Tracking Control of an unknown switched nonlinear system based on adaptive backstepping with nonlinear disturbance observer

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Abstract

In this paper, an adaptive control schemes to solve the output tracking control problem of a class of nonlinear switched systems in presence of the disturbance is proposed. First, a nonlinear disturbance observer (NDO) is designed and the backstepping scheme is constructed based on the standard Lyapunov function method for tracking purpose. With the propose scheme, the existence of a standard Lyapunov function for all subsystems with unknown parameters infers the global uniform asymptotic stability of this more generic switched system the switching parameters used in the switching system are defined differently for each subsystem. Analyzing the system’s stability proved that the closed-loop signal boundedness under arbitrary switching is well ensured. It is shown that the proposed adaptive anti-disturbance control scheme based on a nonlinear disturbance observer is a suitable control approach for a class of nonlinear switched systems.

Keywords: nonlinear switched system, disturbance adaptive control, nonlinear system.

1. Introduction

With the progress of control techniques, a switched system under arbitrary switching is stable asymptotically, and common Lyapunov function exists for all subsystem [1,2,3]. A backstepping control scheme was proposed to stabilize a class of switched nonlinear systems with unknown parameters in strict-feedback form [4,5]. Backstepping is a new-developed technique proposed to control uncertain systems that have an internal and external disturbance. By applying this method for the systems with mismatching conditions, the performance of a system is improved [6,7]. In the past few decades, the research main goal is to conquer the difficulties of internal and external disturbances
and improve system performance. Disturbance uncertainty relation of the fin stabilizer system causes uncertainties that may be modeled or repeated. Various linear and nonlinear control methodologies have been proposed for strict feedback systems, including robust control [8], sliding mode control for the nonlinear system [9, 10], a fuzzy neural technique based robust control [11, 12]; and automatic disturbances rejection method of control [13] to eliminate or reduce such kind of uncertainties. A variable structure disturbance estimation technique has been proposed for PWM-based DC-DC buck power converter systems with mismatched disturbances [13] and a nonlinear disturbance observer propose a special kind of friction, i.e., Coulomb friction [14]. The systems with high nonlinearity and coupled dynamics are not appropriate for applying the linear disturbance observation method because of these restrictions. Researches developed studies and investigation of NDO for nonlinear systems in the past years [15, 16, 17, 18, 19]. Considering the flexible air-breathing hypersonic vehicle (FAHV) the Integrated Sliding Mode method, a backstepping approach with disturbance observer (DO), has been proposed [20].

In this paper, the switching unknown parameters are different for each subsystem. The disturbance in all subsystems is considered identical. The scheme is designed based on the common Lyapunov function approach for all subsystems, as pointed out in [21, 22, 23, 24]. In this work, the designed nonlinear disturbance observer is estimated disturbance and switching unknown parameters are transformed into their common bound via backstepping control design, and using the adaptive method. The transformed bound of switching parameters is then estimated online. With the scheme, a common Lyapunov function for all subsystems is constructed successfully, and the boundedness of all signals in the closed-loop signal under arbitrary switching is well guaranteed. The innovation of this article is the use of NDO with observer gains for switched system with unknown function. Although the disturbance is the same for all subsystems, but the nonetheless observer gains is different for every subsystems. The adaptive control with backstepping technique based NDO is applied for solving the tracking problem and estimation of the all disturbance with proposed disturbance observer is ensured.

This article content is organized as follow: In section 2 the nonlinear control problem is formulated, the nonlinear disturbance observer is defined for system in section 3. In section 4, the adaptive nonlinear control with backstepping method based NDO under arbitrary switching is designed. The constructing of the proposed controller is based on common Lyapunov function, the stability analyzing of the designed control is illustrated. according to the result of this section, and the closed-loop signal boundness of switched system is ensured in this part. Section 5 is described the effectiveness of the designed adaptive control based NDO through simulation. Finally, in the section 6 the conclusion of all paper is illustrated.

2. Problem statement

This paper takes into account a class of switched nonlinear systems with uncertain parameters and disturbance as follows:

\[
\begin{align*}
\dot{x}_1 &= x_{i+1} + f_{n,\delta(t)}(x_n) + \varphi_i(x_1, x_2, \cdots, x_i) \\
\vdots
\end{align*}
\]

\[
\begin{align*}
\dot{x}_n &= g_{n,\sigma(t)} + f_{n,\delta(t)}(x_n) + T(x) + d \\
y &= x_1
\end{align*}
\]

(2.1)

Where \( f_{n,\delta(t)}(x_n) = \theta_{\sigma(t)}^T \varphi_n(x_n), \ var \ = (x_1, x_2, \cdots, x_n)^T \in \mathbb{R}^i, i = 1, 2, \cdots, n \) is the system state \( u \in \mathbb{R} \) is control input and \( y \in \mathbb{R} \) denotes system output. \( \varphi_i(x_1, x_2, \cdots, x_i) \in \mathbb{R}^p \) for \( i = 1, 2, \cdots, n \), \( T(x) \)
is known smooth nonlinear function \( \theta_{s}(t, \in \mathbb{R}^{n}) \) an unknown switching parameters with switching signal \( \sigma; [0]u \mathbb{R}^{+} \rightarrow M = \{1, 2, \ldots, n\} \) \( g_{n, \sigma(t)} \) is positive constant.

\( d \) is the disturbance which is considered the same for all subsystems.

Assumption 1: The reference trajectory \( y(t) \) and disturbance \( (d) \) and their first \( n \)-order time derivatives are known, smooth, and bounded.

In this paper adaptive tracking problem and disturbance rejection problem is studied for a class of switched nonlinear system with completely unknown function. NDO is considered with different gain value for any system adaptive control scheme is designed based common Lyapunov function and in this strategy switching parameters are firstly transformed in to their common bound and with using adaptive law the transformed bound is estimated respectively such that all the closed-loop signals are guaranteed to be bounded and the reference signal is tracked by the output of system.

3. Nonlinear disturbance observes

The nonlinear disturbance observer for the system \( (2.1) \) is written by combining the internal disturbance and external disturbance as follows:

\[
d_{k} = \dot{x}_{n} - f_{(n, \delta(t))}(x_{n}) - g_{(n, \sigma(t))}u(t) - T(x) \tag{3.1}
\]

Where \( k = \sigma(t) [0, \infty] \rightarrow M = \{1, 2, \ldots, m\} \) and the disturbance observer can be proposed as follow in \( [10] \).

\[
\hat{d}_{k} = -L_{d}(x_{1}, x_{2})\dot{d}_{k} + L_{d}(x_{1}, x_{2}) (\dot{x}_{2} - \theta_{\sigma}^{T}w(x_{n}) - g_{\sigma}u(t) - T(x)) \tag{3.2}
\]

Where \( L_{d}(x_{1}, x_{2}) = a, a > 0 \), consider for designing parameter Defining the variable \( z_{k} = \hat{d}_{k} - p(x) \) and inserting \( z_{k} \) in to the above equation gives

\[
\dot{z}_{k} + \dot{p}(x) = -L_{d_{k}}(x)z_{k} + L_{d_{k}}(x)p(x) + L_{d_{k}}(x) = \frac{\delta p_{k}(x)}{\sigma(x)} \tag{3.3}
\]

\( L_{d_{k}}(x) \) is NDO gain value then:

\[
\dot{p}(x) = -L_{d_{k}}(x) \tag{3.4}
\]

The tracking error dynamics can be computed

\[
\dot{e}_{d_{k}} = \hat{d}_{d_{k}} - \dot{d} = \dot{z} + \dot{p}_{k}(x) - \dot{d} \tag{3.5}
\]

\[
\dot{e}_{d_{k}} = \hat{d}_{d_{k}} - \dot{d} = \dot{z} - L_{d_{k}}(x) - \dot{d} \tag{3.6}
\]

\[
\begin{align*}
\dot{e}_{d_{k}} &= -L_{d_{k}}(x)z - L_{d_{k}}(x) \left[ \theta_{\sigma}^{T}w(x_{n}) - g_{\sigma}(t)u(t) - T(x) + p(x) \right] + \dot{p}(x) - \dot{d} \\
&= -L_{d_{k}}(x)z - L_{d_{k}}(x) \left[ x' + (p(x) - d) \right] + \dot{p}(x) - \dot{d} \\
&= -L_{d_{k}}(x)z - L_{d_{k}}(x) \left[ \dot{x} + e_{d_{k}} - z \right] + \dot{p}_{k}(x) - \dot{d} \\
&= -L_{d_{k}}(x)e_{d_{k}} - L_{d_{k}}(x) \left[ \dot{x} + \dot{p}_{k}(x) - \dot{d} \right] \\
&= -L_{d_{k}}(x)z - L_{d_{k}}(x) \left[ \theta_{\sigma}^{T}w(x) + g_{(n, \sigma(t))}(x)u + T(x) + p_{k}(x) \right] + \dot{p}_{k}(x) - \dot{d} \\
&= -L_{d_{k}}(x)z - L_{d_{k}}(x) \left[ x' + g(p_{k}(x) - d) \right] + \dot{p}_{k}(x) - \dot{d}
\end{align*}
\]
Using (3.4) in (3.6), disturbance tracking error dynamics are governed by

\[ \dot{e}_d = -L_{d_k}(x)e_d - \dot{d} \]  

(3.7)

With assuming the observer gain \( L_{d_k}(x) \) has been designed to satisfy

\[ e_d^T L_{d_k}(x)e_d \leq -\alpha |e_d|^2 \]  

(3.8)

To analyze how the NDO in (3.1) approaches disturbance tracking the Lyapunov candidate function for any \( k \in M \) \( V_d = \sum_{k=1}^{i} \frac{1}{2} e^2_{d_k} \) is considered taking the derivative of \( V_d \) along trajectories of (3.7) yield.

\[ \dot{v}_d = \sum_{j=1}^{i} -e_{d_k}^T L_{d_k}(x)e_d - e_{d_k}^T \dot{d} \]  

(3.9)

Assuming that there exists a positive real constant \( w \) such that \( |\dot{d}(t)| \leq w \) for all \( t \geq 0 \), considering (3.8) and (3.9) it results in

\[ \dot{v}_d \leq -\sum_{j=1}^{i} \alpha d|ed|^2 \leq -\zeta d|ed|^2 \]  

(3.10)

Where \( \zeta \) is positive constant.

4. Adaptive control scheme

The adaptive backstepping technique will be considered to design an adaptive control scheme that can guarantee a common Lyapunov function (CLF) for all subsystems. Firstly, we define the error variable.

\[ Z_1 = y - y_r \]  

(4.1)

\[ z_i = x_i - a_{i-1} - y_r^{(i-r)}, \quad i = 1, 2, \ldots, n \]  

(4.2)

Where for \( 1 \leq i \leq n-1 \), \( \alpha_i \) is the common virtual control to be constructed at the ith step that

\[ \alpha_1 = -\lambda_1 z_1 - \frac{z_1}{2} - \frac{1}{2a_i^2} z_1 \hat{\theta} w_1^T w_1 \]  

(4.3)

\[ \alpha_i = -\lambda_i z_i - \frac{z_i}{2} - z_{i-1} - \frac{1}{2a_i^2} z_i \hat{\theta} w_i^T w_i + r \sum_{j=1}^{i-1} \left[ \frac{\delta \alpha_i - 1}{\delta x_i} x_{j+1} + \frac{\delta \alpha_{i-1}}{\delta y_r^{(j-r)}} y_r^{(j-r)} + \frac{\delta \alpha_{i-1}}{\delta y_r^{(j-r)}} g_r^{(j-r)} \right] \]  

(4.4)

\[ + \frac{\delta \alpha_{i-1}}{\delta \theta} \left( \sum_{j=1}^{i-1} \frac{r}{2a_i^2} z_j w_j^T w_j \right) + r \sum_{j=2}^{i} \frac{z_j}{2a_i^2} w_j^T w_j - \frac{\delta \alpha_{i-1}}{\delta \theta} \sigma \cdot \hat{\theta} \]  

(4.5)

for \( i = n \), \( v_i \) and \( a_i \) is required to be the following form:

\[ \alpha_i = \frac{1}{g_{n,k}} (v_i + \hat{d}_k) \]  

(4.6)
where

\[ v_i = -\lambda_i z_i - \frac{z_i}{2} - z_{i-1} - \frac{1}{2a_i^2} z_i \dot{\theta} w_i^T w_i + r \sum_{j=1}^{i-1} \left[ \delta \alpha_{i-1} \frac{1}{\delta x_i} x_{j+1} + \frac{\delta \alpha_{i-1}}{\delta y_i^{(j)}} y_i^{(j)} + \frac{\delta \alpha_{i-1}}{\delta y_r^{(j-1)}} g_r^{(j)} \right] \]

\[ + \frac{\delta \alpha_{i-1}}{\delta \theta} \left( \sum_{j=1}^{i-1} \frac{r}{2a_i^2} z_j^2 w_j^T w_j \right) + \frac{r}{2a_i^2} \sum_{j=2}^{i} z_j \frac{\delta \alpha_{j-1}}{\delta \theta} w_i^T w_i - \frac{\delta \alpha_{i-1}}{\delta \theta} \sigma \cdot \hat{\theta} \] (4.7)

Where, \( c_i, a_i \) and \( r \), are positive design parameters, \( \lambda_i = c_i + g_{i,\text{max}}, g_{i,\text{max}} = \max \{ g_{i,k} : k \in M \} \), \( \hat{\theta} = \max i \in M \{ \| \theta_i \| \} \). \( \hat{\theta} \) denotes the estimate of the unknown constant \( \theta \) and the function \( w_i, i = 1, 2, \ldots, n \) are given by

\[ w_1 = \varphi_1, \quad w_i = \varphi_i - \sum_{j=1}^{i-1} \frac{\delta \alpha_{i-1}}{\sigma a_j} \varphi_j, \quad i = 1, 2, \ldots, n \] (4.8)

The parameter update law and adaptive control law are chosen as

\[ \dot{\hat{\theta}} = \sum_{i=1}^{n} \frac{r}{2a_i^2} z_i w_i^T w_i - \sigma_0 \hat{\theta} \] (4.9)

\[ u_k = \frac{1}{g_{n,k}} (v_n + \hat{d}_k) \] (4.10)

Where, \( \sigma_0 \) is the positive design parameter.

For showing the stability of the proposed method, the following Lyapunov function is considered:

\[ v = \sum_{i=1}^{n} \frac{1}{2} z_i^2 + \frac{1}{2r} \hat{\theta}^2 + \frac{1}{2} \hat{d}^2 \] (4.11)

Where \( \hat{\theta} = \theta - \hat{\theta} \) and \( \hat{d} = \hat{d} - d \),

\[ \frac{d}{dt} \left( \sum_{i=1}^{n} \frac{1}{2} z_i^2 \right) = \sum_{i=1}^{n-1} z_i z_{i+1} + \sum_{i=1}^{n} \theta_i w_i z_i - \sum_{i=2}^{n} z_i \left( \sum_{j=1}^{i-1} \frac{\delta \alpha_{i-1}}{\delta g_r^{(j-1)}} g_r^{(j)} + \sum_{j=1}^{i-1} \frac{\delta \alpha_{i-1}}{\delta x_j} x_{j+1} \right) \]

\[ + \sum_{i=1}^{n} z_i \alpha_i - z_n (\hat{d}) - \sum_{i=2}^{n} z_i \frac{\delta \alpha_{i-1}}{\delta \hat{\theta}} \] (4.12)

By applying the Cauchy – Schwarz inequality \[25\], we obtain that

\[ \sum_{i=1}^{n} \theta_i^2 w_i z_i \leq \sum_{i=1}^{n} \frac{1}{2a_{i,\text{min}}} z_i^2 \theta w_i^T w_i + \sum_{i=1}^{n} \frac{1}{2} \alpha_i^2 \] (4.13)

Substituting the virtual control function given by (4.3)-(4.10) and (4.12), the below statement obtains.

\[ \frac{d}{dt} \left( \sum_{i=1}^{n} \frac{1}{2} z_i^2 \right) \leq - \sum_{i=1}^{n} c_i z_i^2 + \sum_{i=1}^{n} \frac{1}{2a_{i,\text{min}}} z_i^2 \hat{\theta} w_i^T w_i - z_n \hat{d} + \sum_{i=1}^{n} \frac{1}{2} \alpha_i^2 + vr \] (4.14)
\[ V_r = - \sum_{i=1}^{n} z_i \frac{\delta \alpha_i - 1}{\delta \hat{\theta}} + \sum_{i=2}^{n} z_i \frac{\delta \alpha_i - 1}{\delta \hat{\theta}} \sum_{j=1}^{i-1} \frac{r}{2a_{j, \text{min}}} z_j^2 w_j^T w_i + \sum_{i=2}^{n} z_i \frac{\delta \alpha_i - 1}{\delta \hat{\theta}} - \sum_{i=2}^{n} z_i \frac{\delta \alpha_i - 1}{\delta \hat{\theta}} \sigma_0 \hat{\theta} \]  

Next, we further prove \( V_r = 0 \) using the parameter update law \( \hat{\theta} \) designed in (4.9) the first term on the right side of (4.15) can be expanded and rearranged as

\[ - \sum_{i=2}^{n} z_i \frac{\delta \alpha_i - 1}{\delta \hat{\theta}} = - \sum_{i=2}^{n} z_i \frac{\delta \alpha_i - 1}{\delta \hat{\theta}} \left( \sum_{j=1}^{i-1} \frac{r}{2a_{j, \text{min}}} z_j^2 w_j^T w_i + \sum_{j=1}^{i-1} \frac{r}{2a_{j, \text{min}}} z_j^2 w_j^T w_i \right) + \sum_{i=2}^{n} z_i \frac{\delta \alpha_i - 1}{\delta \hat{\theta}} \sigma_0 \hat{\theta} \]

Substituting (4.8) into (4.7), it follows that \( V_f = 0 \).

According to the assumption and using that fact

\[ - z_2 \dot{d} \leq \frac{1}{2} z_2^2 + \frac{1}{2} d^2 \]  

And by computing the time derivative of the second and third term on the right side of (4.15)

\[ \frac{d}{dt} \left( \frac{1}{2r} \theta^2 \right) = - \frac{1}{r} \dot{\theta} \leq - \sum_{i=1}^{n} \frac{r}{2a_i, \text{min}} z_i^2 w_i^T w_i \leq - \frac{1}{2} \dot{\theta}^2 + \frac{1}{2} \theta_i^2 \]

So, the following theorem can be obtained.

**Theorem 4.1.** Consider the switched nonlinear system (2.1) under arbitrary switching, the stability of a switched system under an arbitrary switching signal is well ensured if the adaptive control scheme (4.3)-(4.8) is applied.

Therefore, the closed-loop system satisfies the following desired properties

- All the closed-loop signals of the system are globally bounded
- The ultimate bound for the steady-state performance of tracking error is \( \lim_{t \to \infty} |z_1(t)| \leq \sqrt{\frac{2b_0}{a_0}} \)

where

\[ a_0 = \min \{2c_j, \sigma_0 \ 1 \leq j \leq n\} \]  

\[ b_0 = \frac{\sigma_0}{2r} \theta^2 + \frac{1}{2} d^2 + \sum_{j=1}^{n} \frac{1}{2} a_j \]
\textbf{Proof}. By substituting $V_r$ in to (4.14) and combining with adaptive law (4.3)-(4.9), we have

$$
\dot{v} \leq -\sum_{i=1}^{n} c_i z_i^2 + \frac{\sigma_0}{r} \hat{\theta} + d d + \sum_{i=1}^{n} \frac{1}{2} u_i^2 \leq -av + b
$$

(4.22)

According to the comparison principle, one gets

$$
V_n(t) \leq \left( V_n(0) - \frac{b_0}{a_0} \right) e^{-at} + \frac{b_0}{a_0} \quad \forall \; t \geq 0
$$

(4.23)

\[\square\]

5. Simulation results

The model system, which we study, is a gyroscope, which has attributes of great potential to navigational aeronautical and space engineering.

The equation governing the motion of the gyro after necessary transformation is such that.

$$
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= T(x) + B \sin x_1 + f \sin wt \sin x_1 + \theta_\sigma (x_2 - x_2^3) + d(t) + g(n, \sigma(t)) u_\sigma(t) \\
y &= x_1
\end{align*}
$$

(5.1)

Where $T(x) = -\alpha^2(1-\cos x_1) \frac{\alpha^2}{\sin^2 x_1}$

Dynamical behavior including the chaotic motion for the following parameters $a^2 = 1$, $w = 2$, $f = 35.5$, $g_{2,1} = 1$, $g_{2,2} = 1.5$ and $u(0) = 0$ two subsystems are considered in this example, $\sigma(t) \in M = \{1, 2\}$ for simulation we select $\theta_1 = 0.4$, $\theta_2 = 0.1$, $c_1 = 1.25$, $c_2 = 2$, $r_1 = 10$, $r_2 = 5$, $a_1 = a_2 = 0.5$, $\hat{\theta}(0) = 0.2$, $\beta_1 = \beta_2 = 0.025$.

In this paper for any subsystem the time history of the chaotic gyro with initial conditions of $(a)(x_1, y_1) = (1, -1)$ and $(b)(x_d, y_d) = (1, -1/2)$.

According to theorem 4.1, an adaptive law $\hat{\theta}$ and the control law

$$
\hat{\theta} = \sum_{i=1}^{n} \frac{r}{2a_i} z_i^2 w^T w - B \hat{\theta}
$$

(5.2)

$$
u_1 = -\frac{1}{g_{2,1}} (v_2 + \hat{d}_1)
$$

(5.3)

$$
u_2 = -\frac{1}{g_{2,2}} (v_2 + \hat{d}_2)
$$

(5.4)

Where, $\lambda_2 = c_1 + g_{2,1}$, $\lambda_1 = c_1 + 1$.

The nonlinear adaptive law $\hat{\theta}$ in (3.5) contains all system state in this case, the term $-(\delta a_{i-1}/\delta \hat{\theta}) \hat{\theta}$ generated at the ith step of backstepping based nonlinear disturbance procedure cannot be canceled directly by virtual controller $\alpha_i$, as $\alpha_i$ is only allowed to include $x_1, \ldots, x_i$ for the subsequent step of back stepping with NDO design. As seen (4.11), such an issue is well considered via the use of rearranging operation. The simulation results are illustrated in Figure 1 for $e_d = \hat{d} - d$ and in Figure 2 for $e_1 = x_1 - x_{1d}$ and in Figure 3 $e_2 = x_2 - x_{2d}$. Figure 4 illustrate the evolution of switching.
signal of system. In these figures, it can be seen that the NDO achieves good disturbance attenuation ability and good tracking performance is achieved, also synchronization error will converge to zero.

This paper proposes an adaptive control scheme with a nonlinear disturbance observer (NDO). The system has both uncertain and known parameters. The scheme estimates the bound on switching parameters to construct a common Lyapunov function for all subsystems in the presence of disturbance. It is shown that by designing the nonlinear compensation appropriately, the steady-state response of tracking error can be obtained on system output. Simulation studies of a gyro switched system have been carried out to demonstrate the proposed NDO method’s validity. The Result has shown that the proposed method obtains much better disturbance rejection ability and tracking control against model uncertainties.

Figure 1: Estimation error \((d - \hat{d})\)

Figure 2: The graph of error \(e_1\) between two chaotic gyros with active control.
6. Conclusion

This paper proposes an adaptive control scheme with a nonlinear disturbance observer (NDO). The system has both unknown parameters. The scheme estimates the bound on switching parameters to construct a common Lyapunov function for all subsystems in the presence of disturbance. It is shown that by designing the nonlinear compensation appropriately, Gains Zrovo’s the steady-state response of tracking error can be obtained on system output. Simulation studies of a gyro switched system have been carried out to demonstrate the proposed NDO method’s validity. The Result has shown that the proposed method obtains much better disturbance rejection ability and tracking control against model uncertainties.
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