Int. J. Nonlinear Anal. Appl. 12 (2021) No. 1, 37–44
ISSN: 2008-6822 (electronic)
http://www.ijnaa.semnan.ac.ir
http://dx.doi.org/10.22075/i10.22075/IJNAA.2021.4653

A New Technique of Reduce Differential Transform Method to Solve Local Fractional PDEs in Mathematical Physics

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Abstract

In this manuscript, we investigate solutions of the partial differential equations (PDEs) arising in mathematical physics with local fractional derivative operators (LFDOs). To get approximate solutions of these equations, we utilize the reduce differential transform method (RDTM) which is based upon the LFDOs. Illustrative examples are given to show the accuracy and reliable results. The obtained solutions show that the present method is an efficient and simple tool for solving the linear and nonlinear PDEs within the LFDOs.

Keywords: Local fractional RDTM; Diffusion equation, Klein-Gordon equation, Schrodinger equation, Nonlinear gas dynamic equation, Local fractional derivative operators
2010 MSC: 35R11; 74H10.

1. Introduction

There are many fractional differential equations, which are very helpful and applicable in engineering and mathematical physics such as diffusion equation, Klein-Gordon equation, Laplace equation, Schrodinger equation and nonlinear gas dynamic equation. Some various techniques have recently been developed to solve linear and nonlinear PDEs with LFDOs such as, LFFDM [1, 2, 3], LFADM [3, 4], LFSEM [5, 6], LFLTM [7, 8], LFFSM [9, 10], LFVIM [11, 12, 13], LFDTM [14, 15], LFLDM [16], LFHPM [17], and LFLVIM [18]

The local fractional RDTM is powerful approximate method for various kinds of linear and nonlinear PDEs with LFDOs. The solution procedure of the LFRDTM is much less and simpler than that in other numerical methods. The solution obtained by the LFRDTM is an infinite power

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Received: xx 2020 *Revised:* xx 2020

series for initial value problems, which can be, in turn, expressed in a closed form, the exact solution. Our aim is to extend the applications of the proposed method to obtain the analytical approximate solutions to PDEs with LFDOs.

2. LFRDTM

As in [19, 20], the basic definition of reduced differential transform with local fractional operator is proposrd as follows:

2.1. Definition

If $\varphi(\eta,\kappa)$ is a Lf function, then the LF spectrum function

$$\Phi_\xi(\eta)=\frac{1}{\Gamma(1+\xi\vartheta)}\left[\frac{\partial^{\xi\vartheta}\varphi(\eta,\kappa)}{\partial\kappa^{\xi\vartheta}}\right]_{\kappa=\kappa_0}, \tag{2.1}$$

is RDT of the function $\varphi(\eta,\kappa)$ via LFDO, where $\xi=0,1,\ldots,n$.

2.2. Definition

The inverse of RDT of $\Phi_\xi(\eta)$ via LFDO is defined as follows:

$$\varphi(\eta,\kappa)=\sum_{\xi=0}^{\infty}\Phi_\xi(\eta)(\kappa-\kappa_0)^{\xi\vartheta}. \tag{2.2}$$

From (2.1) and (2.2) we get

$$\varphi(\eta,\kappa)=\sum_{\xi=0}^{\infty}\frac{(\kappa-\kappa_0)^{\xi\vartheta}}{\Gamma(1+\xi\vartheta)}\left[\frac{\partial^{\xi\vartheta}\varphi(\eta,\kappa)}{\partial\kappa^{\xi\vartheta}}\right]_{\kappa=\kappa_0}, \tag{2.3}$$

From (2.3), it is obvious that the LFRDT is derived from the LF Taylor theorems.

If $\kappa_0=0$, then (2.1) and (2.2) become

$$\Phi_\xi(\eta)=\frac{1}{\Gamma(1+\xi\vartheta)}\left[\frac{\partial^{\xi\vartheta}\varphi(\eta,\kappa)}{\partial\kappa^{\xi\vartheta}}\right]_{\kappa=0}, \tag{2.4}$$

$$\varphi(\eta,\kappa)=\sum_{\xi=0}^{\infty}\Phi_\xi(\eta)\kappa^{\xi\vartheta}. \tag{2.5}$$

The following theorems that can be deduced from (2.1) and (2.2) are presented below:

2.3. Theorem

Suppose that $\Phi_\xi(\eta)$, $\Psi_\xi(\eta)$ and $\Theta(\eta)$ are RDT with LFDOs of the functions φ, ψ and θ respectively, then

1. 2. If $\varphi=\psi+\theta$ then

$$\Phi_\xi(\eta)=\Psi_\xi(\eta)+\Theta_\xi(\eta).$$

3. If $\varphi=\psi\theta$ then

$$\Phi_\xi(\eta)=\sum_{l=0}^{\xi}\Psi_l(\eta)\Theta_{\xi-l}(\eta).$$

4. If $\varphi = a\psi$, where a is a constant, then

$$\Phi_\xi(\eta) = a\Psi_\xi(\eta).$$

5. If $\varphi = \dfrac{\partial^{n\vartheta}\psi}{\partial\kappa^{n\vartheta}}$ then

$$\Phi_\xi(\eta) = \frac{\Gamma(1+(\xi+n)\vartheta)}{\Gamma(1+\xi\vartheta)}\Psi_{\xi+n}(\eta).$$

6. If $\varphi = \dfrac{\eta^{n\vartheta}}{\Gamma(1+n\vartheta)}\dfrac{\kappa^{m\vartheta}}{\Gamma(1+m\vartheta)}$ then

$$\Phi_\xi(\eta) = \frac{\eta^{n\vartheta}}{\Gamma(1+n\vartheta)}\frac{\delta_\vartheta(\xi-m)}{\Gamma(1+m\vartheta)},$$

where

$$\delta_\vartheta(\xi-m) = \begin{cases} 1, & \xi = m, \\ 0, & \xi \neq m. \end{cases}$$

7. If $\varphi = \dfrac{\partial^{n\vartheta}\psi}{\partial\eta^{n\vartheta}}$ then

$$\Phi_\xi(\eta) = \frac{\partial^{n\vartheta}\Psi_\xi(\eta)}{\partial\eta^{n\vartheta}}.$$

For illustration of the methodology of the presented method, we write the PDE within LFDO as:

$$\begin{aligned} &L_\vartheta\left[\varphi\right] + R_\vartheta\left[\varphi\right] + N_\vartheta\left[\varphi\right] = \omega, \\ &\varphi(\eta,0) = \phi(\eta). \end{aligned} \tag{2.6}$$

where $L_\vartheta = \frac{\partial^\vartheta}{\partial\kappa^\vartheta}$ and R_ϑ are linear LFDO, N_ϑ is nonlinear LFDO and $\omega(\eta,\kappa)$ is an inhomogenuous term.

By taking the LFRDT on both sides of (2.6), we have

$$\begin{aligned} &\frac{\Gamma(1+(\xi+1)\vartheta)}{\Gamma(1+\xi\vartheta)}\Phi_{\xi+1}(\eta) = \Omega_\xi(\eta) - R_\vartheta\left[\Phi_\xi(\eta)\right] + N_\vartheta\left[\Phi_\xi(\eta)\right], \\ &\Phi_0(\eta) = \phi(\eta). \end{aligned} \tag{2.7}$$

where $\Phi_\xi(\eta)$ and $\Omega_\xi(\eta)$ are RDT with LFDOs of the functions $\varphi(\eta,\kappa)$ and $\omega(\eta,\kappa)$ respectively.

3. Applications of Local Fractional RDTM to Solve PDEs Arising in Mathematical Physics

3.1. Example

Let us start with local fractional diffusion equation given in the following form:

$$\frac{\partial^\vartheta\varphi(\eta,\kappa)}{\partial\kappa^\vartheta} - \frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}\frac{\partial^{2\vartheta}\varphi(\eta,\kappa)}{\partial\eta^{2\vartheta}} = 0, \tag{3.1}$$

with initial value

$$\varphi(\eta,0) = \frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}, \tag{3.2}$$

Applying the local fractional RDTM for (3.1), we have

$$\Phi_{\xi+1}(\eta) = \frac{\Gamma(1+\xi\vartheta)}{\Gamma(1+(\xi+1)\vartheta)}\left[\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}\frac{\partial^{2\vartheta}\Phi_\xi(\eta)}{\partial\eta^{2\vartheta}}\right], \tag{3.3}$$

where

$$\Phi_0(\eta) = \frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}. \tag{3.4}$$

Hence, from (3.3) and (3.4), we give the components as follows:

$$\begin{aligned}
\Phi_1(\eta) &= \frac{1}{\Gamma(1+\vartheta)}\left[\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}\frac{\partial^{2\vartheta}\Phi_0(\eta)}{\partial\eta^{2\vartheta}}\right] \\
&= \frac{1}{\Gamma(1+\vartheta)}\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}, \\
\Phi_2(\eta) &= \frac{\Gamma(1+\vartheta)}{\Gamma(1+2\vartheta)}\left[\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}\frac{\partial^{2\vartheta}\Phi_1(\eta)}{\partial\eta^{2\vartheta}}\right] \\
&= \frac{1}{\Gamma(1+2\vartheta)}\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}, \\
\Phi_3(\eta) &= \frac{\Gamma(1+2\vartheta)}{\Gamma(1+3\vartheta)}\left[\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}\frac{\partial^{2\vartheta}\Phi_2(\eta)}{\partial\eta^{2\vartheta}}\right] \\
&= \frac{1}{\Gamma(1+3\vartheta)}\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}, \\
&\vdots
\end{aligned}$$

Therefore, $\varphi(\eta,\kappa)$ is evaluted as follows

$$\begin{aligned}
\varphi(\eta,\kappa) &= \sum_{\xi=0}^{\infty}\Phi_\xi(\eta)\kappa^{\xi\vartheta} \\
&= \frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}E_\vartheta(\kappa^\vartheta),
\end{aligned} \tag{3.5}$$

which is exactly the same as that obtained by LFLVIM [18].

3.2. Example

Let us consider Klein–Gordon equation within local fractional operators:

$$\frac{\partial^{2\vartheta}\psi}{\partial\kappa^{2\vartheta}} - \frac{\partial^{2\vartheta}\psi}{\partial\eta^{2\vartheta}} - \psi = 0, \quad 0 < \vartheta \leqslant 1 \tag{3.6}$$

is presented and its initial valuses are defined as follows:

$$\psi(\eta,0) = \frac{\eta^\vartheta}{\Gamma(1+\vartheta)}, \frac{\partial^\vartheta\psi(\eta,0)}{\partial\kappa^\vartheta} = 0. \tag{3.7}$$

Implementing the RDTM via local fractional derivative to (3.6), we have

$$\frac{\Gamma(1+(\xi+2)\vartheta)}{\Gamma(1+\xi\vartheta)}\Psi_{\xi+2}(\eta) - \frac{\partial^{2\vartheta}\Psi_\xi(\eta)}{\partial\eta^{2\vartheta}} - \Psi_\xi(\eta) = 0, \tag{3.8}$$

which equivalent to the following formula

$$\Psi_{\xi+2}(\eta) = \frac{\Gamma(1+\xi\vartheta)}{\Gamma(1+(\xi+2)\vartheta)}\left[\frac{\partial^{2\vartheta}U_{\xi}(\eta)}{\partial\eta^{2\vartheta}} + \Psi_{\xi}(\eta)\right], \tag{3.9}$$

where

$$\Psi_0(\eta) = \frac{\eta^{\vartheta}}{\Gamma(1+\vartheta)}, \Psi_1(\eta) = 0. \tag{3.10}$$

Following (3.9) and (3.10):

$$\begin{aligned}
\Psi_2(\eta) &= \frac{1}{\Gamma(1+2\vartheta)}\left[\frac{\partial^{2\vartheta}\Psi_0(\eta)}{\partial\eta^{2\vartheta}} + \Psi_0(\eta)\right] \\
&= \frac{1}{\Gamma(1+2\vartheta)}\frac{\eta^{\vartheta}}{\Gamma(1+\vartheta)}, \\
\Psi_3(\eta) &= \frac{\Gamma(1+\vartheta)}{\Gamma(1+3\vartheta)}\left[\frac{\partial^{2\vartheta}\Psi_1(\eta)}{\partial\eta^{2\vartheta}} + \Psi_1(\eta)\right] \\
&= 0, \\
\Psi_4(\eta) &= \frac{\Gamma(1+2\vartheta)}{\Gamma(1+4\vartheta)}\left[\frac{\partial^{2\vartheta}\Psi_2(\eta)}{\partial\eta^{2\vartheta}} + \Psi_2(\eta)\right] \\
&= \frac{1}{\Gamma(1+4\vartheta)}\frac{\eta^{\vartheta}}{\Gamma(1+\vartheta)},
\end{aligned}$$

$$\begin{aligned}
\Psi_5(\eta) &= \frac{\Gamma(1+3\vartheta)}{\Gamma(1+5\vartheta)}\left[\frac{\partial^{2\vartheta}\Psi_3(\eta)}{\partial\eta^{2\vartheta}} + \Psi_3(\eta)\right] \\
&= 0, \\
\Psi_6(\eta) &= \frac{\Gamma(1+4\vartheta)}{\Gamma(1+6\vartheta)}\left[\frac{\partial^{2\vartheta}\Psi_4(\eta)}{\partial\eta^{2\vartheta}} + \Psi_4(\eta)\right] \\
&= \frac{1}{\Gamma(1+6\vartheta)}\frac{\eta^{\vartheta}}{\Gamma(1+\vartheta)}, \\
&\vdots
\end{aligned}$$

Therefore:

$$\begin{aligned}
\psi(\eta,\kappa) &= \sum_{\xi=0}^{\infty}\Psi_{\xi}(\eta)\kappa^{\xi\vartheta} \\
&= \frac{\eta^{\vartheta}}{\Gamma(1+\vartheta)}cosh_{\vartheta}(\kappa^{\vartheta}),
\end{aligned} \tag{3.11}$$

which is exactly the same as that obtained by LFSEM [6].

3.3. Example

Consider the local fractional Schrodinger equation given in the following form:

$$\frac{\partial^{\vartheta}\varphi(\eta,\kappa)}{\partial\kappa^{\vartheta}} = -\frac{h_{\vartheta}}{2mi^{\vartheta}}\frac{\partial^{2\vartheta}\varphi(\eta,\kappa)}{\partial\eta^{2\vartheta}}, \tag{3.12}$$

with initial value

$$\varphi(\eta,0)=\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}, \tag{3.13}$$

Applying the local fractional RDTM for (3.12) and (3.13), we have

$$\frac{\Gamma(1+(\xi+1)\vartheta)}{\Gamma(1+\xi\vartheta)}\Phi_{\xi+1}(\eta)=-\frac{h_\vartheta}{2mi^\vartheta}\frac{\partial^{2\vartheta}\Phi_\xi(\eta)}{\partial\eta^{2\vartheta}}, \tag{3.14}$$

which reduces to

$$\Phi_{\xi+1}(\eta)=\frac{\Gamma(1+\xi\vartheta)}{\Gamma(1+(\xi+1)\vartheta)}\left[-\frac{h_\vartheta}{2mi^\vartheta}\frac{\partial^{2\vartheta}\Phi_\xi(\eta)}{\partial\eta^{2\vartheta}}\right], \tag{3.15}$$

where

$$\Phi_0(\eta)=\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}. \tag{3.16}$$

Hence, from (3.15) and (3.16), we give the components as follows:

$$\begin{aligned}
\Phi_1(\eta) &= \frac{1}{\Gamma(1+\vartheta)}\left[-\frac{h_\vartheta}{2mi^\vartheta}\frac{\partial^{2\vartheta}\Phi_0(\eta)}{\partial\eta^{2\vartheta}}\right]=-\frac{h_\vartheta}{2mi^\vartheta}\frac{1}{\Gamma(1+\vartheta)},\\
\Phi_2(\eta) &= \frac{\Gamma(1+\vartheta)}{\Gamma(1+2\vartheta)}\left[-\frac{h_\vartheta}{2mi^\vartheta}\frac{\partial^{2\vartheta}\Phi_1(\eta)}{\partial\eta^{2\vartheta}}\right]=0,\\
\Phi_3(\eta) &= 0,\\
&\vdots
\end{aligned}$$

and so on. Therefore, $\varphi(\eta,\kappa)$ is evaluted as follows

$$\begin{aligned}
\varphi(\eta,\kappa) &= \sum_{\xi=0}^{\infty}\Phi_\xi(\eta)\kappa^{\xi\vartheta}\\
&= \frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}+i^\vartheta\frac{h_\vartheta}{2m}\frac{\kappa^\vartheta}{\Gamma(1+\vartheta)}
\end{aligned} \tag{3.17}$$

which is exactly the same as that obtained by LFSEM [5].

3.4. Example

Let us consider the following nonlinear gas dynamics equation involving LFDOs:

$$\frac{\partial^\vartheta\varphi}{\partial\kappa^\vartheta}+\varphi\frac{\partial^\vartheta\varphi}{\partial\eta^\vartheta}+\varphi\left[\varphi-1\right]=0,\quad 0<\vartheta\leqslant 1 \tag{3.18}$$

with the initial value conditions as follows:

$$\varphi(\eta,0)=E_\vartheta(-\eta^\vartheta). \tag{3.19}$$

Applying the LFRDT to both sides of equation (3.18), we have

$$\frac{\Gamma(1+(\xi+1)\vartheta)}{\Gamma(1+\xi\vartheta)}\Phi_{\xi+1}(\eta)+\sum_{l=0}^{\xi}\Phi_l(\eta)\frac{\partial^\vartheta}{\partial\eta^\vartheta}\Phi_{\xi-l}(\eta)+\sum_{l=0}^{\xi}\Phi_l(\eta)\Phi_{\xi-l}(\eta)-\Phi_\xi(\eta)=0, \tag{3.20}$$

which equivalent to the following formula

$$\Phi_{\xi+1}(\eta) = -\frac{\Gamma(1+\xi\vartheta)}{\Gamma(1+(1+\xi)\vartheta)}\left[\sum_{l=0}^{\xi}\Phi_l(\eta)\frac{\partial^\vartheta}{\partial\eta^\vartheta}\Phi_{\xi-l}(\eta) + \sum_{l=0}^{\xi}\Phi_l(\eta)\Phi_{\xi-l}(\eta) - \Phi_\xi(\eta)\right]. \tag{3.21}$$

From equation (3.19), we obtain

$$\Phi_0(\eta) = E_\vartheta(-\eta^\vartheta). \tag{3.22}$$

Therefore, from equations (3.21) and (3.22), we give the components as follows

$$\begin{aligned}
\Phi_1(\eta) &= -\frac{1}{\Gamma(1+\vartheta)}\left[\Phi_0(\eta)\frac{\partial^\vartheta}{\partial\eta^\vartheta}\Phi_0(\eta) + \Phi_0(\eta)\Phi_0(\eta) - \Phi_0(\eta)\right] \\
&= -\frac{1}{\Gamma(1+\vartheta)}\left[-E_\vartheta(-2\eta^\vartheta) + E_\vartheta(-2\eta^\vartheta) - E_\vartheta(-\eta^\vartheta)\right], \\
&= \frac{1}{\Gamma(1+\vartheta)}E_\vartheta(-\eta^\vartheta), \\
\Phi_2(\eta) &= -\frac{\Gamma(1+\vartheta)}{\Gamma(1+2\vartheta)}\left[\Phi_0(\eta)\frac{\partial^\vartheta}{\partial\eta^\vartheta}\Phi_1(\eta) + \Phi_1(\eta)\frac{\partial^\vartheta}{\partial\eta^\vartheta}\Phi_0(\eta) + 2\Phi_0(\eta)\Phi_1(\eta) - \Phi_1(\eta)\right] \\
&= \frac{1}{\Gamma(1+2\vartheta)}E_\vartheta(-\eta^\vartheta), \\
\Phi_3(\eta) &= \frac{\Gamma(1+2\vartheta)}{\Gamma(1+3\vartheta)}\left[\Phi_0\frac{\partial^\vartheta}{\partial\eta^\vartheta}\Phi_2 + \Phi_1\frac{\partial^\vartheta}{\partial\eta^\vartheta}\Phi_1 + \Phi_2\frac{\partial^\vartheta}{\partial\eta^\vartheta}\Phi_0 + 2\Phi_0\Phi_2 + \Phi_1\Phi_1 - \Phi_2\right] \\
&= \frac{1}{\Gamma(1+3\vartheta)}E_\vartheta(-\eta^\vartheta),
\end{aligned}$$

and so on. Hence, the solution of equation (3.18) is

$$\begin{aligned}
u(\eta,\kappa) &= \sum_{\xi=0}^{\infty}U_\xi(\eta)\kappa^{\xi\vartheta} \\
&= E_\vartheta(\kappa^\vartheta - \eta^\vartheta),
\end{aligned} \tag{3.23}$$

4. Conclusion

In this work, the local fractional RDTM was utilized for the PDEs arising in mathematical physics within LFDOs such as diffusion, wave, Schrodinger and nonlinear gas dynamic equations. The local fractional RDTM introduces a significant improvement in the fields over existing techniques because it takes less calculations and the number of iteration is less compared by other methods.The present method is shown that is an effective method to obtain the analytical approximate solutions for the PDEs within LFDOs.

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