 = \frac{1}{2}s_3^2 \tag{3.30}$$

$$w_d = u + a\dot{u}_d - a\lambda_4 e_4 - ad_4 sign(s_4) \tag{3.31}$$

It is easily shown that this control law results in the following:

$$\dot{V}_4 \leq |s_4|(|\Delta_4| - d_4) \leq 0 \tag{3.32}$$

where

$$V_4 = \frac{1}{2}s_4^2 \tag{3.33}$$

$$s_4 = e_4 + \int \lambda_4 e_4 dt \tag{3.34}$$

$$e_4 = u - u_d \tag{3.35}$$

Now the following theorem is presented.

Theorem 3.1. *Considering the dynamic system* (2.1)-(2.3) *and applying the control laws* (3.4), (3.16)*,* (3.28) *and* (3.31)*, the closed-loop signals are bounded and the tracking errors e_1, e_2, e_3 and e_4 converge to zero, asymptotically.*

Proof . The Lyapunov function candidate is defined as follows:

$$V = \sum_{i=1}^{4} V_i = \sum_{i=1}^{4} \frac{1}{2} s_i^2 \tag{3.36}$$

where (3.7), (3.19), (3.30) and (3.33) define V_1, V_2, V_3 and V_4, respectively. The following is concluded according to (3.11), (3.23), (3.29) and (3.32):

$$\dot{V} \leq -\sum_{i=1}^{4} |s_i| \underbrace{(|\Delta_i| - d_i)}_{\alpha_i} \tag{3.37}$$

Assume that $\Gamma_i(s_i)$ is a positive semi definite differentiable function satisfying $\Gamma_i(s_i) \leq \alpha_i |s_i|$ Thus, $\sum_{i=1}^{4} \Gamma_i(s_i) \leq \sum_{i=1}^{4} \alpha_i |s_i|$. The following results from (3.37)

$$F(t) = \sum_{i=1}^{4} \Omega_i(s_i) \leq -\dot{V} \tag{3.38}$$

Integrating it with respect to time yields

$$\int_0^t F(\tau) d\tau \leq V(0) - V(t) \tag{3.39}$$

The following is concluded from boundedness of $V(0)$ and $V(t)$ being non-increasing and bounded

$$\lim_{t \to \infty} \int_0^t F(\tau) d\tau \leq \infty \tag{3.40}$$

$\dot{s}_1$ and $\dot{s}_2$ are bounded according to (3.6) and (3.18). And the same holds for $\dot{s}_3$ and $\dot{s}_4$. $d(\Omega_i)/ds_i$ is bounded. Thus, $\dot{\Omega}_i = \dot{s}_i d(\Omega_i)/ds_i$ is bounded and it follows from (3.38) that $\dot{F}$ is also bounded. Barbalat's lemma [17] guarantees convergence of the tracking error. □

Remark 3.2. *(Barbalat's lemma [17]): Assume that $f(t)$ has a finite time limit as $t \to \infty$ and $\dot{f}(t)$ is uniformly continuous (in other words, $\ddot{f}(t)$ is bounded), then $\dot{f}(t) \to 0$ as $t \to \infty$.*

Proof . If $f(t)$ in Barbalat's lemma is as follows

$$f(t) = \int_0^t F(\tau) d\tau \tag{3.41}$$

It follows from (3.40) that $f(t)$ has a finite time limit as $t \to \infty$. According to the above discussion, boundedness of $\ddot{f}(t)$ results from boundedness of $\dot{F}(t)$. Hence, it can be concluded from Barbalat's lemma that as $t \to \infty$, $\dot{f}(t) = F(t) \to 0$. Thus, (3.38) results in asymptotic convergence of $\Omega_i(s_i)$ to zero indicating the asymptotic convergence of the sliding surfaces s_i to zero and asymptotic convergence of e_1, e_2, e_3 and e_4 to zero. □

Remark 3.3. *To reduce the chattering problem resulting from the sign function, the proportional-integrator structure proposed in [17] or the modification proposed in can be used.*

Table 1: The model parameters

Bergman minimal model	
$P_1(min)^{-1}$	0
$P_2(min)^{-1}$	0.0123
$P_3(min)^{-1}$	8.2×10^{-8}
$n(min^{-1})$	0.2659
I_b	7
G_b	70
$x_1(0)$	200
$x_3(0)$	50

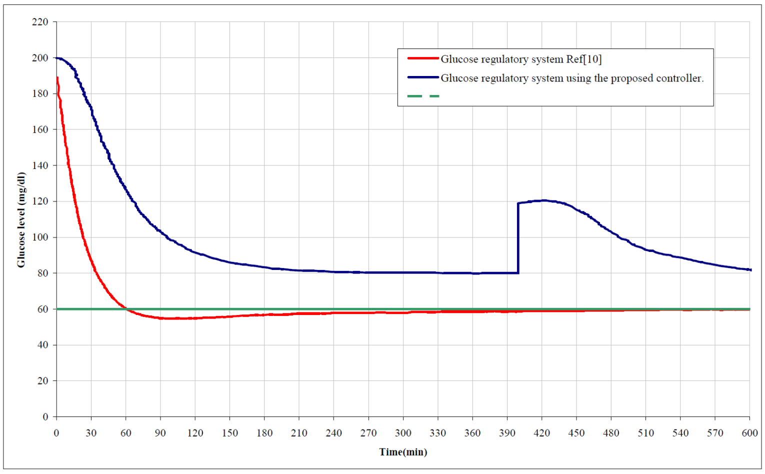

Figure 1: Concentration of Glucose

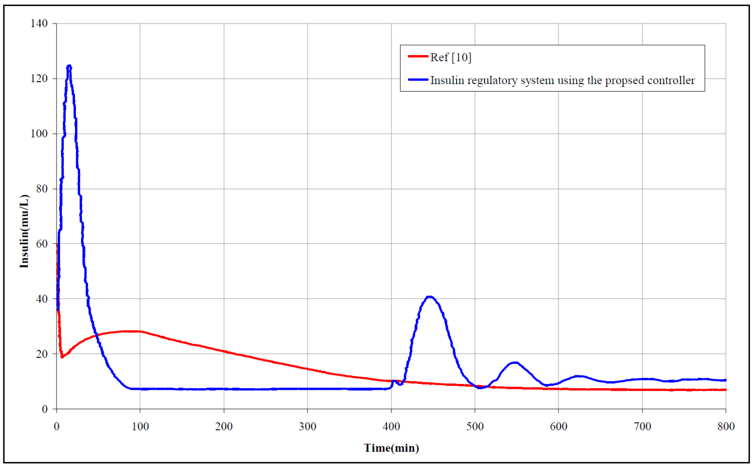

Figure 2: Concentration of Plasma insulin

4. Simulation Results

Parameters of the model described in [11] are given in Table 1. The controller's parameters are $\lambda_1 = 0.015$, $\lambda_2 = 0.05$, $\lambda_3 = 0.14$, $\lambda_4 = 0.1$. To study robustness of the controller against uncertainty, 10 percent uncertainty is applied to P_2 and P_3. Moreover, the control system is affected by the external disturbance $D(t) = 20\exp(-0.5t)$ at $t = 400$ (min). Fig. 1 represents the blood glucose level. It can be seen that concentration of the blood glucose can be reduced by the controller from 200 (mg/dl) to the desired level of 80 (mg/dl). Comparison with the controller proposed in [10, 13, 14] shows superiority of the controller presented in this study because there is a clear steady state in the response of the proposed controller, while the design presented in [10] has no steady state in its response.

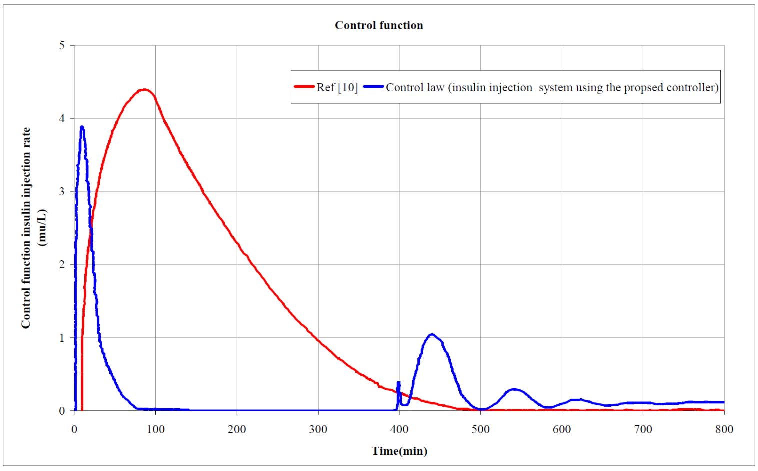

Figure 3: Control law (insulin injection with pump)

Fig. 2 shows concentration of the plasma insulin measured in (mU/L). It can be seen that values of this signal are acceptable. Fig. 3 shows he control signal $w(t)$ which is the insulin injection with pump. It can be seen that the control signal is bounded and no chattering is present.

5. Conclusion

This paper presented a robust controller for regulating blood glucose in type I diabetes using back-stepping sliding mode controller. In this design, external disturbance and parametric uncertainty are also considered. This design reduces the tracking error by integrating the tracking error in the sliding surface. The proposed controller outperforms previous designs. Simulation results verify the satisfactory performance of the proposed controller.

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