

# Prediction of Renewable Energy Production Using Grey Systems Theory

D. Darvishi Salookolaei<sup>a\*</sup>, P. Babaei<sup>a</sup>, S. Heydari gorji<sup>b</sup>

<sup>a</sup>Department of Mathematics, Payame Noor University, Tehran, Iran,

<sup>b</sup>Department of Management, Payame Noor University, Tehran, Iran.

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## Abstract

Due to the reduction of renewable energy resources such as fossil fuels, the energy crisis is one of the most critical issues in today's world. The application of these resources brings about many environmental pollutions that lead to global warming. Therefore, various countries have attempted to reduce potential damage and use renewable energies by the introduction and promotion of renewable energies as an essential strategy to reduce  $CO_2$  emissions and to find alternatives to fossil energy in the transportation and electricity generation sectors. This study attempts to predict the production process of renewable energies in Iran by 2025 and study the characteristics of this energy and its usage in the world and Iran. Since there are very few data in this field, four grey prediction models are used including GM(1,1), DGM(2,1), Grey Verhulst and FGM(1,1) models. According to the three indices of the error values of MSE, RMSE, and MAPE, all the predictions done by the methods above are among the best prediction methods. By examining the results achieved, FGM(1,1) method was the best model concerning its less error than other models and has estimated 16740.45 MW for renewable energy production in 2025.

**Keywords:** Prediction, Grey system, Absolute prediction error, Renewable Energy, GM(1,1).

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## 1. Introduction

An increase in global demand for energy from fossil fuels, such as oil and gas, plays a crucial role in the abundance of greenhouse gas emissions such as carbon dioxide and the resultant air pollution. One of the most important reasons for this increase is the immense growth in the world population and the advancement of technology and its accompanying problems, such as poverty, environmental

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\*Corresponding author

Email address: d\_darvishi@pnu.ac.ir, p.babaeivalohi@gmail.com, heydari1863@gmail.com (D. Darvishi Salookolaei<sup>a\*</sup>, P. Babaei<sup>a</sup>, S. Heydari gorji<sup>b</sup>)

issues caused by greenhouse gas emissions [1, 2]. Because of the energy crisis, all countries in the world seek to reduce energy consumption and find new resources to meet their needs [3]. Therefore, transition to renewable energy resources as alternative sources that have high capacity and are reliable, economical, and eternal is a necessity for the future [2, 4].

Renewable energies are divided into solar energy, wind energy, biomass energy, wave energy, etc. Furthermore, due to an increase in oil prices in global markets, the need for energy supply from other sources is felt significantly. Generally, energy is supplied from two available sources, e.g., renewable energy and non-renewable energy. Renewable energy resources such as solar energy, wind energy, tidal energy, and geothermal energy are infinite and without limitations. In contrast, the application of non-renewable energy resources is associated with their reduction and completion. The evidence shows that although Iran's potential to use renewable resources is very high, so far, they have not been appropriately applied. The consumption level of different types of energy in the world is shown in Fig. 1 [5].

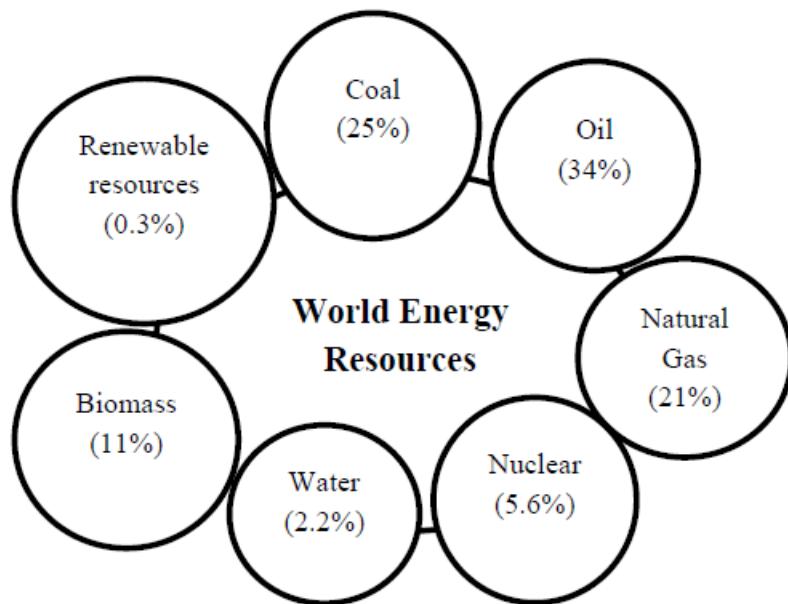


Figure 1: The amount of consumption of energy resources available in the universe

Historical data on renewable energies consist of a range of finite samples, and prediction methods require extensive data analysis. For example, conventional regression analyses with the genetic algorithm are not inadequate, nor can we use the ARIMA prediction models because these methods have limited number of data (i.e., at least 50 and preferably 100 observations or more) and cannot provide reliable results for low observations. Therefore, in this study, the grey system theory has been utilized, which can well deal with lack of reliability and incomplete information in systems. The grey system theory was initially used to control systems. By expanding its applications to use in other fields such as managerial decision, social and economic research, weather resources prediction has been developed. The grey system theory is mainly used in systems with incomplete information, uncertain behavioral patterns, and inaccurate scientific mechanisms that can be used to conduct comprehensive analyses and develop long-term predictions. Among the useful feature of this system, we can refer to use at least four data to make predictions. In this paper, GM(1, 1), DGM(2, 1), Grey Verhulst, and FGM(1, 1) models are also used, and the accuracy of the predictions has also been calculated. Finally, the optimal prediction method is determined in four models.

The present study is organized as follows: in the remainder, literature review and empirical studies conducted in this field are presented. In the Section 2, grey system theory is introduced. Different grey prediction methods and the ways of evaluating the prediction errors have been proposed in the Section 3. In the Section 4, the results and discussion about the predictions and calculated error of the methods of this research will be presented. Finally, the Section 5 will be devoted to conclusions and suggestions.

### *1.1. Population and Energy Status in the World*

Energy has a vital role in human life, economy and its development in communities. The existence of high energy resources in different parts of the world will satisfy the needs of people and economic development of different countries. Increasing the population with a very rapid trend in the world, especially in parts of Africa and Asia, coupled with an increase in the growth of life and the growing industrialization trend of societies and economic advancements in countries like China and India has led to a high increase in demand for energy and its high consumption throughout the world [1, 2]. The world's population in 2014 was about 7282.46 million people [2] which is predicted to reach 8.5 billion people by 2030, 9.7 billion people by 2050, and 11.2 billion people by 2100 [1], indicating that the population will rise by 2 billion people in each generation and by 83 million people in each year [1, 2]. The energy production process shows that approximately 77.9% of the world's electricity is supplied by fossil fuels, including oil, gas, coal, and nuclear energy [2]. With the current trend up to 2030, the production of primary energy resources is predicted to be about 655 quadrillion BTU, which is less than the world's demand for that year and generates energy shortages. The electricity production process in the world by 2030 shows that the current trend requires production of about 28500 billion kwh of electricity [6]. One of the most important problems of fossil fuels is the emission of all kinds of greenhouse gases that affect the lives of humans around the world, and about 80% of the world's carbon dioxide is dependent on the production and use of fossil fuels [1]. These gases pollute ecosystems [2] and lead to the death of 4 to 7 million humans worldwide [7]. In addition to having a special capacity for economic growth, renewable energies will lead to the diversification of the country's energy basket will promote energy security and can play a major role by reducing air pollution in environmental preservation.

### *1.2. Necessity and Limitation of Renewable Energy Consumption in Iran*

Among the ten most important challenges in the world over the next 50 years, the lack of energy and how to supply it, due to the finitude of fossil energy resources in the future, is one of the most fundamental and important issues in the future due to the finitude of fossil energy resources in the future . Iran produces about 1.75% of carbon dioxide ( $CO_2$ ) in the world, which represents high consumption of fossil energies [1]. Therefore, transition to renewable energy resources as alternative sources that have high capacity and are reliable, economical, and eternal is a necessity for the future [2]. The main constraints of the development of renewable energies in Iran include extremely high initial costs, technological problems, lack of financial resources to pay the difference in the purchase cost of renewable electricity and non-renewable electricity, high amounts of fossil fuel resources compared to renewable energy resources, and the existence of rich oil resources in Iran. Guaranteed purchase of electricity and products is a common way to encourage investment in renewable energies, as the short-term perspective in the industry is the main concern of investors in renewable energies. An increase in extra taxes for polluting industries could also be another way of paying more attention to renewable energies in some countries; however, these methods are very challenging. One of the most important reasons why the development of renewable energies has not blossomed in Iran, is Iran's policies toward energy, the constraints of financial resources as well as the lack of effective and advanced technologies for generating renewable electricity.

### 1.3. Research background

There are several studies about energy consumption in various countries and for different time periods by applying different techniques and parameters; however, this study investigates the prediction process of renewable energies that is the important point for predicting renewable energies in using four models of grey technique to investigate the prediction. The grey system theory was proposed by J. Deng in 1982 and later by Hang to solve uncertainties. If the clear and transparent information of a system is embodied in a white color and completely unknown information of a system in black, then the information about most of the systems in nature is not white (i.e., well-known) or black (i.e., not completely unknown), but a mixture of the two, i.e., grey. Such systems are called grey systems whose main feature is incompleteness of the information about that system [8]. Studies on the grey system theory can be divided into grey relational analysis, grey model construction, grey prediction, grey decision-making, and grey control. Grey prediction is one of the most important components of the grey system theory and is useful in solving uncertain problems with small and incomplete examples. Over the last three decades, the grey model is widely used in fields such as agriculture, industry, society, economy, transportation, geology, water and meteorology, environment, education, and health care. First, we briefly review various studies in the field of energy prediction in the world and then review the history of studies carried out in the field of grey prediction.

In a study entitled “a global review of Enhanced Geothermal System (EGS)”, Lu[9] estimates the world’s available geothermal potential to generate 1200 Giga watts of electricity, and predicted that 70 Giga watts of power would be produced out of this amount by 2050. In a study for the future of geothermal energy in Turkey and the world, Melikoglu [10] reported that Turkey’s available geothermal potential for generating electricity is 4500 megawatts and this country is considered the world’s seventh-rich country in this regard. Turkey plans to supply 30% of its total energy consumption from renewable energy resources by 2023, out of which the contribution of geothermal energy is 600 megawatts. In a study about the direct use of geothermal energy, Lund and Boyd [11] reported that 28, 58, 72, 78, and 82 countries have directly used this energy in 1995, 2000, 2005, 2010, and 2015, respectively, that by the end of 2014, the quantity of world usage has reached 70885 megawatts, i.e. a growth rate of 46.2% compared to 2010. The same amount of geothermal energy usage has saved up to 352 million barrels of oil and prevented 149.1 million tons of carbon dioxide emissions.

In this study, the important point to perform predictions in the field of renewable energies is to use four models of grey technique to examine the prediction that is very practical in modeling phenomena that have uncertainty and complexity. There are numerous studies on grey predictions in different fields, some of which will be referred to here. In a study entitled “application of fractional order-based grey power model in water consumption prediction”, Yuan et al. [12] have employed a grey optimized model (i.e., IAGO) to predict water consumption in Wuhan, China. This model can best extract the grey data hidden in the original data. Meanwhile, to improve the accuracy of the model, an algorithm is introduced to optimize the model parameters. In an article entitled “grey System analysis in the field of medicine and health”, Zhang et al. [13] predict the degree of infection of the respiratory system of the hospital using the GM(1, 1) grey model and providing theoretical foundations for the futuristic study of the management of respiratory tract infections of the hospital. Zheng et al. [14] have proposed a new approach in line with the relationship between  $CO_2$  emissions and economic growth using the grey system theory. The structural parameters of the model were used by employing a particle swarm optimization (PSO) to improve the accuracy of the optimized model. In their study, Sang-Bing, Tsai et al. [15] predicted the growth trends of renewable energy consumption in China, three grey prediction models, e.g., GM(1,1), NGM(1,1) and the Verhulst grey models according to theoretical and scientific theories.

## 2. Grey System Theory

The theory of grey systems was proposed in 1982 by Deng, and it was later employed by Hang for solving uncertainty problems. During late 1960s, Hang performed many studies on the prediction and control of economic and fuzzy systems when facing systems with many uncertainties. The indices of these systems could be roughly described by fuzzy mathematics and/or statistics and probabilities. To solve these systems optimally, Deng published an article under the title of “The Controlling Problems of Grey Systems” in 1982 to introduce grey systems theory. The major advantage of the grey systems theory is its need for low volume of data. In fact, grey systems theory has been posited as an effective method for solving the problems with discrete data and imperfect information [16]. The term “grey” in the grey system theory is a combination of black and white wherein black indicates indefinite information and white indicates perfect information. Grey points to incomplete information; in other words, the information that is somewhat clear and somewhat uncertain. These systems are called grey systems. To get more familiar with the theory of grey systems, the interested reader can refer to the studies in [17]. The grey system theory includes the following fields: grey generating, grey relational analysis, grey forecasting, grey decision making, and grey control. The majority of the prediction methods need a large number of data and statistical methods that are utilized to investigate the system properties. Moreover, systematic investigation is very difficult for the exogenous confusion in the system and the complex mutual interrelationships between the system and the peripheral environment. Grey prediction model, as the core of the grey system theory, features the advantage of creating a model using few and uncertain data and it is an appropriate instrument for the prediction of systems with complex, uncertain and disorganized structure. The grey prediction model is a lot more applicable and simpler than other prediction methods. Dervishi et al. [18] Used different models of grey prediction to estimation of rainfall in Iran. Also, they used the GM(1,1) and DGM(2,1) models to forecasting electricity consumption in Mazandaran province in Iran [19]. Darvishi and Babaei [20] used the GM(1,1) and fractional order accumulation into grey model. By using the grey prediction, grey linear programming problem with uncertain value of price product solve and optimal production was calculated. In these studies and the others, it is seen that grey system theory-based approaches can achieve good performance characteristics when applied to real-time systems. The reason behind it is that grey predictors adapt their parameters to new conditions as new outputs become available. Therefore, grey predictors are more robust with respect to noise and lack of modeling information when compared to conventional methods [21].

## 3. Grey Prediction Model

Grey prediction model investigates the preliminary data assisted by the grey differential equation to extract the rules governing the system. The model creates a dynamic and continuous differential equation from the series of discrete data to actualize time-series prediction. Each grey model is expressed in the form of GM (n,m) wherein n is the order of the differential equation and m determines the number of variables.

### 3.1. GM (1,1) Model

The Grey forecasting model GM (1,1) is a time series prediction model encompassing a group of differential equations adapted for parameter variance as well as a first-order differential equation. In this section, we focus on the grey prediction model, GM(1,1), which has been applied in many aspects of social and natural science, including decision-making, finance, economics, engineering and meteorology. GM(1,1) is the most applied models of time-series prediction model, and it is basically

an exponential model [22]. Liu and Deng studied the range suitable for GM(1,1) based on a simulated test. The area of validity, the area to be used carefully, the area not suitable for use and the prohibited area of GM(1,1) have been divided clearly according to the threshold of the developing coefficients (Liu et al., 2016). To smooth the randomness, the primitive data obtained from the system to form the GM(1,1) is subjected to an operator, named Accumulating Generation Operator (AGO) [23]. The operator reveals the internal order pattern of the data or the trends of the data series. Then, the differential equation operationalizes system prediction in n stages. Finally, the prediction values and Inverse Accumulated Generating Operator (IAGO) are applied to figure out the main data estimates [24]. The procedure of GM(1,1) grey prediction model can be summarized as follows:

**Step 1:** Let  $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}$  denote a non-negative sequence of original data, where  $n$  is the length of the raw data sequence and  $n \geq 4$ .

**Step 2:** The new cumulative data sequence  $X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)\}$ , which is the Accumulated Generating Operator (AGO) (Deng, 1982) of  $x^{(0)}$ , is obtained as  $x^{(1)}(k) = \sum_{i=1}^k x^{(0)}(i)$ ,  $k = 1, 2, 3, \dots, n$ . The data sequence could weaken the randomness of  $x^{(0)}$ . It is obvious that it is monotonically increasing.

**Step 3:** The generated mean sequence of is defined as:

$$Z^{(1)} = \{z^{(1)}(1), z^{(1)}(2), \dots, z^{(1)}(n)\} \quad (3.1)$$

where  $Z^{(1)}(k)$  is the mean value of adjacent data, i.e.

$$Z^{(1)} = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1), k = 2, 3, \dots, n. \quad (3.2)$$

The least-square estimate sequence of the grey difference equation of GM(1,1) is defined as follows:

$$Z^{(1)} = \alpha x^{(1)}(k) + (1 - \alpha)x^{(1)}(k-1), k = 2, 3, \dots, n. \quad (3.3)$$

$$x^{(1)}(k) + az^{(1)}(k) = b \quad (3.4)$$

Where  $a$  is the development coefficient, and  $b$  is the input grey coefficient or grey parameter.

**Step 4:** Define the first-order differential equation of sequence  $x^{(1)}$  as:

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b. \quad (3.5)$$

In above-mentioned equitation,  $t$  denotes the independent variables,  $a$  represents the grey developed coefficient of GM(1,1) model, and  $b$  is the grey controlled variable of the GM(1,1) model.

**Step 5:** Utilizing the least-squares estimation, we can derive the estimated first-order AGO sequence  $x_p^{(1)}(k+1)$  and the estimated inversed AGO sequence  $x_p^{(0)}(k+1)$  as follows,

$$x_p^{(1)}(k+1) = \left[ x^{(0)}(1) - \frac{b}{a} \right] e^{-ak} + \frac{b}{a}, \quad (3.6)$$

$$x_p^{(0)}(k+1) = x_p^{(1)}(k+1) - x_p^{(1)}(k). \quad (3.7)$$

Where  $k = 1, 2, 3, \dots, n$ .

Parameters  $a$  and  $b$  can be conducted by the least square estimation methods as the following equations:

$$[a, b]^T = [B^T B]^{-1} B^T y, \quad (3.8)$$

where  $y = [x^{(0)}(2), x^{(0)}(3), \dots, x^{(0)}(n)]^T$

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}. \quad (3.9)$$

To obtain the predicted value of the primitive data at a time  $(k+1)$ , IAGO is used to establish the following grey model:

$$x_p^{(0)}(k+1) = x_p^{(1)}(k+1) - x_p^{(1)}(k) = \left(x^{(0)}(1) - \frac{b}{a}\right)(1 - e^a)e^{-ak}, \quad k = 1, 2, \dots, n. \quad (3.10)$$

The predicted value of the primitive data at a time  $(k+h)$  can be obtained as follows:

$$x_p^{(0)}(k+h) = \left(x^{(0)}(1) - \frac{b}{a}\right)(1 - e^a)e^{-a(k+h-1)}, \quad k = 1, 2, \dots, n. \quad (3.11)$$

In large data areas, the grey system prediction method based on small data mining as a new force suddenly rises, which becomes an effective tool for valuable information extraction from a mass of data. It is a significant job to build more normal model testing standards based on the grey system prediction model testing method and statistical testing theory [25].

### 3.2. Fourier Residual Modification GM(1,1)

One of the methods used to increase the accuracy of the grey model is the remaining modification method by the Fourier series (Kayacan et al., 2010).

**Step 1.** Error sequence  $\varepsilon^{(0)}$  is defined as:

$$\varepsilon^{(0)} = \{\varepsilon^{(0)}(2), \varepsilon^{(0)}(3), \dots, \varepsilon^{(0)}(n)\} \quad (3.12)$$

$$\varepsilon^{(0)} = x^{(0)}(k) - x_p^{(0)}(k) \quad k = 2, 3, \dots, n. \quad (3.13)$$

where  $x^{(0)}(k)$  and  $x_p^{(0)}(k)$  represent the actual value and predicted value of the GM(1,1) method, respectively.

**Step 2.** Used the Fourier series for modifying the error values of GM(1,1) as the following equation:

$$\varepsilon^{(0)}(k) = \frac{1}{2}a_0 + \sum_{i=1}^z \left[ a_i \cos\left(\frac{2\pi i}{T}k\right) + b_i \sin\left(\frac{2\pi i}{T}k\right) \right] \quad k = 2, 3, \dots, n. \quad (3.14)$$

In the above equation  $T = n - 1$  is the period of alternation and  $z = [\frac{n-1}{2}] - 1$  denotes the minimum frequency of the Fourier series expansion.

**Step 3.** By applying the least-squares method, the parameters  $a_0$ ,  $a_i$  and  $b_i$  can be obtained.

$$\begin{aligned} C &= (P^T P)^{-1} P^T \varepsilon^{(0)} \\ \varepsilon^{(0)} &= P \times C \end{aligned} \quad (3.15)$$

$$P = \begin{bmatrix} \frac{1}{2} & \cos\left(2\frac{2\pi}{T}\right) & \sin\left(2\frac{2\pi}{T}\right) & \cos\left(2\frac{2\pi}{T}\right) & \sin\left(3\frac{2\pi}{T}\right) & \dots & \cos\left(2\frac{2\pi z}{T}\right) & \sin\left(2\frac{2\pi z}{T}\right) \\ \frac{1}{2} & \cos\left(3\frac{2\pi}{T}\right) & \sin\left(3\frac{2\pi}{T}\right) & \cos\left(3\frac{2\pi}{T}\right) & \sin\left(3\frac{2\pi}{T}\right) & \dots & \cos\left(3\frac{2\pi z}{T}\right) & \sin\left(3\frac{2\pi z}{T}\right) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{1}{2} & \cos\left(n\frac{2\pi}{T}\right) & \sin\left(n\frac{2\pi}{T}\right) & \cos\left(n\frac{2\pi}{T}\right) & \sin\left(n\frac{2\pi}{T}\right) & \dots & \cos\left(n\frac{2\pi z}{T}\right) & \sin\left(n\frac{2\pi z}{T}\right) \end{bmatrix}. \quad (3.16)$$

And

$$C = [a_0, a_1, b_1, a_2, b_2, \dots, a_z, b_z] \quad (3.17)$$

**Step 4.** Finally, the main prediction series is modified as follows:

$$x_{pf}^{(0)}(k) = x_p^{(0)}(k) + \varepsilon^{(0)}(k) \quad k = 2, 3, \dots, n. \quad (3.18)$$

### 3.3. DGM(2,1) Model

The DGM(2,1) model [26] is a single sequence second-order linear dynamic model and is fitted by differential equations. Assume an original series to be  $X^{(0)}$

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}, \quad (3.19)$$

a new sequence  $X^{(1)}$  is generated by the accumulated generating operation (AGO).

$$X^{(1)}(k) = \{x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)\}, \quad (3.20)$$

where

$$X^{(1)}(k) = \sum_{i=1}^k x^{(0)}(i), \quad k = 1, 2, \dots, n. \quad (3.21)$$

Setting up a second-order differential equation:

$$\frac{d^2x^{(1)}(t)}{dt^2} + a \frac{dx^{(1)}(t)}{dt} = b. \quad (3.22)$$

Where

$$[a, b]^T = [B^T B]^{-1} B^T y, \quad (3.23)$$

$$y = \begin{bmatrix} (x^{(0)}(2) - x^{(0)}(1)) \\ (x^{(0)}(3) - x^{(0)}(2)) \\ \vdots \\ (x^{(0)}(n) - x^{(0)}(n-1)) \end{bmatrix}. \quad (3.24)$$

$$B = \begin{bmatrix} -x^{(0)}(2) & 1 \\ -x^{(0)}(3) & 1 \\ \vdots & \vdots \\ -x^{(0)}(n) & 1 \end{bmatrix}. \quad (3.25)$$

According to Eq. (3.22), we have:

$$x_p^{(1)}(k+1) = \left[ \frac{b}{a^2} - \frac{x^{(0)}(1)}{a} \right] e^{-ak} + \frac{b}{a}(k+1) + \left( x^{(0)}(1) - \frac{b}{a} \right) \left( \frac{1+a}{a} \right). \quad (3.26)$$

The prediction values of the original sequence can be obtained by applying inverse AGO to  $x^{(1)}$ . Namely,

$$x_p^{(0)}(k+1) = x_p^{(1)}(k+1) - x_p^{(1)}(k) = \left[ \frac{b}{a^2} - \frac{x^{(0)}(1)}{a} \right] (1 - e^{-ak}) e^{-ak} + \frac{b}{a}, \quad k = 1, 2, \dots, n-1. \quad (3.27)$$

### 3.4. Grey Verhulst Model

As for non-monotonic wavelike development sequences, or saturated sigmoid sequences, one can consider establishing a grey Verhulst model. The main purpose of the Verhulst Model is to limit the whole development for a real system, and it is effective in describing some increasing processes. The Grey Verhulst model can be defined as [16]:

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b(x^{(1)})^2 \quad (3.28)$$

Grey difference equation of (3.28) is

$$x^{(0)}(k) + az^{(1)}(k) = b(z^{(1)}(k))^2. \quad (3.29)$$

Similar to the GM(1,1) model

$$[a, b]^T = [B^T, B]^{-1} B^T y \quad (3.30)$$

where  $y = [x^{(0)}(2), \dots, x^{(0)}(n)]^T$

$$B = \begin{bmatrix} -z^{(1)}(2) & (z^{(1)}(2))^2 \\ -z^{(1)}(3) & (z^{(1)}(3))^2 \\ \vdots & \vdots \\ -z^{(1)}(n) & (z^{(1)}(n))^2 \end{bmatrix}. \quad (3.31)$$

The solution of  $x^{(1)}(t)$  at time  $k$ :

$$x_p^{(1)}(k+1) = \frac{ax^{(0)}(1)}{bx^{(0)}(1) + (a - bx^{(0)}(1))e^{ak}} \quad (3.32)$$

Applying the IAGO, the solution of  $x^{(0)}(t)$  at time k:

$$x_p^{(0)}(k) = \frac{ax^{(0)}(1)(a - bx^{(0)}(1))}{(bx^{(0)}(1) + (a - bx^{(0)}(1))e^{a(k-1)})} \times \frac{(1 - e^a)e^{a(k-2)}}{(bx^{(0)}(1) + (a - bx^{(0)}(1))e^{a(k-2)}}. \quad (3.33)$$

### 3.5. Model Evaluation Scales

It should be noted that most predictions do not exactly match reality, and should try to minimize the forecast error. There are various techniques for forecasting, each of which has its own application. To compare the model precision, there are three common tools such as Mean Absolute Error (MAE), Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE). As a judgment method, prediction error determination indicates the success of a prediction model. This study adopted three criteria to evaluate the performance of the grey renewable energy forecasting model. RMSE is a part of a standard for evaluating the prediction precision. Standard deviation designates an example of the differences between the real and estimated values. The relative root-mean-square error is defined as:

$$RMSE = \sqrt{\frac{\sum_{k=1}^n (x^{(0)}(k) - x_p^{(0)}(k))^2}{n}}. \quad (3.34)$$

Where  $x^{(0)}(k)$  denotes the observed cumulative renewable energy at time  $t$ ,  $x_p^{(0)}(k)$  is the forecast cumulative renewable energy at time  $t$ . The RMSE represent a quantitative judgment of model performance.

MAE measures the difference between the real and the estimated values and it is expressed as below:

$$MAE = \frac{1}{n} \sum_{k=1}^n |x^{(0)}(k) - x_p^{(0)}(k)| \quad (3.35)$$

MAPE indicates the mean value of the prediction error ratio and it is explained as below:

$$MAPE = \frac{1}{n} \sum_{k=1}^n \left| \frac{x^{(0)}(k) - x_p^{(0)}(k)}{x^{(0)}(k)} \right| \times 100 \quad (3.36)$$

To compare the prediction power of MAPE, there are four considered regions. If these values are below 10%, the model prediction power could be envisaged excellent. Values between 10 and 20 percent are suggestive of good prediction and values in a range from 20% to 50% are indicative of an acceptable prediction. Values above 50% are expressive of imprecise prediction [27].

#### 4. Results and Discussion

In this research, annual renewable energy production data in Iran have been used which have been adapted from the global statistics site ([www.irena.org/resource](http://www.irena.org/resource)). The collected values were simulated using a time series of renewable energy in the 2012-2018 periods. For the 2019-2025 period, the prediction has been carried out based on four grey models, e.g., GM(1,1), FGM(1,1), DGM(2,1), and Grey Verhulst. The values obtained are given in the second column, and the predicted values are given in the third to the sixth columns of Table1. Then, based on MAE, RMSE, and MAPE error evaluation method, the accuracy of the prediction methods has been compared. The results of the calculations are summarized in tables1 and 2. As shown in Table 1, the first and second columns correspond to the year and the actual values of renewable energy. The third column of Table corresponds to the predicted values by the GM(1,1) model.

The fourth column of Table 1 is associated with the predicted values of the FGM(1,1) model. By applying the remaining Fourier series method, the predicted value is closer to the actual value. To increase the accuracy of the GM(1,1) model, the FGM(1,1) method has been applied. The DGM(2,1) method, which is a second-order linear dynamic model, is presented in the fifth column of Table 1. The last model studied is the Grey Verhulst model. The last column of Table 1 contains its predicted values. According to the values obtained in Table1, in 2025, the amount of renewable energy in Iran by the following grey models, e.g., GM(1,1), FGM(1,1), DG M(2,1), and Grey Verhulst, is estimated to be 16705.15, 16740.45, 14812.66, and 14661.15 megawatts, respectively.

##### 4.1. Comparison of Prediction Models

In the prediction studies, a variety of evaluation measures have been used by the authors. In this study, as shown in Table 1, four grey prediction models have been exploited, e.g., GM(1,1), FGM(1,1), DGM(2,1), and Grey Verhulst. Any prediction method that has less mean percentage error in the prediction will have more accuracy in prediction and can be used to predict future renewable energy production. In Table 2, the accuracy of prediction performed by different models is discussed.

As shown in Table 2, in the GM(1,1) model, the MSE, RMSE, and MAPE error values are 62.53, 74.19, and 0.55, respectively. Furthermore, in the FGM(1,1) model, the MSE, RMSE, and MAPE error values are 2.18, 2.17, and 0.018, respectively. These results show that FGM(1,1) a higher accuracy than the GM(1,1) method. For the DGM(2,1) model, the MSE, RMSE, and MAPE errors are 185.75, 198.75, 1.67, respectively. According to table2, for the Grey Verhulst model, the MSE, RMSE, and MAPE errors are 27.61, 30.98, and 0.24, respectively. According to the error values of the four models mentioned in Table2, all methods are considered suitable methods for prediction. The FGM(1,1) method is the best model since it has less error than other models. The chart below shows the actual and predicted values by the four models.

Table 1: Actual and estimated renewable energy production values

Year	Actual value(M.W)	GM(1,1)	FGM(1,1)	DGM(2,1)	Grey Verhulst
2012	9858	9858	9858	9858	9858
2013	10380	10498.13	10382.07	10144.55	10401.42
2014	10955	10912.49	10952.79	10692.68	10918.91
2015	11452	11343.19	11454.19	11205.18	11406.84
2016	11824	11790.89	11821.8	11684.36	11862.6
2017	12263	12256.27	12265.19	12132.39	12284.6
2018	12675	12740.01	12672.8	12551.3	12672.19
2019	-	13242.85	13283.15	12942.98	13345.45
2020	-	13765.53	13876.53	13309.19	13633.34
2021	-	14308.84	14339.75	13651.6	13890.98
2022	-	14873.59	14882.52	13971.75	14120.41
2023	-	15460.64	15393.43	14271.09	14323.81
2024	-	16070.85	15954.93	14550.97	14503.43
2025	-	16705.15	16740.45	14812.66	14661.5

Table 2: Comparative analysis of forecasting error

Model	MAE	RMSE	MAPE
GM(1,1)	62.53	74.19	0.55
FGM(1,1)	2.18	2.17	0.018
DGM(2,1)	185.75	198.75	1.67
GreyVerhulst	27.61	30.98	0.24

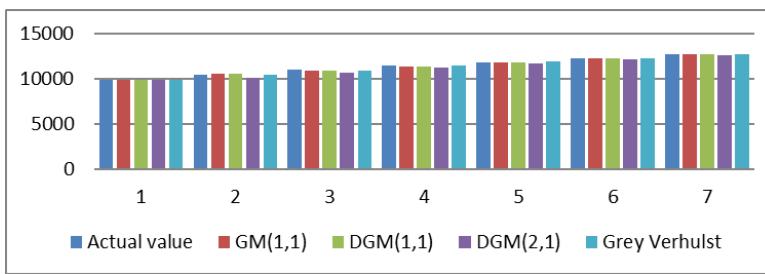


Figure 2: Comparison of actual values with predicted values

According to the results obtained, the FGM(1,1) model comparison has the least error compared with the other three models, and has the highest accuracy; hence, it is more appropriate for predictions. According to the evaluation of the error rate of the proposed prediction models, the FGM(1,1), Grey Verhulst, GM(1,1), and DGM(2,1) respectively have the least error.

## 5. Conclusion

Non-renewable energy or fossil energy resources are exhaustible. They destroy the environment in the stages of extraction, exploration, as well as improper consumption. As a result, factors such as

population explosion and the promotion of living standards require different energy resources more than ever. Moreover, the diversity of using different energies will make the country more reliable in terms of energy supply, so special attention is paid to renewable energies. With the current trend of energy consumption in the world and population growth, extraction from inexhaustible sources is highly increasing and will reduce these resources in the future. Considering the environmental issues around the use of these resources and their pollution, it is essential to move toward renewable sources. The whole world is now moving toward it. Renewable energy consumption in Iran is an emerging technology. In this paper, according to available data from the growth trend of renewable energy, this type of energy has been investigated using four grey prediction models, e.g., GM(1,1), FGM(1,1), DGM(2,1), and grey Verhulst. The comparison of predictions as per the grey sample and regression analysis using MSE, RMSE, and MAPE indices, shows that the FGM(1,1) model has the least prediction error and thus the highest accuracy and is more suitable for prediction. Consequently, it is predicted that in 2025, 16740.45 megawatts of renewable energy will be produced in Iran. Therefore, it is necessary to provide the required financial resources for the development of renewable energy resources.

## References

- [1] A. Shahsavari and M. Akbari, Potential of solar in developing countries for reducing energy-related emissions, *Renewable and Sustainable ENERGY Reviews*, 90 (2018) 275-291.
- [2] N. Kannan and D. Vakeesan, Solar energy for future world: A review, *Renewable and Sustainable Energy Reviews*, 62(2016) 1092-1105.
- [3] N. Mousavi, M. Mohebbi and M. Teimouri, Identifying the most applicable renewable energy systems of Iran, *International Journal of Scientific and Technology Research*, 6(30)(2017)51-59.
- [4] B. Kumar, A study on global solar PV energy developments and policies with special focus on the top ten solar PV power producing countries, *Renewable and sustainable Energy Reviews*, 43(2017) 621-634.
- [5] M. A. Green, Third-generation photovoltaics: solar for 2020 and beyond, *physical E: Low dimensional Systems and Nanostructures*, 14(1-2)(2002)65-70.
- [6] International Energy Agency, International Energy Statistics, Available from: <http://www.Eia.gov/cfapps/ipdproject/IEDIndex3.cfm>(2018).
- [7] M.Z. Jacobson, et al., 100% Clean and Renewable Wind, Water, and Sunlight All-Sector Energy Roadmaps for 139 Countries of the World, 1(1)(2017)108-121.
- [8] J.L. Deng, The control problems of grey systems, *System Control Letter*, 5(1982)288–294
- [9] S.M. Lu, A global review of the enhanced geothermal system, *Renewable and Sustainable Energy Reviews*, 81(2018)2902-2921.
- [10] M. Melikoglu, Geothermal energy in turkey and around the world: A review of the literature and an analysis based on Turkey's Vision 2023 energy targets, *Renewable and Sustainable Energy Reviews*, 76(2017) 485-492.
- [11] J.W. Lund and T. L. Boyd, Direct utilization of geothermal energy worldwide review, *Geothermics*, 60(2016) 66-93.
- [12] Y. Yuan, H. Zhao, X. Yuan, L. Chen, and X. Lei, Application of fractional order-based gray power model in water consumption prediction, *Environmental Earth Sciences*, 78(8)(2019)266, 1-8.
- [13] L. Zhang, H. Tang, and M. He, Gray System Analysis in the Field of Medicine and Health, School of Public Health, China Medical University, Shenyang, China(2019).
- [14] Zheng-Xin Wang, Li. Qin, Modelling the nonlinear relationship between CO<sub>2</sub> emissions and economic growth using a PSO algorithm-based grey Verhulst model, *Journal of Cleaner Production*, 207(10)(2019) 214-224.
- [15] Sang -Bing, Tsai., Youzhi Xue, Jianyu Zhang, Quan Chen, Yubin Liu, Jie Zhou., Weiwei Dong. (2016). Models for forecasting growth trends in renewable energy, *Renewable and Sustainable Energy Reviews*, 77, 1169-1178.
- [16] K. L. Wen and Y. F. Huang, The development of grey Verhulst toolbox and the analysis of population saturation state in Taiwan–Fukien, *International Conference on Systems, Man and Cybernetics*, Netherlands, 6(2004). 5007–5012.
- [17] K. Li and T. Zhang, Forecasting electricity consumption using an improved grey prediction model, *Information*, 9(2018)1–18.
- [18] D. Darvishi, P. Babaei and S. Liu, Application of Grey System Theory in Rainfall Estimation, Control and Optimization in Applied Mathematics, 2(2)(2017a)15-31.

- [19] D. Darvishi, P. Babaei and S. Liu, Grey prediction model for forecasting electricity consumption, International Journal of Applied Operational Research, 7(4) (2017b) 1-9.
- [20] D. Darvishi and P. Babaei , Grey prediction in linear programming problems, International Journal of Applied Operational Research, 9(1)(2018).11-18.
- [21] E. Kayacan, B. Ulutas and O. Kaynak, Grey system theory-based models in time series prediction, Expert Systems with Applications, 37 (2010)1784–1789.
- [22] S. Liu, Y. Yang, N. Xie and J. Forrest, New progress of grey system theory in the new millennium, Grey Systems: Theory and Application, 6(1)(2016) 2-31.
- [23] J.L. Deng, Introduction to grey system theory, The Journal of Grey System, 1(1)(1989)1–24.
- [24] S. Sorooshian, A. AghaKouchak, P. Arkin, J. Eylander, E. Foufoula-Georgiou, R. Harmon, M.H. Jan ,H. Bisher Imam, R. Kuligowski, B. Skahill and G. Skofronick-Jackson, Advancing the remote sensing of precipitation, Bulletin of the American Meteorological Society. 92(10)(2011)1271–1272.
- [25] S. Liu, Y. Yang and J. Forrest ,Grey Data Analysis, Springer, Singapore (2017).
- [26] J. Li ,The optimization DGM(2,1) model and its application, Journal of Grey System, 24(2)(2012) 181–186.
- [27] C.D. Lewis, Industrial and Business Forecasting Methods: A Practical Guide to Exponential Smoothing and Curve Fitting, Butterworth Scientific London, UK(1982).