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The Analysis of Multifractal System, Long-Term Memory, and Weak Form Efficiency of the Returns in Tehran Stock Exchange

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Abstract

Studying and recognizing the behaviour of securities returns has always been the focus of investors and researchers since the dawn of capital markets. Based on the statistics of the last decade, the stock market has been one of the major centres for making investment and getting high returns. Analysis and forecasting the price of financial assets has always been an intriguing topic in both scientific and practical disciplines which created various challenges for financial analysis. Chaos theory and fractal analysis are the latest theories in this regard. The present study has reviewed the information on Tehran Stock Exchange (TSE) companies' returns between 2014 and 2018 in monthly intervals to measure the multifractal system, long-term memory (LTM), and weak-form efficiency of the stock return variable.

The aim of the study has been addressed by using Hurst's rescaled range (R/S) statistic model. Any R/S larger than 0.5 indicated a correlation between future stock returns and previous returns and that previous data influence the market. Consequently, it is not a random market but has an LTM, a fractal dimension, and is relevant to the Efficient Market Hypothesis (EMH) that confirms these variables.

Keywords: Stock returns, multifractal systems, Chaos Theory and Hurst indices.

1. Introduction

When reviewing the financial markets data, it is essential to gain information regarding the analysis of time series that govern price trends and stock returns. Two main components in determining the value of an asset are the probability of losing or gaining an asset (risk) along with the return rates.

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This study has briefly reviewed the EMH and proposed the Fractal Markets Hypothesis (FMH) as an alternative that addresses the inefficiency of EMH. The FMH explains why the statistical structure of this market is similar to the EMH. The FMH and its efficiency are approved by using a multifractal analysis system. Based on what has been mentioned, this question raised that ”Whether the stock return on the Tehran Stock Exchange (TSE) inherits a poor efficiency, multifractal system, and long-term memory (LTM)?”

Therefore, according to the explanations and based on the research conducted by Mensi et al. [13], the study questions are as follows:

i. ”Do the companies in TSE have poor return efficiency?”

ii. ”Do the companies in TSE have LTM?”

iii. ”Do the companies in TSE have a multifractal system?”

2. Theoretical foundations and study background

The capital market not only distributes risk but also enables individuals to diversify their investments; hence, reduces the PPM risk. The total risk is decreased because the benefits of other investments offset the loss in some investments. The capital market also reduces the risk of illiquidity. Liquidity refers to the ease with which an asset, or security, can be converted into ready cash without affecting its market price. Transaction costs and information asymmetry reduce liquidity and increase risk. Financial markets help reduce the risk of illiquidity by disseminating and analyzing information and facilitating transactions.

Complex capital market issues can generally be divided into macroeconomic and microeconomic categories. Macroeconomic issues affecting the capital market include the impact of business cycles, economic indicators, and government monetary and financial policies. Capital market deals with issues such as consumer confidence index (CCI), informed and uninformed investors, novices and experienced users, and behavioural issues such as loss aversion, overconfidence and overreaction within microeconomic issues.

2.1. Efficient market and random walk hypothesis

Louis Bachelier founded financial market research. In his PhD thesis ”The Theory of Speculation” (1900), he maintained that statistical methods should be used to analyze the random behaviour of stocks and bonds. The focus of this study was the fair market, in which any given price could change in any direction with the same probability of occurrence. As a result, the mathematical expectation of any speculation will be zero. Under these conditions, the probability of exceeding the real price is independent of the calculated price value, and the probability curve of the total price will be symmetric [19].

Contrary to Bachelier’s significant contribution, market analysis faced plodding progress. After the unprecedented stock crisis of 1929, scientists valued this theory, and random market research led to the competitiveness of the investment and interpreted the ”prices” as the best future information sign. Unfortunately, the researchers did not consider ”prices” as a determinant economic model. Fama [20] stated that ”the main impetus for the development of this theory stems from the evidence of the 1950s and 1960s that random walks could approximate the behavior of ordinary stocks and prices.” After encountering this evidence, economists were forced to make sensible suggestions. Finally, by 1970, Fama [20] proposed a theoretical framework for the random market called the ”efficient market hypothesis (EMH)". This theory states that a market is efficient when all available information is quickly disseminated in the market and quickly reflected in stock returns.

Fama stated that the market would be efficient if it has the following conditions:

- The stock transaction includes no transfer rate.
- All information is freely available to all market participants.
- Stock returns reflect the information available in the share market [20].

In a friction-free economy, investors get the expected competitive returns in the market, and all the costs and benefits associated with stock values are reflected in their returns. According to Fama's theory, competition between investors allows the stock market to shape stock returns according to the best economic expectations of the future.

So if prices deviate from their core value, market participants will rectify it. Finally, prices reach a competitive equilibrium according to the information. The random walk hypothesis is an expanded version of the EMH. Random patrol hypothesis state that random information is the only factor in changing prices. Accordingly, in the absence of new information, there is no reason for price changes. Today prices are the best source to forecast tomorrow's prices. As a result, the possibility of change occurs through a game of chance, such as tossing a coin.

The next price change is independent of past events, and their mathematical form includes random variables that have an independent and uniform distribution.

If there is a high number of transactions, the distribution of random variables will be independent, uniform, and according to central limit theorem (CLT) will be Gaussian or normal.

The normal distribution has the following characteristics. First, the whole distribution can be determined by the first two torques of the mean and variance, which determines the position and dispersion, respectively. Second, the sum of standard random variables has a normal distribution. Brownian motions statistically express the random walk hypothesis. Brownian motions are defined as social processes that have three main characteristics, including time homogeneity, independent development, and continuity in the path.

Brownian motion is a process that is:

i. Statistically consistent, which means the price of the production process is constant over time, so if the process X_t is at $t > 0$ then the process $X_{t_0+t} - X_{t_0}$ has the same distribution function for $t > 0$.

ii. Increasing the process for separate time frames has a two-sided argument.

iii. The process at successive frequencies is decreasing, meaning the majority of the price fluctuations are small, and they change constantly.

This method differentiates processes that have sudden jumps and assumes that the smallest motion from t0 to t can be described as the following formula:

$$X_t - X_{t_0} \approx e^*(t - t_0)^H$$

"e" is a standard normal random variable that has a mean of zero and its variance is a ratio of the differential distances of T^2 and $H = 0.5$. In other words, to obtain X_t in Brownian motion, the random number of "e" is multiplied by $(t - t_0)^H$ and the result is added to the position of X^{t_0}.

EMH and random walk hypothesis have been used in many empirical studies. While some of them have been confirmed, many others have been rejected. This being said, these theories are a barrier to pricing and risk management models, so neoclassical theories are considered as the governing paradigms of economics and finance and the basis of thought on Wall Street.

2.2. Financial market complexity

The economic order of arises from the interaction of complex phenomena that contribute to the emergence of economic processes, any simple, economic phenomena do not necessarily arise from rational behaviours and expectations. We are living in a world of diverse knowledge, attitudes, and reciprocal behaviours (Russer 2004). Consequently, contrary to the neoclassical assumptions, the market complexity defines economics as an unbalanced system with nonlinear and unexpected behaviours resulting from heterogeneous individuals with complex dependencies like chaos theory.

At a certain point, the system becomes stable, but this state disappears by a sudden event. As a result, the system never reaches a single equilibrium point and instead has multiple equilibrium points that are far from the dynamic equilibrium point.

Complexity writers use the sand mass model to describe the cyclical behaviour of economic systems such as financial markets. In this regard, asset prices eventually rise due to changes in some internal or external factors of the system. This is due to changes in technology or new patterns of behaviour. The speculative bubbles can grow until the market reaches a critical point and inevitably lead to a price explosion. Similar to the sand mass model, the distribution of these events has a more significant variance than the random shedding of sand grains.

Under these turbulent conditions, it is unfeasible that companies behave optimally and have enough information to meet rational and neoclassical expectations or assumptions. As a result, the theory assumes the market participants with limited knowledge and incomplete insights into the future. Furthermore, companies are always changing and learning from the emerging environment. Thus, one of the vital keys of this paradigm is an economic model that is a system, in which expectations and dynamics feedback affect each other.

Therefore, companies with natural cognitive limitations will try to learn, adapt, and update parameters according to observable behaviours. As can be seen, the complexity of economics discriminates between two neoclassical perspectives. While the first perspective concludes that economics is a closed linear system that tends to reach equilibrium, the second perspective regards market participants as rational individuals with rational expectations. It provides more rational assumptions about the behaviour of individuals and economic systems. This gives economists a better understanding of market dynamics. Besides, this has led to the creation and relativity of methods and the emergence of new opportunities to discover complex phenomena in economics.

Table 1: Comparison between neoclassical and complex economics

	Neoclassical	**Complex**
Physics discipline	**Newtonian physics**	**Physics of thermodynamics, complexity, and biology**
System	Closed, linear and balanced	Open, dynamic, nonlinear, out of balance, and with characteristics of self-organization and adaptability to the environment
Agent	Homogeneous and transparent market prices	Heterogeneous with two-way and unlimited interactions. Direct interaction allows feedback on the relationship.
Expectations	Rational	Comparative

Finally, complex economics abandons neoclassical assumptions such as the uniqueness of equilibrium, the rationality of decision-makers, polarity, and a system that provides a more realistic description of the market. This study also reviews how turbulence theory and fractal science will contribute to the development of financial theories.

2.3. Fractal definition

Fractals have specific, measurable properties and features which are described by the objectives of the model. Identical to the Sierpiński triangle, where each small triangle is equivalent to a larger triangle, the primary characteristic of a fractal is self-similarity. In reality, self-similarity is a qualitative process that each scale is similar to other scales, not equivalent or comparable. Each branch of the tree is qualitatively similar to the other branches, but each branch is unique. These self-similar properties made the fractal to maintain a fixed scale [11].

Fractal geometry symbolizes a recurring pattern in objects and images, that is, if each image or shape with this property is divided into smaller parts (based on the fractal scale), each of these smaller parts is itself a scaled-down copy of the primary form, which is a type of critical self-organization the systematic perspective. The purpose of fractal geometry is to calculate and find this geometric dimension in order to anticipate the behaviour of nature and its dynamical patterns [18].

2.4. Chaos theory and fractal science in finance

Chaos theory and fractal science aim to investigate nonlinear and non-periodic behaviours in both physics and finance. Any system with sensitivity to the initial conditions tends to follow a path that is created by strong gravity. While irregular behaviours are local system characteristics, distinct patterns exist in market behaviour. That is why these new paradigms entered the financial space. These approaches see the market as a state of practical randomness and global certainty that can only be seen in its fractal structure. Therefore, applying the fractal view to the market yields a better understanding of market dynamics. In the financial crisis of 2008, many dominant economic theories and financial perspectives were challenged. The EMH of 5 was temporarily unresponsive in times of crisis. Directly, the technical analysis exists in the fractal theory as the price movements under the fact that history repeats itself. Thus, this framework is formed by analyzing the investor horizon, the role of liquidity, and the impact of information in the business cycle [15].

2.5. Chaos theory and in capital markets

In systematic framework models, researchers try to use simplified relationships and anticipate how the system will react to this situation; therefore, the reason and effect in these models are explicitly defined [14].For example, the primary assumption of capital asset pricing models is that the ROA is either a linear or proportional function of the risk factor. The two hypotheses of dependence and linear relation allow the researcher to formulate a simple mathematical model to explain this relational model. Another common assumption of simplified models is that any self-contained system moves toward equilibrium. However, the applicability of a model is significantly reduced, and many problems cannot be overcome under simplified model assumptions. New science has made it possible to study the dynamic behaviour of nonlinear models under the name of chaotic behaviour. Perhaps the most significant contribution of chaos theory is motivating for studying the complex behaviour of dynamic systems [16].

For instance, if the stock market return chart is plotted on an hourly, daily, monthly, or yearly basis without any indication of time, it is unlikely to identify a pattern based on clean periods. However, chaos theory indicates that chaotic time series often have irregular cycles and assertive process behaviour. In other words, periodic patterns can be identified, but their start and end times are unpredictable, so the transition from one stage to another is unpredictable and sudden. Accordingly, chaotic behaviour is an integral part of a system, but if there is a transparent, predictable, and constant pattern in the market behaviour; then this pattern is the reason for the existence of long-term memory in the market and the absence of chaotic behaviour [2].

2.6. Fractal dimensions

A basic method for calculating the fractal dimension is to cover the curve with circles of radius "r". Then count them and increase the radius:

$$N * (2 * r)^d = 1$$
$$N =$$
$$r =$$

since the scale is linear, its fractal dimensions are equal to 1. Any random walk process that has a 50-50 chance of increasing or decreasing then has a fractal dimension of 1.5. If the fractal dimension is between 1 and 1.5, the time direction is more than one line and less than the random walk model. This random walk process is flatter than a straighter gear line.

$$d = \log(N)/\log\left(\frac{1}{2 * r}\right)$$

Fractal dimensions can be resolved, such as the slope of log/log. We can increase the radius for time series such as increasing time and then count the number of circles needed to cover the curve. Therefore, the fractal dimension of a time series is a function of its time scale. Fractal dimensions of time series are critical because they indicate whether the process is a state between completely definite (line with a fractal dimension of 1) and random (line with a fractal dimension of 1.5).

The fractal dimensions of a line alternate between 1 and 2. If it is $1.5 < d < 2$, then it is much jagged than random time series [6].

2.7. Return on Investment

Return on Investment (ROI) is a driving force that motivates and rewards investors. ROI is essential for investors since it is the primary goal of any investment process. Measuring a portfolio performance is the only logical way (before risk assessment) that investors can do to compare alternative and distinctive investments. To better understand the performance of an investment, it is necessary to measure the actual return (related to the past), since the study of past returns has a significant role in estimating and predicting future returns (Roodpashti and Pedram, 2012).

2.8. Research background

Mensi et al. [13] investigated poor EMH, LTM, and multifractal systems in European stock markets. The results indicated that these markets have LTM in both short-term and long-term periods, especially in the latter one. The results also showed that the Greece Stock Market (ASE) is more EMH than other studied stock markets and that Portugal and Ireland have inefficiencies in their stock markets.

Biglar et al. [5] have analyzed the effect of the fractal systems of the stock market network on stock returns. The study population included all 382 companies involved in the TSE; finally, by using data analysis, 349 companies with complete information profile between 2013 and 2016 were selected as the study sample. The construction of the stock exchange network of TSE was done and was analyzed according to the created network and its fractal property. The test results suggested that the TSE is a non-fractal network that is made by using stocks and relationship among the shares as peaks and edges, respectively.

Khajavi [2] conducted an experimental analysis of fractal dimensions on the stock prices of companies listed on the TSE. The statistical sample was price analysis between 2003 - 2012. This research investigated the random time series of prices by R/S analysis and Hurst exponent. The R/S analysis is a robust nonlinear method to study random time series and differentiate them from non-random time series. The most crucial advantage of R/S analysis is that it is independent of the type of time series distribution. The findings of this study showed that the time series of the price is not independent and random and has an LTM.

3. Methods

This is an experimental and practical study; since it examines the observations of time series and stock returns. It is also a descriptive study regarding the methods of implementation and data acquisition. The statistical population of the study included all companies listed on the TSE between 2014 to 2018.

The present study has used the systematic sampling method, which has been selected from the statistical population of the sample companies according to the following conditions and limitations:

1. The company should be listed on the stock exchange before 2015 and was actively present in the stock exchange until the end of 2019.
2. The company should not be an investment company, holdings, or financial intermediaries.
3. The financial year of the company should end on March 20, and there should be no change in the financial year during the research period.
4. The company should not have a trading interval for more than one month.
5. Financial information about companies must be available.

It should be noted that the information regarding the returns of TSE and companies has been extracted monthly from 2015 to 2019. After these restrictions were applied, 141 companies were selected as a statistical sample, and the data were analyzed by R and EVIEWS software.

The study hypothesis and its goals were formulated according to the mentioned statistical model as follows:

1. The stock returns of companies in the TSE had low efficiency.
2. The stock returns of companies in the TSE had an LTM.
3. The stock returns of companies in the TSE had multifractal systems.

4. Study variables

The stock returns are the independent variable of this study which is relatively calculated from the first and last price by the following formula:

$$\text{Total return on the last (first) share price } = \frac{\text{Bonus benefits + Preemptive rights benefits + gross cash dividend share + share price difference}}{\text{Stock prices at the end (beginning) of the fiscal year}}$$

The dependent variable of this study is the existence of fractal dimensions, according to Hurst exponent. Hurst was aware of Einstein's (1908) work on Brownian motions. Brownian motions are considered as a prototype of a random walk. According to Einstein's findings, the distances covered by random components increase by the second root of time, in other words:

$$R = T^{0.5}, \quad R : covereddistance, \quad T : timeindex$$

The above relationship is used in financial economics to measure annual fluctuations by using the standard deviation (SD) of monthly returns, multiplying them, and taking the second root. It is assumed that the efficiency segment increases with the second root of time. Using this feature, Hurst investigated the randomness of the Nile River flood with the following mathematical equations:

$$\begin{aligned} X_m &= (x_1 + x_2 + \cdots + x_n)/n \\ S_n &= \sum_{r=1}^{m} \sqrt{(x_r - x_m)^2} \\ Z_r &= (x_r - x_m) \\ Y_1 &= (z_1 + z_r) \\ R_n &= \max(Y_1, Y_2, \cdots, Y_n) - \min(Y_1, Y_2, \cdots, Y_n) \end{aligned}$$

The index n for R_n states that this is the modified range for $x_1, x_2, \cdots, x_n$, because y is modified to mean of zero. The maximum y is greater than or equal to zero, and its minimum is less than or equal to zero, R_n is always non-negative. Now if we replace n with T, we get equation $R = T^{0.5}$, which is an independent time series. This method is used to increase the value of n as a high equation only for time series that have Brownian motion.

Since Brownian series usually have a mean of zero and a variance of one, the equation must be generalized for independent systems to be used in time series that do not have Brownian motion. Hurst considered the following equation to be a generalized version of the above equation:

$$(R/S)_n = cn^H$$

Where n is constant in $(R/S)_n$ for the value of R/S for $x_1, x_2, \cdots, x_n$ and $C = a$. The R/S value in the above equation returns to the revised range because it has a mean of zero. Generally, the value of the R/S scale when time increases is equal to n with power H, where H is the symbol of the Hurst exponent. This is the first connection between the Hurst model phenomenon and fractal geometry. All fractal scales share common property and are based on the power law.

Rescaling the modified interval by dividing by the SD was an outstanding achievement. Hurst was the first person who did this for the first time, and in this way, he compared different phenomena. As we will see, rescaling allows us to compare different periods. Inflation was a dilemma for comparing stock returns data in 1920 and 1980, which was minimized by rescaling of the R/S method.

It is possible to compare different phenomena with different time intervals by rescaling the data with a mean of zero and SD of one. Hurst even predicted a re-standardization of group theory in physics. Rescaling interval analysis can describe time series that do not have scale specifications, which is another fractal property.

The Hurst view can be calculated by the linear slope created by the LO. $G(R/S)$ plot versus $LOG(n)$. Suppose the system is distributed independently then $H = 0.5$. The Hurst River reached $H = 0.9$, according to the Nile data. This means that the rescaled interval increases at a faster rate than the square root of time, and the system covers a greater distance from the random process; therefore, to cover a greater distance the correlated annual fluctuations of the river flood interact with each other.

4.1. Interpreting the Hurst exponent

According to the basic theory, $H = 0.5$ is considered as an independent process. The R/S analysis is non-parametric and therefore, does not need to be distributed as a function. The $0.5 \leq H \leq 1.00$ is a stable time-series and is characterized by the effect of LTM. Theoretically, it coveys that everything that happens today will infinitely affect the future. Given the chaos theory, there is a correlation between future events and the initial conditions, so the LTM occurs regardless of the time scale.

All daily changes are correlated with changes in the coming days, and thus, all weekly changes are correlated with changes in the coming weeks with no specified time scale. This is the critical feature of a fractal time series. The $0 \leq H \leq 0.5$ marks the instability. An unstable system covers a distance of less than one random state.

5. Results

The central indices such as mean and scatter indices such as SD, kurtosis, and skewness are calculated for different variables and presented in the table below.

Table 2: Descriptive statistics for the variable of stock return

Variable name	Number	Average	Mean	Maximum	Minimum	SD	skewness	kurtosis
stock return	1920	0.24209	0.25000	0.950	-0.998	0.396976	-0.353	-0.280

The research variable is calculated monthly. Therefore, we have $141 * 5 * 12 = 8460$, in which the number "141" indicates the number of participated companies in the study, and the number "5" indicates the period of research (2014-2018). Therefore, the number of observations of this research for the variable is equal to 8460 years-months. The compatibility of the return indices in the TSE with normal distribution will be investigated in the next segment. Three test methods have been used in this study to confirm the normality of the stock variable distribution completely. For this purpose, the following methods have been used are Kolmogorov Smirnov, Anderson-Darling, and Chi-square. The test results are presented in Table.3. These tests have been confirmed at different levels, such as (99%, 98%, 95%, 90%, and 80%).

Table 3: Normal distribution statistics of study variables

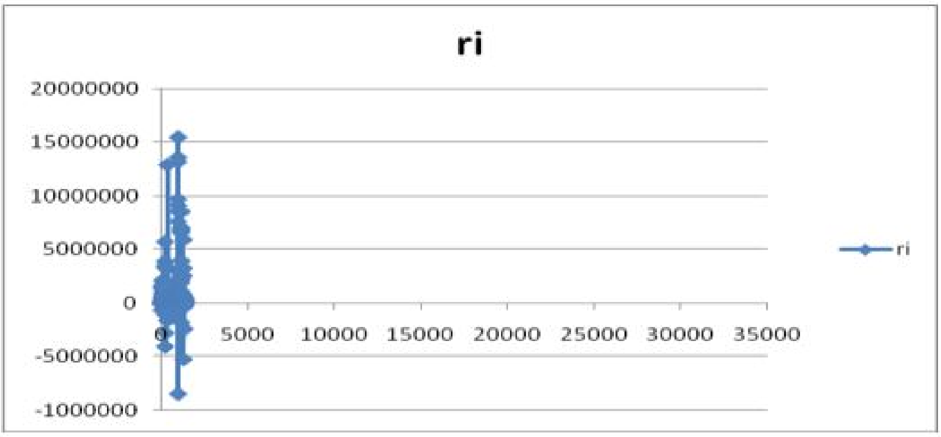
Index	Kolmogorov Smirnov	Anderson-Darling	Chi-square
Stock returns	0.68	27.5	19.1

As can be inferred, the null hypothesis is not acceptable at all confidence intervals where there is a normal distribution. In this section, the modified R/S method algorithm is first examined on a random time series. Plot V will be a straight line for the $\log(n)$ if the process is independent and random. On the other hand, if the R/S scale grows faster than the square root of time ($H > 0.5$), the graph will have an ascending slope, consequently, if the process has ($H < 0.5$) the graph will have a downward slope.

The Hurst exponent grows as the time intervals increase (Multifractal system). In the shorter or longer intervals in is probable to experience data fluctuations. Any more frequent sampling can reduce or even eliminate the effect of oscillation.

Table 4: Description of Hurst view

HR.	
Mean	0.86107
Standard Error	0.31201
Sample Variance	1.31281
H	0.7145
p-value	0.000



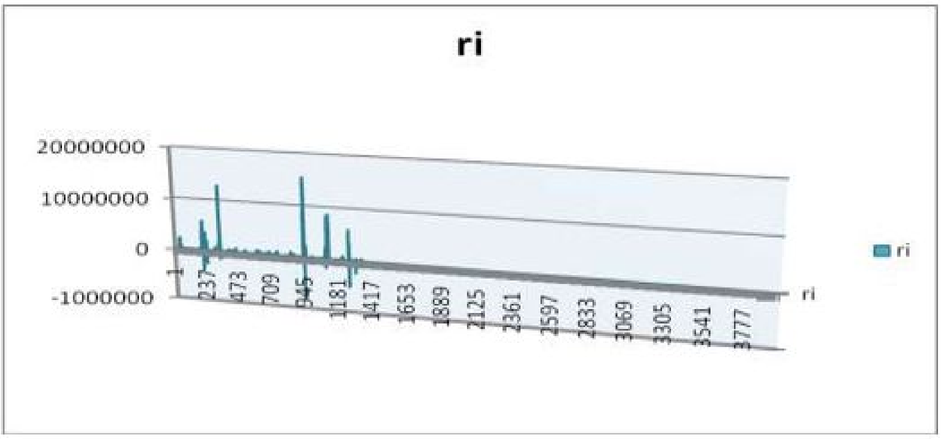


Figure 1: Fluctuations in stock returns over the research period

If the system is distributed independently (or randomly), then H will be equal to 0.5, therefore based on the leading theory, $H = 5.0$ is considered as an independent process. The R/S analysis is non-parametric and therefore, does not require a distribution function, and $0.5 \leq H \leq 1.00$ relates to a stable time series. This stable time series is determined by the effect of LTM and has a descending slope. Therefore, considering that the fractal dimension of stock returns is 0.7145, and it is more significant than 0.5, then the hypothesis is confirmed, and the LTM effect determines this stable time series. Theoretically, it implies that everything that happens today will affect the future forever.

As the dynamics of the chaos theory suggest, there is a correlation between future events and the initial conditions; therefore, this LTM occurs regardless of the time scale. All annual changes are correlated with changes in the upcoming years with no specified time scale, which is the key feature of a fractal time series. As a result, the price of the TSE can be studied by fractal properties which verifies the fifth and sixth hypotheses of the study.

5.0.1. Results of estimating the random walk model

The random step hypothesis requires identical variance ratios for all of the selected q-values. The basis of the multiple variance ratio tests can be seen in the following equation:

$$P_r\left[\max(|Z_1|,\cdots,|Z_m|)\le Z_{a^*/2}\right]\ge(1-a)$$
$$a^*=1-(1-a)^1/m$$

Z_i values are independent and have a stock standard distribution. This is the same standardized sequence that has been calculated in the Chow and Denning (CD) multiple variance ratio test.

Table 5: The calculated values of $z(q)$ and $z^*(q)$ in the Chow and Denning (CD) multiple variance ratio test ($\alpha = 0.05, SMM = 2.569$)

The time period	Variable name	z(q)	z*(q)	MAE
Five years	Stock returns	0.3221	0.3811	0.3123

As it is evident, the maximum absolute value of $z(q)z^*(q)$ is less than the critical value, so it is significant according to the critical value of SMM. Consequently, the random walk will be confirmed per homogeneity of variance, and based on the multiple CD test, the hypothesis of random walk is confirmed for all research variables per homogeneity and heterogeneity of variance.

5.0.2. The results of the RUN test

Null hypothesis and alternative hypothesis have been investigated for this price range as follow:

H_0 = simulated series are not random series. , H_1 = the observed series are random series.

The non-randomness and the alternative hypothesis of the series were investigated, and a significance level of 5% was considered for the study. Because the probability values for the research variables are less than 50.5, therefore, the hypothesis of random series is accepted for all research variables.

Table 6: The results of RUN test for variables

Hurst exponent	results	p-value	z statistic for RUN test	variable	Time period
Five years	Stock returns	-1.51	51.0	simulated series are random series	0.52

The results of comparing the fractal dimension of stock returns, and their simulated fractal dimensions verify a significant difference between these dimensions over a five-year time period. There was no significant difference in all long-term comparisons which indicate the existence of economic inflation and high liquidity in recent years, it has resulted in fluctuations in the capital market that were unpredictable even with random walk method. Accordingly, it is concluded that stock returns have a poor efficiency which verified the first hypothesis of the research.

6. Discussion and conclusion

Due to the uncertainty of investor in determining the effective factors in the investment process such as the exact amount of stock returns, the present study attempted to analyze the poor market efficiency, multifractal systems, and LTM in the TSE. While fluctuations are normal in every stock market, the anticipation of stock returns can select a favorable combination of them and reduce the fluctuations. Moreover, the anticipation of important stock market indicators is a step towards increasing and clarifying the information in the capital market.

The fractal market hypothesis not only examines the efficiency of a financial market but also evaluates its stability and shock absorption properties. Various articles have studied the hypothesis of fractal markets of international financial markets, but few of them have been conducted on the TSE. The present study aimed to investigate this hypothesis on stock returns for companies listed on the TSE. This study has utilized a modified version of the R/S analysis method to estimate Hurst exponent, which is a measure for LTM detection in time series, from 2015 to 2019 for 141 companies.

The results demonstrated chaotic effects with a fractal dimension of 0.6 for the stock return variable. This means that according to the EMH and logical expectations of investors from the market, they were not able to defeat it and gain returns. Therefore. The fractal market hypothesis is preferable to the EMH in case of the TSE.

Moreover, investors were unable to trade invaluable assets due to stock returns. Therefore, in order to achieve higher than average market returns, they had to accept the risk associated with volatile assets. The policy that governs this finding is that although new shocks and information affect stock returns in the short term, these changes have long-lasting effects in long-term periods. Therefore, if policymakers can reduce short-term fluctuations by adopting appropriate policies, they will take a critical step in market efficiency which make the stock market as an essential and reliable tool for the optimal allocation of the country's financial resources, where investors may benefit from both long- and short-term horizons. The study results are consistent with the findings of Khajavi et al. [2], and Mensi et al. [13].

According to the results, the multifractal model can be suggested to different user groups for the following purposes:

- Since the stock returns of companies have low efficiency, LTM, and multifractal systems, investors are advised to pay attention to the financial indicators to gain more profit and returns. Therefore, the legal aspects of emerging markets regarding the obligation to disclose public and private information and support for investors is a proposal that is highly emphasized by experts. Since the efficiency of the country's capital market has been challenged and the emergence and presence of large quasi-government sectors in the country, the stock market is out of competition, and some actors have gained considerable influence in it. Therefore, rational traders are advised to follow market behaviour along with technical analysis in their forecasts. This creates rational bubbles that require specific identification tests.

- The results of the research show that despite the recent efforts to create information transparency in the TSE, the situation of the stock exchange has not changed. Since the information transparency is the core of EMH, more serious steps must be taken in this regard.

- Given that primary duty of managers is boosting the capital of shareholders to the maximum extent possible, managers should pay special attention to financial variables and their effect on stock returns of the companies and implement strategies to increase stock returns in the future.

- The involvement of financial analysts and intermediaries in stock trading should be possible so that they can provide more transparency in the presentation of financial records and stock returns.

- Investors, analysts and other stakeholders are advised to pay more attention to the quality of published information, especially returns, and include it in their decision-making models.

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