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Analytical Solution for the Time Fractional Newell-Whitehead-Segel Equation by Using Modified Residual Power Series Method

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Abstract

The Newell-Whitehead-Segel equation is an important model arising in biology, finance, fluid-mechanics and some more processes. Various researchers worked on approximate solution of this model by using different methods. In this paper, the Newell-Whitehead-Segel equation of fractional order is solved by using a generalized Taylor series formula together with residual error function, which is named the residual power series method (RPSM). The illustrative examples are presented to demonstrate the accuracy and effectiveness of the proposed method.

Keywords: Functional residual power series, Newell-Whitehead-Segel equation of fractional order, Caputo fractional derivative.
2010 MSC: 68M10

1. Introduction

Today, the fractional differential equations are more and more important in many fields such as mathematics and dynamic system [1, 3], signal processing [4], control theory [5], and economics [6]. The person who firstly proposed fractional differential equation was Leibniz and L' Hopital in 1695. Diethelm and Ford [7] proposed an analytical questions of existence and uniqueness of solutions by fractional differential equations. Recently, various methods have been proposed to solve nonlinear fractional differential equations such as Adomian decomposition method [8, 9], differential transform

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method [10], homotopy perturbation method [11], variational iteration method [12, 13], homotopy analysis method [14, 16], homotopy analysis transform method [17, 20], fractional variation iteration method [21, 24] etc.

Recently, an analytical method based on power series expansion without linearization, discetization, or perturbation has been introduced and successfully applied to many kinds of fractional differential equations arising in nonlinear and dynamic problems. The method was named residual power series method (RPSM) that has been widely used in different fields [25, 35]. Furthermore, in [29, 36], residual power series method used to solve the time fractional differential equations.

In this paper, an analytical solution of the time fractional Newell-Whitehead-Segel equations is proposed by a modified version of this method. In natural phenomena, non-equilibrium systems are usually shown in many extended states: uniform, oscillatory, chaotic and pattern states. Many stripes patterns such as ripples in sand, stripes of seashells arise in a variety of spatially extended systems which can be modeled by a set of equations called amplitude equations. One of the most well-known amplitude equation is the Newell-Whitehead-Segel equations that describe the dynamical behavior near the bifurcation point of the Rayleigh-Benard convention of the binary fluid mixtures [37].

The Newell-Whitehead-Segel equation [38, 39] of integer order is

$$\frac{\partial U}{\partial t} = k\frac{\partial^2 U}{\partial x^2} + aU - bU^q \tag{1.1}$$

The classical Newell-Whitehead-Segel equations have been studied by Laplace Adomian decompositions method [40], differential transform method [41], reduced differential transform method [42], Adomian decomposition method [43, 44], homotopy perturbation method [45, 47], iterative method [48],variational iterative method [49], finite difference scheme [50], etc. In this paper, we consider the fractional model of Newell-Whitehead-Segel equation of the form

$$\frac{\partial^\alpha U}{\partial t^\alpha} = k\frac{\partial^2 U}{\partial x^2} + aU - bU^q, 0 < \alpha \leq 1 \tag{1.2}$$

where α is a parameter which describes the order of the time fractional derivative.

The fractional derivative has been taken in Caputo sense. If we take $\alpha = 1$ the fractional Newell-Whitehead-Segel of Eq.(1.2) reduces to the classical Newell-Whitehead-Segel Eq.(1.1). In fractional differential equations, the general response expression contains a parameter describing the order of the fractional derivative that can be varied to obtain various responses.

The time-fractional Newell-Whitehead-Segel equations describe particle motion with memory in time. Spase-fractional derivative arises when variations are heavy-tailed and describes particle motion that accounts for variation in the flow field over the entire system. Therefore, the study of the time fractional Newell-Whitehead-Segel of Eq.(1.2) is very important.

Recently, some methods have been proposed to solve Newell-Whitehead-Segel equation of fractional order such as homotopy analysis Sumudu transform method [51] and variational iteration method [52].

This paper is organized as follows. In Section 2, some basic definitions about the Caputo and Modified Residual Power Series Method (MRPS) method are introduced. In Section 3, we use the MRPS method to solve the time fractional Newell-Whiteheah-Segel. Numerical results and discussions are presented by graphics in Section 4. Finally, the conclusions are given in Section 5.

2. Modified residual power series method

In this section, some definitions and theorems regarding the fractional derivatives are reviewed systematically.

Definition 2.1. *[3] Let $f(x) : [0, +\infty) \to \mathbb{R}$ be a function and n be the upper positive integer of $\alpha(\alpha > 0)$. The Caputo fractional derivative is defined by*

$$D^{\alpha} f(t) = \begin{cases} \dfrac{1}{\Gamma(n-\alpha)} \int_0^x (t-\tau)^{n-\alpha-1} d\tau & n-1 < \alpha < n, \\ \dfrac{d^n f(x)}{dx^n} & \alpha = n \in N. \end{cases} \tag{2.1}$$

Theorem 2.2. *[3] The Caputo fractional derivative of the power function satisfies*

$$D^{\alpha} x^q = \begin{cases} \dfrac{\Gamma(q+1)}{\Gamma(q+1-\alpha)} x^{q-\alpha} & \alpha \le q, \\ 0 & \alpha > q. \end{cases} \tag{2.2}$$

The important theorems that are related to the fractional power series were presented by El-Ajou et al. [25, 26]. These theorems are constructed by using Caputo fractional derivatives.

We present the following definition and some properties the fractional power series which are used in this paper. More details can be found in [31, 33].

Definition 2.3. *[25, 26] For $0 \le n-1 < \alpha \le n$ a power series expansion of the form below:*

$$\sum_{m=0}^{\infty} c_m (t-t_0)^{m\alpha} = c_0 + c_1 (t-t_0)^{\alpha} + c_2 (t-t_0)^{2\alpha} + ..., \qquad t \ge t_0 \tag{2.3}$$

is called fractional power series about $t = t_0$, where is variable and c_m are constants called the coefficients of the series.

Theorem 2.4. *[25] Suppose that f has a fractional power series (FPS) representation at $t = t_0$ of the form*

$$f(t) = \sum_{m=0}^{\infty} c_m (t-t_0)^{m\alpha}, \quad 0 \le n-1 < \alpha \le n, t_0 \le t < t_0 + R. \tag{2.4}$$

If $D^{m\alpha} f(t) \in (t_0, t_0 + R)$ then coefficients c_m of (2.4) are given by the formula complexity

$$c_m = \frac{D^{m\alpha} f(t_0)}{\Gamma(m\alpha + 1}, \qquad m = 0, 1, 2, ... \tag{2.5}$$

where $D^{m\alpha} = D^{\alpha}.D^{\alpha}...D^{\alpha}$ (m-times) and R is the radius of convergence.

Theorem 2.5. *[25] Let $u(x) \in C([x_0, x_0 + R))$ and $D^{i\alpha} u(x) \in C((x_0, x_0 + R))$ for $i = 0, 1, ..., m+1$ where $0 \le m-1 \le \alpha \le m$. Then*

$$I^{(m+1)\alpha} D^{(m+1)\alpha} u(x) = \frac{D^{(m+1)\alpha}(\overline{w})}{\Gamma((m+1)\alpha + 1)} (x-x_0)^{(m+1)\alpha+1}, \quad x_0 \le \overline{w} \le x \le x_0 + R. \tag{2.6}$$

Theorem 2.6. *[25] Let $u(x) \in C([x_0, x_0 + R))$ and $D^{i\alpha} u(x) \in C((x_0, x_0 + R))$ and $D^{i\alpha} u(x)$ can be differentiate $m-1$ with respect to x for $i = 0, 1, ..., m+1$ where $0 \le m-1 \le \alpha \le m$. Then,*

$$u(x) = \sum_{k=0}^{m} \frac{D^{k\alpha}(\overline{w})}{\Gamma(k\alpha + 1)} (x-x_0)^{(m+1)\alpha+1}, \quad x_0 \le \overline{w} \le x \le x_0 + R. \tag{2.7}$$

Theorem 2.7. *[25] Let $|D^{(m+1)\alpha}u(x)| \in A$ on $x_0 \leq x \leq s$ where $m-1 \leq \alpha \leq m$. Then, the reminder R_m satisfies*

$$|R_m| \leq \frac{A}{\Gamma((m+1)\alpha+1)}(x-x_0)^{(m+1)\alpha+1}, \quad x_0 \leq x \leq s. \tag{2.8}$$

Where $R_m = u(x) - \sum_{k=0}^{m} \dfrac{D^{k\alpha}(x_0}{\Gamma(k\alpha+1)}(x-x_0)^{k\alpha}$.

Definition 2.8. *[26] A power series of the form*

$$\sum_{m=0}^{\infty} f_m(x)(t-t_0)^{k\alpha} = f_0(x) + f_1(x)(t-t_0)^{\alpha} + f_2(x)(t-t_0)^{2\alpha} + \dots \tag{2.9}$$

for $0 \leq n-1 < \alpha \leq n$ and $t \geq t_0$ is called multiple fractional power series about $t = t_0$ where t is a variable and f_m are functions of x that called the coefficients of the series.

Theorem 2.9. *[25, 26] Suppose that $u(x,t)$ has a multiple power series representation at $t = t_0$ of the form*

$$u(x,t) = \sum_{m=0}^{\infty} f_m(x)(t-t_0)^{k\alpha}, \quad 0 \leq n-1 < \alpha \leq n, \quad t \leq t_0 < t_0 + R. \tag{2.10}$$

If $D_t^{m\alpha}(x,t)$ are continuous on $I \times (t_0, t_0+R)$, $m = 0,1,2,\dots$ then coefficients $f_m(x)$ of (2.7) are given as

$$f_m(x) = \frac{D_t^{m\alpha}u(x,t_0)}{\Gamma(m\alpha+1}, \quad m = 0,1,2,\dots \tag{2.11}$$

where $D_t^{m\alpha} = \dfrac{\partial^{m\alpha}}{\partial t^{m\alpha}} = \dfrac{\partial^{\alpha}}{\partial t^{\alpha}}.\dfrac{\partial^{\alpha}}{\partial t^{\alpha}}.\dfrac{\partial^{\alpha}}{\partial t^{\alpha}}...\dfrac{\partial^{\alpha}}{\partial t^{\alpha}}$ (m - times) and $R = min_{c\in I}R_c$, in which R_c is the radius of convergence of the following fractional power series

$$\sum_{m=1}^{\infty} f_m(c)(t-t_0)^{m\alpha}.$$

3. Solution of the time fractional newell-whitehead-segel equation by MRPS method

In this section, an analytical solution of Newell-Whitehead-Segel equation (NWS) of fractional order is proposed by the Modified Residual Power Series method (MRPS). Consider the following class of fractional initial value NWS equation of the form

$$\frac{\partial^{\alpha}U}{\partial t^{\alpha}} = K\frac{\partial^2 U}{\partial x^2} + aU - bU^q, \quad 0 < \alpha \leq 1 \tag{3.1}$$

$$u(x,0) = f_0(x) \tag{3.2}$$

Using the MRPS method, the solution problems of (3.1) - (3.2) can be written the fractional power series form as

$$u(x,t) = \sum_{n=0}^{\infty} f_n(x)\frac{t^{\alpha}}{\Gamma(1+n\alpha)}. \tag{3.3}$$

In order to obtain the approximate value of (2.2), the form of the ith series $u(x,t)$ is proposed. Then the truncated series $u_i(x,t)$ is defined by:

$$u_i(x,t) = \sum_{n=0}^{i} f_n(x) \frac{t^{\alpha}}{\Gamma(1+n\alpha)}, \tag{3.4}$$

where $t = 0, u(x,0) = f_0(x)$ is considered to be the 1st MRPS approximate solution of $u(x,t)$. To find the values of the MRPS-coefficients $f_n(n = 1, 2, 3, ...)$, we solve the fractional differential equation

$$D_t^{(n-1)\alpha} Res_n(u(x,0)) = 0, \quad n = 1, 2, 3, ... \tag{3.5}$$

We define the ith residual function of Newell-Whitehead-Segel equation of fractional order as follows:

$$Res_i(x,t) = D_t^{\alpha} u_i(x,t) - k u_{i,xx}(x,t) - a u_i(x,t) + b(u_i(x,t))^q, \tag{3.6}$$

or,

$$\begin{aligned} Res_i(x,t) = & \sum_{n=0}^{i} f_n(x). \frac{\partial^{\alpha}}{\partial t}(\frac{t^{\alpha}}{\Gamma(1+n\alpha)}) - k(\sum_{n=0}^{i} f_n''(x)(\frac{t^{\alpha}}{\Gamma(1+n\alpha)})) \\ & - a(\sum_{n=0}^{i} f_n(x)(\frac{t^{\alpha}}{\Gamma(1+n\alpha)})) + b(\sum_{n=0}^{i} f_n(x)\frac{t^{\alpha}}{\Gamma(1+n\alpha)})^q. \end{aligned} \tag{3.7}$$

Here, we present two examples to show the efficiency of the proposed method. We use Matlab software to generate the results in this section.

Example 3.1. *Consider a linear time-fractional Newell-Whitehead-Segel equation*

$$D_t^{\alpha} U = U_{xx} - 2U, \quad 0 < \alpha < 1, \tag{3.8}$$

with initial condition

$$U(x,0) = e^x. \tag{3.9}$$

The exact solution of Eqs. (3.8) - (3.9) *is*

$$U(x,t) = e^x E_{\alpha}((-t)^{\alpha}). \tag{3.10}$$

When $\alpha = 1$, *the exact solution of Eqs.* (3.8) – (3.9) *is* $U(x,t) = e^{x-t}$. $Res_i(x,t)$ *is the ith residual function of* (3.8), *which is defined as*

$$Res_i(x,t) = D_t^{\alpha} u_i(x,t) - u_{i,xx}(x,t) + 2u_i(x,t). \tag{3.11}$$

Step 1. For $i = 1$, *the residual function of the time fractional Newell-Whitehead-Segel equation can be written as*

$$Res_1(x,t) = D_t^{\alpha} u_1(x,t) - u_{1,xx}(x,t) + 2u_1(x,t). \tag{3.12}$$

Where $u_1(x,t)$ *can be written by* (3.4) *as*

$$u_1(x,t) = f_0(x) + f_1(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} \tag{3.13}$$

Then we get:

$$\begin{aligned} Res_1(x,t) &= D_t^\alpha u_1(x,t) - u_{1,xx}(x,t) + 2u_1(x,t) \\ &= f_1(x) - f_0''(x) - f_1''(x)\frac{t^\alpha}{\Gamma(1+\alpha)} + 2f_0(x) + 2f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)} \\ &= f_1(x) - e^x - f_1''(x)\frac{t^\alpha}{\Gamma(1+\alpha)} + 2e^x + 2f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}. \end{aligned} \tag{3.14}$$

For $t = 0$ we have:

$$Res_1(x,t) \mid_{t=0} = f_1(x) - e^x + 2e^x. \tag{3.15}$$

Then according to $Res_1(x,t) \mid_{t=0} = 0$, we have:

$$f_1(x) = -e^x. \tag{3.16}$$

Step 2. For $i = 2$, the residual function of the time fractional Newell-Whitehead-Segel equation can be written as

$$Res_2(x,t) = D_t^\alpha u_2(x,t) - u_{2,xx}(x,t) + 2u_2(x,t), \tag{3.17}$$

with the condition

$$u_2(x,t) = f_0(x) + f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}. \tag{3.18}$$

Therefore, we can obtain

$$\begin{aligned} Res_2(x,t) &= D_t^\alpha u_2(x,t) - u_{2,xx}(x,t) + 2u_2(x,t) \\ &= \lceil f_1(x) + f_2(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\rceil - \lceil f_0''(x) + f_1''(x)\frac{t^\alpha}{\Gamma(1+\alpha)} + f_2''(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\rceil \\ &+ 2\lceil f_0(x) + f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\rceil. \end{aligned} \tag{3.19}$$

Then we solve $D_t^\alpha Res_2(x,0) = 0$; thus,

$$f_2(x) = e^x. \tag{3.20}$$

Step 3. For $i = 3$, the residual function of the time fractional Newell-Whitehead-Segel equation can be written as

$$Res_3(x,t) = D_t^\alpha u_3(x,t) - u_{3,xx}(x,t) + 2u_3(x,t). \tag{3.21}$$

With the condition

$$u_3(x,t) = f_0(x) + f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + f_3(x)\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}. \tag{3.22}$$

Then we can get:

$$\begin{aligned} Res_3(x,t) &= D_t^{\alpha} u_3(x,t) - u_{3,xx}(x,t) + 2u_3(x,t) \\ &= \lceil f_1(x) + f_2(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} + f_3(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\rceil \\ &- \lceil f_0''(x) + f_1''(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} + f_2''(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + f_3''(x)\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}\rceil \\ &+ 2\lceil f_0(x) + f_1(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + f_3(x)\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}\rceil. \end{aligned} \tag{3.23}$$

Then we solve $D_t^{\alpha} Res_3(x,0) = 0$ *thus,*

$$f_3(x) = -e^x. \tag{3.24}$$

To continue this process, we have:

$$D_t^{\alpha} Res_i(x,0) = 0 \longrightarrow f_i(x) = (-1)^i f_0(x). \tag{3.25}$$

So, the analytical solution of the MRSP method is equal to the exact method.

Example 3.2. *Consider a nonlinear time fractional Newell-Whitehead-Segel equation*

$$D_t^{\alpha} U = U_{xx} + 2U - 3U^2, \quad 0 < \alpha \leq 1, \tag{3.26}$$

with the initial condition

$$U(x,t) = \lambda. \tag{3.27}$$

When $\alpha = 1$, *the exact solution of Eqs.* (3.25) - (3.26) *is* $U(x,t) = \dfrac{-\frac{2}{3}\lambda e^{2t}}{-\frac{2}{3} + \lambda - \lambda e^{2t}}$.

$Res_i(x,t)$ *is the ith residual function of* (3.25), *which is defined as*

$$Res_i(x,t) = D_t^{\alpha} u_i(x,t) - u_{i,xx}(x,t) + 2u_i(x,t) + 3(u_i(x,t))^2. \tag{3.28}$$

Step 1. For $i = 1$, *the residual function of the nonlinear time fractional Newell-Whitehead-Segel equation can be written as*

$$Res_1(x,t) = D_t^{\alpha} u_1(x,t) - u_{1,xx}(x,t) + 2u_1(x,t) + 3(u_1(x,t))^2. \tag{3.29}$$

Where $u_1(x,t)$ *can be written by* (3.3) *as*

$$u_1(x,t) = f_0(x) + f_1(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} \tag{3.30}$$

Then we get

$$\begin{aligned} Res_1(x,t) &= D_t^{\alpha} u_1(x,t) - u_{1,xx}(x,t) + 2u_1(x,t) + 3(u_1(x,t))^2 \\ &= f_1(x) - f_0''(x) - f_1''(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} - 2f_0(x) - 2f_1(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} \\ &+ 3(f_0(x) + f_1(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)})^2 \\ &= f_1(x) - f_1''(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} - 2\lambda - 2f_1(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} \\ &+ 3(\lambda + f_1(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)})^2. \end{aligned} \tag{3.31}$$

For $t = 0$ we have

$$Res_1(x,t) \mid_{t=0} = f_1(x) - 2\lambda + 3\lambda^2. \tag{3.32}$$

Then according to $Res_1(x,t) \mid_{t=0} = 0, f_0(x) = \lambda$, we have

$$f_1(x) = 2\lambda - 3\lambda^2. \tag{3.33}$$

Step 2. For $i = 2$, the residual function of the nonlinear time fractional Newell-Whitehead-Segel equation can be written as

$$Res_2(x,t) = D_t^{\alpha} u_2(x,t) - u_{2,xx}(x,t) + 2u_2(x,t) + 3(u_2(x,t))^2. \tag{3.34}$$

With the condition

$$u_2(x,t) = f_0(x) + f_1(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}. \tag{3.35}$$

Therefore, we can attain

$$\begin{aligned} Res_2(x,t) &= D_t^{\alpha} u_2(x,t) - u_{2,xx}(x,t) + 2(u_2(x,t))^2 \\ &= \lceil f_1(x) + f_2(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)}\rceil - \lceil f_0''(x) + f_1''(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} + f_2''(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\rceil \\ &- 2\lceil f_0(x) + f_1(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\rceil \\ &+ 3(\lceil f_0(x) + f_1(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\rceil)^2. \end{aligned} \tag{3.36}$$

Next, we solve $D_t^{\alpha} Res_2(x,0) = 0$ thus,

$$f_2(x) = -6\lambda(2\lambda - 3\lambda^2). \tag{3.37}$$

Proceeding in this manner the enduring components can be obtained using Matlab software. Hence we find the solution as $U(x,t) = lim_{n\to\infty} U_n(x,t)$.

Now by taking $\alpha = 1$ we get the solution of classical nonlinear Newell-Whitehead-Segel equation as

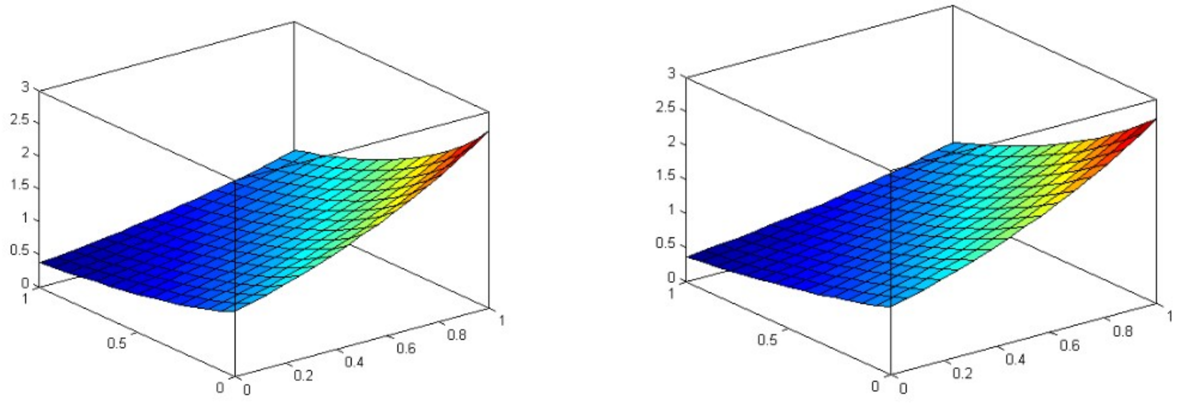
$$U(x,t) = \lambda + (2\lambda - 3\lambda^2)\frac{t}{1!} + 2(1-3\lambda)(2\lambda - 3\lambda^2)\frac{t^2}{2!} + 6\lambda(2-3\lambda)^2\frac{t^3}{3!} + ... \tag{3.38}$$

Which converge very fast to the following exact solution:

$$U(x,t) = \frac{-\frac{2}{3}\lambda e^{2t}}{-\frac{2}{3} + \lambda - \lambda e^{2t}}. \tag{3.39}$$

4. Result and discussion

In this section, the approximate analytical solution of the time fractional Newell-Whitehead-Segel equation by using modified residual power series method is calculated. We can compare the exact solution of the linear and nonlinear Newell-Whitehead-Segel equations with the analytical approximate solution by graphics.

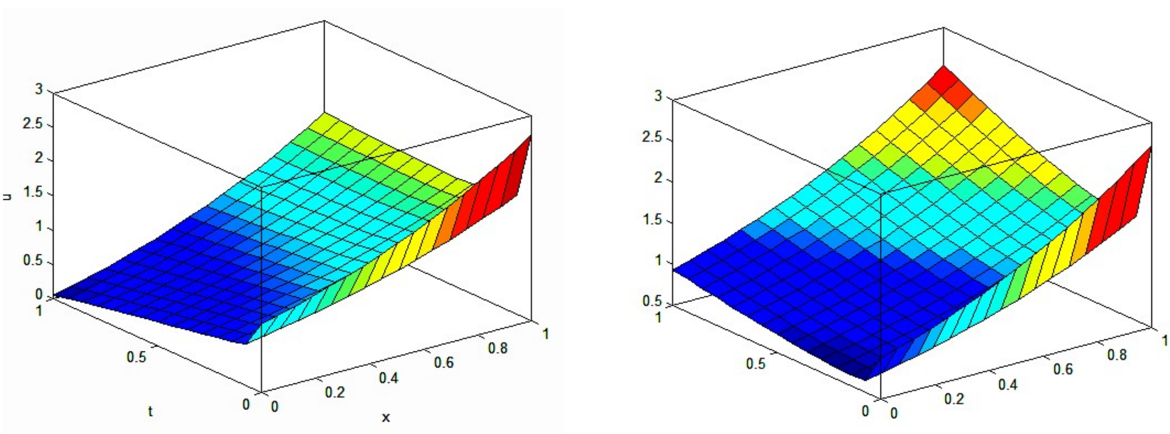


(a) The approximate solution with $\alpha = 0.9$ (b) The exact solution with $\alpha = 1$

Figure 1: 3D graphics of the exact and approximate solutions of Example 3.1.

In Figure 1, the approximate and the exact solutions of the time fractional linear Newell-Whitehead-Segel equation are presented by drawing three-dimensional graphics.

Figure 1(a) presents the approximate solution, where $\alpha = 0.9$, and Figure 1(b) presents the exact solution at $\alpha = 1$. From the mentioned figures, we can see that, when α approaches 1, the approximate solution is close to the exact solution. So, we can conclude that when approaches 1, the three-dimensional graphic is accurate and when α approaches 0, the three-dimensional graphic is inaccurate. In such phenomena, one can say that when α approaches 0, the solution bifurcates or admits chaotic behavior.



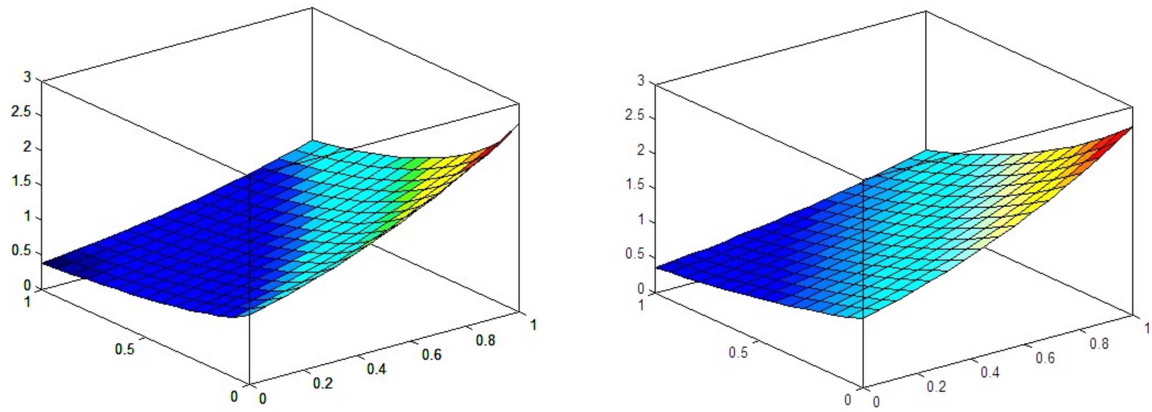
(a) The approximate solution with $\alpha = 0.5$ (b) The exact solution with $\alpha = 0.25$

Figure 2: 3D graphics of the approximate solutions of Example 3.1.

In Figure 2, the three-dimensional graphics show the influence of different α on analytical solutions. Figure 2(a) presents the approximate solutions when $\alpha = 0.5$, and Figure 2(b) presents approximate solutions when $\alpha = 0.25$.

In Figure 3, the three-dimensional graphics show the approximate solutions when $\alpha = 0.8$ and $\alpha = 1$ (Figures 3(a) and 3(b), respectively).

In Figure 2 and 3, we find that larger the value of α is, the smoother is the plane. As parameter



(a) The approximate solution with $\alpha = 0.8$ (b) The exact solution with $\alpha = 1$

Figure 3: 3D graphics of the approximate solutions of Example 3.1.

α increases, the graphics get closer and closer to the exact solution of the graphic.

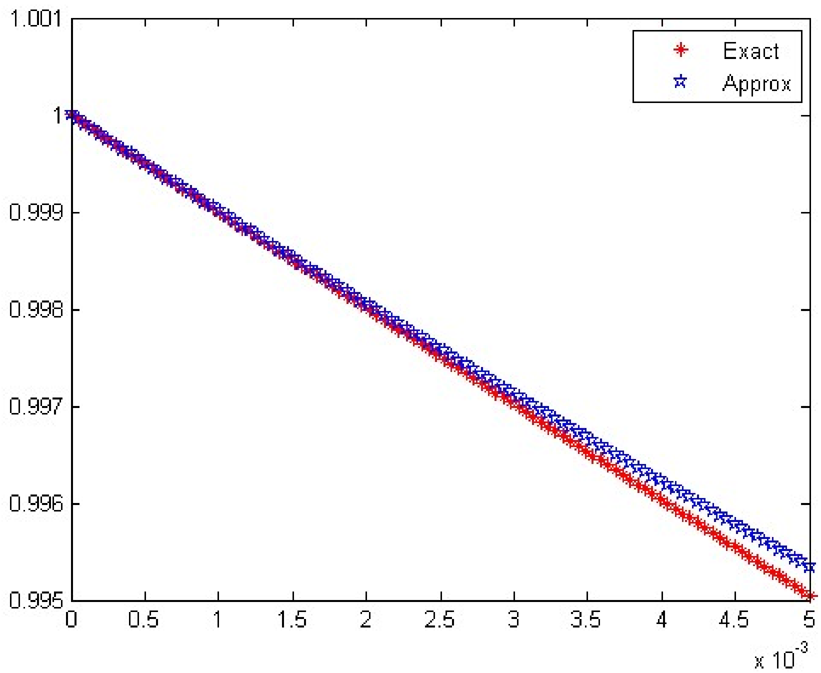


Figure 4: The exact and approximate solutions of Example 3.2

The numerical results obtained by using MRPS and the exact solution are presented through Figure 4 at $\alpha = 1$ and $\lambda = 1$ for different values of t. Figure 4 shows that the results obtained with the help of MRPS are approximately same to exact solution. It is also to be noticed that, when we increase the value of t, then U decreases. It is also worth mentioning that only three iterations are used to compute the results for Figure 4. Hence, if we increase the number of iterations then, efficiency and accuracy can be dramatically enhanced.

5. Conclusion

In this paper, we discuss an analytical solution of the fractional model of Newell-Whitehead Segel equation by using the modified residual power series method (MRPS). Results show that the analytical solutions by the modified residual power series method are close to the exact solution. It is observed that the MRPS method is simple but powerful technique used to solve such nonlinear equations. In general, MRPS method is an effective and convenient method to finding the analytical solution of the time fractional Newell-Whitehead-Segel equation. This technique can be extended to other applications in science and engineering.

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