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Stability of fuzzy orthogonally $*$-n-derivation in orthogonally fuzzy C^*-algebras

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Abstract

In this paper, using fixed point methods, we prove the fuzzy orthogonally $*$-n-derivation on orthogonally fuzzy C^*-algebra for the functional equation

$$f(\frac{\mu x+\mu y}{2}+\mu w)+f(\frac{\mu x+\mu w}{2}+\mu y)+f(\frac{\mu y+\mu w}{2}+\mu x)=2\mu f(x)-2\mu f(y)-2\mu f(w).$$

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1. Introduction

The stability problem of functional equations originated from the question of Ulam [20] concerning the stability of group homomorphisms. Hyers [10] gave the first affirmative partial answer to the question of Ulam for Banach spaces. Hyers' theorem was generalized by Th.M. Rassias [17] for linear mappings by considering an unbounded Cauchy difference. Park et al. proved stability homomorphisms and derivations in Banach algebras, Banach ternary algebras, C^*-algebras, Lie C^*-algebras and C^*-ternary algebras [11, 15, 16]. The stability problems of several functional equations have been extensively investigated by a number of authors, and there are many interesting results concerning this problem [4, 8, 9, 12, 18].

In the follwing, we review the basic definitions of orthogonally sets [2, 6] and the definition of fuzzy normed spaces [7, 13], which can be consider the main definition of our paper.

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Definition 1.1. *Let $X \neq \emptyset$ and $\perp \subseteq X \times X$ be an binary relation. If $\perp$ satisfies the following condition*

$$\exists x_0; (\forall y; y \perp x_0) \text{ or } (\forall y; x_0 \perp y),$$

then X is called an orthogonally set (briefly O-set). We denote this O-set by $(X, \perp)$.

Definition 1.2. *Let $(X, \perp)$ be an orthogonally space. A function $f : X \to X$ is called $\perp$-preserving, if $x \perp y$, then $f(x) \perp f(y)$ for all x, y.*

Let $(X, \perp)$ be an O-set. A sequence $\{x_n\}_{n \in \mathbb{N}}$ is called orthogonally sequence (briefly O-sequence) if

$$(\forall n; x_n \perp x_{n+1}) \text{ or } (\forall n; x_{n+1} \perp x_n).$$

Definition 1.3. *Let $(X, \perp, d)$ be an orthogonally metric space (i.e, $(X, \perp)$ is an O-set and (X, d) is a metric space), then $f : X \to X$ is $\perp-$continuous at $a \in X$ if for each O-sequence $\{a_n\}_{n \in \mathbb{N}}$ in X, $a_n \to a$,implies $f(a_n) \to f(a)$. Also, f is $\perp-$continuous on X if f is $\perp-$continuous at each $a \in X$.*

It is abvious to see that every continuous mapping is $\perp-$continuous.

Definition 1.4. *Let $(X, \perp, d)$ be an orthogonally metric space, then X is orthogonally complete (briefly O-complete) if every Cauchy O-sequence is convergent.*

Every complete metric space is O-complete, but the converse is not true.

Definition 1.5. *Let $(X, \perp, d)$ be an orthogonally metric space and $0 < \lambda < 1$. A mapping $f : X \to X$ is said to be orthogonality contraction with Lipschitz constant λ if*

$$d(fx, fy) \leq \lambda d(x, y) \quad if \ x \perp y.$$

Definition 1.6. *Let X be a set. A function $d : X \times X \to [0, \infty]$ is called a generalized metric on X if d satisfies the following conditions:*
(1) $d(x, y) = 0$ if and only if $x = y$ for all $x, y \in X$;
(2) $d(x, y) = d(y, x)$ for all $x, y \in X$;
(3) $d(x, z) \leq d(x, y) + d(y, z)$ for all $x, y, z \in X$.

Theorem 1.7. *[2] Let $(X, d, \perp)$ be an O-complete generalized metric space and $0 \leq \lambda < 1$. Let $T : X \to X$ be $\perp$-preserving, $\perp$-continuous and $\perp$-λ-contraction. Consider the "O-sequence of successive approximations with initial element x_0": x_0, $T(x_0)$, $T^2(x_0)$, ..., $T^n(x_0)$, ... Then, either $d(T^n(x_0), T^{n+1}(x_0)) = \infty$ for all $n \geq 0$, or there exists a positive integer n_0 such that $d(T^n(x_0), T^{n+1}(x_0)) < \infty$ for all $n \geq n_0$. If the second alternative holds, then*
i) the O-sequence of $\{T^n(x_0)\}$ is convergent to a fixed point x^* of T.
ii) x^* is the unique fixed point of T in $X^* = \{y \in X : d(T^n(x_0), y) < \infty\}$.
iii) $d(y, x^*) \leq \frac{1}{1-\lambda} d(y, T(y))$ for all $y \in X^*$.

In the follwing, we use the definition of fuzzy normed spaces to investigate a fuzzy version of the Hyers-Ulam stability for the functional equation in the fuzzy normed algebra setting [1, 3, 14, 19].

Definition 1.8. *Let X be a vector space. A function $N : X \times \mathbb{R} \to [0, 1]$ is called a fuzzy norm on X if*
(N_1) $N(x, t) = 0$ for all $x \in X$ and $t \in \mathbb{R}$ with $t \leq 0$;
(N_2) $x = 0$ if and only if $N(x, t) = 1$ for all $x \in X$ and $t > 0$;
(N_3) $N(cx, t) = N(x, \frac{t}{|c|})$ for all $x \in X$ and $c \neq 0$;
(N_4) $N(x + y, s + t) \geq min\{N(x, s), N(y, t)\}$ for all $x, y \in X$ and $s, t \in \mathbb{R}$;
(N_5) $N(x, .)$ is a non-decreasing function of $\mathbb{R}$ and $lim_{t\to\infty}N(x, t) = 1$ for all $x \in X$ and $t \in \mathbb{R}$;
(N_6) for all $x \in X$ with $x \neq 0$, $N(x, .)$ is continuous on $\mathbb{R}$.
The pair (X, N) is called a fuzzy normed vector space.

Definition 1.9. *Let $(X, \perp, N)$ be a orthogonally fuzzy normed vector space. A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ or converges if there exists $x \in X$ such that*

$$lim_{n\to\infty}N(x_n - x, t) = 1$$

for all $t > 0$. In this case, x is called the limit of the sequence $\{x_n\}$ and we denote it by $N - lim_{n\to\infty}x_n = x$.

Definition 1.10. *Let (X, N) be a fuzzy normed vector space. A sequence $\{x_n\}$ in X is called Cauchy if, for each $\epsilon > 0$ and $t > 0$, there exists an $n_0 \in N$ such that for all $n \geq n_0$ and all $p > 0$, we have $N(x_{n+p} - x_n, t) > 1 - \epsilon$.*

It is well-known that every convergent sequence in a fuzzy normed vector space is a Cauchy sequence. If each Cauchy sequence is convergent, then the fuzzy normed vector space is said to be complete and the complete fuzzy normed vector space is called a fuzzy Banach space. We say that a mapping $f : X \to Y$ between fuzzy normed vector spaces X and Y is continuous at a point $x_0 \in X$ if, for each sequence $\{x_n\}$ converging to $x_0 \in X$, the sequence $f(x_n)$ converges to $f(x_0)$. If $f : X \to Y$ is continuous at each $x \in X$, then $f : X \to Y$ is said to be continuous on X.

Definition 1.11. *A fuzzy normed algebra (X, N) is a fuzzy normed space (X, N) with the algebraic structure such that*
(N_7) $N(xy, ts) \geq N(x, t)N(y, s)$ for all $x, y \in X$ and $t, s > 0$.

Every normed algebra $(X, ||.||)$ defines a fuzzy normed algebra (X, N), where N is defined by

$$N(x, t) = \frac{t}{t + ||x||}$$

for all $t > 0$. This space is called the induced fuzzy normed algebra.

Definition 1.12. *Let (X, N) and (Y, N) be fuzzy normed algebras.*
(1) A C-linear mapping $f : X \to Y$ is called a homomorphism if

$$f(xy) = f(x)f(y)$$

for all $x, y \in X$.
(2) An C-linear mapping $f : X \to X$ is called a derivation if

$$f(xy) = f(x)y + xf(y)$$

for all $x, y \in X$.

Definition 1.13. *Let (U, N) be a fuzzy Banach algebra. Then an involution on $\mathcal{U}$ is a mapping $u \to u^*$ from $\mathcal{U}$ into $\mathcal{U}$ which satisfies the following:*
(a) $u^{**} = u$ for any $u \in \mathcal{U}$;
(b) $(\alpha u + \beta v)^* = \overline{\alpha} u^* + \overline{\beta} v^*$;
(c) $(uv)^* = v^* u^*$ for any $u, v \in \mathcal{U}$.

If, in addition, $N(u^*u, ts) = N(u, t)N(u, s)$ and $N(u^*, t) = N(u, t)$ for all $u \in \mathcal{U}$ and $t, s > 0$, then $\mathcal{U}$ is a fuzzy C^*-algebra.

2. Stability of $*$-n-derivation in orthogonally fuzzy C^*-algebras

Throughout this section, assume that $(A, \|.\|_1, \perp_1)$ with norm N_A and $a \perp_1 b$ if $ab^* = b^*a = 0$ is an orthogonally fuzzy C^*-algebras. For any mapping $f : A \to A$, we define

$$\begin{aligned} \Delta_\mu f(x, y, w) :=& f(\frac{\mu x + \mu y}{2} + \mu w) + f(\frac{\mu x + \mu w}{2} + \mu y) + f(\frac{\mu y + \mu w}{2} + \mu x) \\ &- 2\mu f(x) - 2\mu f(y) - 2\mu f(w) \end{aligned} \tag{2.1}$$

for all $\mu \in \mathbb{T}^1 := \{v \in \mathbb{C} : |v| = 1\}$ and $x, y, w \in A$ with $x \perp y$, $y \perp w$ and $w \perp x$. Note that a C-linear mapping $\delta : A \to A$ is called a fuzzy C^*-algebra derivation on fuzzy C^*-algebra if δ satisfies the following

$$\delta(xy) = y\delta(x) + x\delta(y) \tag{2.2}$$

and

$$\delta(x^*) = \delta(x)^* \tag{2.3}$$

for all $x; y \in A$. We are going to investigate the generalized Hyers-Ulam stability of orthogonally fuzzy C^*-algebra derivation on orthogonally fuzzy C^*-algebra for the functional equation

$$\Delta_\mu f(x, y, w) := 0. \tag{2.4}$$

For a given mapping $f : A \to A$, we define

$$\begin{aligned} D(z_1, z_2, ..., z_n) := f(z_1 z_2 ... z_n) - f(z_1) z_2 z_3 ... z_n - z_1 f(z_2) z_3 ... z_n - \\ ... - z_1 z_2 ... z_{n-1} f(z_n) \end{aligned} \tag{2.5}$$

for all $z_i \in A$, $z_i \perp z_{i+1}$.

Theorem 2.1. *Let $f : A \to A$ be a mapping for which there are functions $\varphi : A^n \to [0, \infty)$ such tha there exists an $L < \frac{1}{2}$ with*

$$\varphi(\frac{x_1}{2}, \frac{x_2}{2}, ..., \frac{x_n}{2}) \leq \frac{L\varphi(x_1, x_2, ..., x_n)}{2} \tag{2.6}$$

$$N_A(\Delta_\mu f(x_1, x_2, x_3), t) \geq \frac{t}{t + \varphi(x_1, x_2, x_3, ..., 0)} \tag{2.7}$$

$$N_A(D(x_1, x_2, ..., x_n), t) \geq \frac{t}{t + \varphi(x_1, x_2, ..., x_n)} \tag{2.8}$$

$$N_A(f(x^*)-f(x)^*,t) \geq \frac{t}{t+\varphi(x,0,...,0)} \tag{2.9}$$

for all $\mu \in \mathbb{T}$, $x_1, x_2, ..., x_n \in A$ *with* $x_i \perp x_{i+1}$ *and* $t > 0$. *Then there exists a unique fuzzy orthogonally* $*$-n-*derivation* $\delta : A \to A$ *such that*

$$N_A(f(x)-\delta(x),t) \geq \frac{(6-6L)t}{(6-6L)t+\varphi(x,x,...,x)} \tag{2.10}$$

for all $x \in A$ *and* $t > 0$.

Proof . Putting $\mu = 1$, $x = x_1 = x_2 = x_3$ and $x_4 = ... = x_n = 0$ in (2.7), we have

$$N_A(3f(2x)-6f(x),t) \geq \frac{t}{t+\varphi(x,x,x,...,0)} \tag{2.11}$$

for all $x \in A$. So

$$N_A(f(x)-2f(\frac{x}{2}),t) \geq \frac{3t}{3t+\varphi(\frac{x}{2},\frac{x}{2},\frac{x}{2},...,0)} \geq \frac{3t}{3t+\frac{L}{2}\varphi(x,x,x,...,0)} \tag{2.12}$$

for all $x \in A$. Consider the set $X := \{g : A \to A\}$ and define the generalized metric d on X, by

$$d(g,h) = \inf\{\mu \in \mathbb{R}^+ : \ N_A(g(x)-h(x),\mu t) \geq \frac{t}{t+\varphi(x,x,x,...,0)}, \ \forall x \in A, t > 0\}.$$

Now, we put the orthogonality relation $\perp$ on X as follows

$$h \perp g \ \Leftrightarrow \ h(x) \perp g(x) \ \textit{or} \ g(x) \perp h(x)$$

for all $x \in A$ and $g, h \in X$. It is easy to show that $(X, d, \perp)$ is an O-complete generalized metric space.

Now, we consider the linear mapping $T : X \to X$ defined by $Tg(x) = 2g(\frac{x}{2})$ for all $x \in A$. Let $g, h \in X$ with $g \perp h$ be such that $d(g,h) = \epsilon$. Then $N_A(g(x)-h(x),\epsilon t) \geq \frac{t}{t+\varphi(x,x,x,...,0)}$ for all $x \in A$ and $t > 0$. Hence

$$\begin{aligned} N_A(Tg(x)-Th(x),L\epsilon t) &= N_A(2g(\frac{x}{2})-2h(\frac{x}{2}),L\epsilon t) \\ &= N_A(g(\frac{x}{2})-h(\frac{x}{2}),\frac{L\epsilon t}{2}) \\ &\geq \frac{\frac{Lt}{2}}{\frac{Lt}{2}+\varphi(\frac{x}{2},\frac{x}{2},\frac{x}{2},...,0)} \\ &\geq \frac{\frac{Lt}{2}}{\frac{Lt}{2}+\frac{L\varphi(x,x,x,...,0)}{2}} = \frac{t}{t+\varphi(x,x,x,...,0)} \end{aligned}$$

for all $x \in A$ and $t > 0$. Thus $d(g,h) = \epsilon$ implies that $d(Tg,Th) \leq L\epsilon$. Hence we see that

$$d(Tg,Th) \leq L \ d(g,h)$$

for all $g, h \in X$ with $g \perp h$, that is, T is a strictly contractive self-mapping of X with the Lipschitz constant L. Now, we show that T is $\perp$-continuous. To this end, let $\{g_n\}_{n \in \mathbb{N}}$ be an O-sequence with

$g_n \perp g_{n+1}$ or $g_{n+1} \perp g_n$ in $(X, d, \perp)$ for all $n \in \mathbb{N}$, which convergent to $g \in X$ and let $\epsilon > 0$ be given. Then there exists $N \in \mathbb{N}$ and $k \in \mathbb{R}^+$ with $k < \epsilon$ such that

$$N_A(g_n(x) - g(x), kLt) \geq \frac{t}{t + \varphi(x, x, x, ..., 0)}$$

for all $x \in A$ and $n \geq N$ and so

$$N_A(2g_n(\frac{x}{2}) - 2g(\frac{x}{2}), kLt) \geq \frac{t}{t + \varphi(\frac{x}{2}, \frac{x}{2}, \frac{x}{2}, ..., 0)}$$

for all $x \in A$ and $n \geq N$. By inequality (2.6) and the definition of T, we get

$$N_A(Tg_n(x) - Tg(x), kLt) \geq \frac{t}{t + \varphi(x, x, x, ..., 0)}$$

for all $x \in A$ and $n \geq N$. Hence

$$d(T(g_n), T(g)) \leq kL < \epsilon$$

for all $n \geq N$. It follows that T is $\perp$-continuous. It follows from (2.12) that

$$N_A(f(x) - 2f(\frac{x}{2}), \frac{Lt}{2}) \geq \frac{3t}{3t + \varphi(x, x, x, ..., 0)} \tag{2.13}$$

for all $x \in A$ and all $t > 0$. This implies that $d(f, Tf) \leq \frac{L}{6}$. By Theorem 1.7, there exists a mapping $\delta : A \to A$ satisfying the following:

• δ is a fixed point of T, that is,

$$\delta(\frac{x}{2}) = \frac{\delta(x)}{2} \tag{2.14}$$

for all $x \in A$. The mapping δ is a unique fixed point of T in the set $Y = \{h \in X : d(g, h) < \infty\}$. This implies that δ is a unique mapping satisfying equation (2.14) such that there exists $\mu \in (0, \infty)$ satisfying

$$N_A(g(x) - H(x), \mu t) \geq \frac{t}{t + \varphi(x, x, x, ..., 0)} \tag{2.15}$$

for all $x \in X$ and $t > 0$.

• $d(T^n f, \delta) \to 0$ as $n \to \infty$. This implies the equality

$$N - \lim_{n\to\infty} 2^n f(\frac{x}{2^n}) = \delta(x) \tag{2.16}$$

for all $x \in X$.

• $d(f, \delta) \leq \frac{d(f,Tf)}{1-L}$ with $f \in X$, which implies the inequality $d(f, \delta) \leq \frac{L}{6-6L}$. This implies that the inequality (2.22) holds. It follows from equations (2.7) and (2.16) that

$$\begin{aligned}
&N_A(\delta(\frac{\mu x_1 + \mu x_2}{2} + \mu x_3) + \delta(\frac{\mu x_1 + \mu x_3}{2} + \mu x_2) + \delta(\frac{\mu x_2 + \mu x_3}{2} + \mu x_1) \\
&- 2\mu\delta(x_1) - 2\mu\delta(x_2) - 2\mu\delta(x_3), t) \\
&= N - \lim_{n\to\infty}(2^n f(\frac{\mu x_1 + \mu x_2}{2^{n+1}} + \frac{\mu x_3}{2^n}) + 2^n f(\frac{\mu x_1 + \mu x_3}{2^{n+1}} + \frac{\mu x_2}{2^n}) + 2^n f(\frac{\mu x_2 + \mu x_3}{2^{n+1}} + \frac{\mu x_1}{2^n}) \\
&- 2^{n+1}\mu f(\frac{x_1}{2^n}) - 2^{n+1}\mu f(\frac{x_2}{2^n}) - 2^{n+1} f\mu f(\frac{x_3}{2^n}), t) \\
&\geq \lim_{n\to\infty} \frac{\frac{t}{2^n}}{\frac{t}{2^n} + \varphi(\frac{x_1}{2^n}, \frac{x_2}{2^n}, \frac{x_3}{2^n}, ..., 0)} \geq \lim_{n\to\infty} \frac{\frac{t}{2^n}}{\frac{t}{2^n} + \frac{L^n}{2^n}\varphi(x_1, x_2, x_3, ..., 0)} \to 1
\end{aligned}$$

for all $\mu \in \mathbb{T}$, $x_1, x_2, ..., x_n \in A$ and $t > 0$. Hence

$$\delta(\frac{\mu x_1 + \mu x_2}{2} + \mu x_3) + \delta(\frac{\mu x_1 + \mu x_3}{2} + \mu x_2) + \delta(\frac{\mu x_2 + \mu x_3}{2} + \mu x_1)$$
$$- 2\mu\delta(x_1) - 2\mu\delta(x_2) - 2\mu\delta(x_3) = 0$$

for all $x_1, x_2, ..., x_n \in A$. So the mapping $\delta : A \to A$ is additive and C-linear. It follows from equations (2.8) that

$$N_A(2^{n^2} f(\frac{x_1 x_2 ... x_n}{2^{n^2}}) - 2^n f(\frac{x_1}{2^n}) x_2 x_3 ... x_n - x_1 2^n f(\frac{x_2}{2^n}) x_3 ... x_n - ...$$
$$- x_1 x_2 ... x_{n-1} 2^n f(\frac{x_n}{2^n}), 2^{n^2} t) \geq \frac{t}{t + \varphi(\frac{x_1}{2^n}, \frac{x_2}{2^n}, ..., \frac{x_n}{2^n})}$$

for all $x_1, x_2, ..., x_n \in A$. Then

$$N_A(2^{n^2} f(\frac{x_1 x_2 ... x_n}{2^{n^2}}) - 2^n f(\frac{x_1}{2^n}) x_2 x_3 ... x_n - x_1 2^n f(\frac{x_2}{2^n}) x_3 ... x_n - ...$$
$$- x_1 x_2 ... x_{n-1} 2^n f(\frac{x_n}{2^n}), t) \geq \frac{\frac{t}{2^{2n}}}{\frac{t}{2^{2n}} + \frac{L^n}{2^n} \varphi(\frac{x_1}{2^n}, \frac{x_2}{2^n}, ..., \frac{x_n}{2^n})}$$
$$\geq \frac{\frac{t}{2^{2n}}}{\frac{t}{2^{2n}} + \frac{L^n}{2^n} \varphi(x_1, x_2, ..., x_n)} \to 1 \quad when \quad n \to \infty$$

for all $x_1, x_2, ..., x_n \in A$ and $t > 0$. So $\delta(x_1 x_2 ... x_n) - \delta(x_1) x_2 x_3 ... x_n - x_1 \delta(x_2) x_3 ... x_n - ... - x_1 x_2 ... x_{n-1} \delta(x_n) = 1$ for all $x_1, x_2, ..., x_n \in A$ and $t > 0$. It follows from equation (2.8) that

$$N_A(2^n f(\frac{x}{2^n}^*) - 2^n f(\frac{x}{2^n})^*, 2^n t) \geq \frac{t}{t + \varphi(\frac{x}{2^n}, 0, ..., 0)} \tag{2.17}$$

for all $x \in A$ and $t > 0$. Then

$$N_A(2^n f(\frac{x}{2^n}^*) - 2^n f(\frac{x}{2^n})^*, t) \geq \frac{\frac{t}{2^n}}{\frac{t}{2^n} + \varphi(\frac{x}{2^n}, 0, ..., 0)} \geq \frac{\frac{t}{2^n}}{\frac{t}{2^n} + \frac{L^n}{2^n} \varphi(x, 0, ..., 0)} \tag{2.18}$$

for all $x \in A$ and $t > 0$. Since $\lim_{n\to\infty} \frac{\frac{t}{2^n}}{\frac{t}{2^n} + \frac{L^n}{2^n} \varphi(x, 0, ..., 0)} = 1$ for all $x \in A$ and $t > 0$, we get $N_A(\delta(\frac{x}{2^n}^*) - \delta(\frac{x}{2^n})^*, t) = 1$ for all $x \in A$ and $t > 0$. Thus $\delta(\frac{x}{2^n}^*) = \delta(\frac{x}{2^n})^*$ for all $x \in A$.

Corollary 2.2. *Let θ, p be non–negative real numbers with $0 < p < 1$. Suppose that $f : A \to A$ is a mapping, such that*

$$N_A(\Delta_\mu f(x_1, x_2, x_3), t) \geq \frac{t}{t + \theta(\|x_1\|_A^p + \|x_2\|_A^p + \|x_3\|_A^p)} \tag{2.19}$$

$$N_A(D(x_1, x_2, ..., x_n), t) \geq \frac{t}{t + \theta(\|x_1\|_A^p \|x_2\|_A^p ... \|x_n\|_A^p)} \tag{2.20}$$

$$N_A(f(x^*) - f(x)^*, t) \geq \frac{t}{t + \theta(\|x\|_A^p)} \tag{2.21}$$

for all $\mu \in \mathbb{T}$, $x_1, x_2, ..., x_n \in A$ and $t > 0$ with and $x_i \perp x_{i+1}$. Then there exists a unique fuzzy orthogonally $*$-n-derivation $\delta : A \to A$ such that

$$N_A(f(x) - \delta(x), t) \geq \frac{t}{t + \theta(\|x\|_A^p)} \tag{2.22}$$

for all $x \in A$ and $t > 0$.

Proof . The proof follows from Theorem 2.1 by taking

$$\varphi(x_1, x_2, x_3, t) := \theta(\|x_1\|_A^p + \|x_2\|_A^p + \|x_3\|_A^p)),$$

$$\varphi(x_1, x_2, ..., x_n, t) := \frac{t}{t + \theta(\|x_1\|_A^p \|x_2\|_A^p ... \|x_n\|_A^p)}$$

for all $x_1, x_2, x_3 \in A$ with $x_i \perp x_{i+1}$. Then we can choose $L = 2^{-p}$ and so the desired conclusion follows. □

References

[1] R.P. Agarwal, Y.J. Cho, R. Saadati, S. Wang, Nonlinear L-fuzzy stability of cubic functional equations. J. Inequal. Appl. 2012, (2012), 1–19.

[2] A. Bahraini, G. Askari, M. Eshaghi Gordji, et al. Stability and hyperstability of orthogonally $*$-m-homomorphisms in orthogonally Lie C^*-algebras: a fixed point approach. J. Fixed Point Theory Appl. 20, (2018), 1-12 .

[3] T. Bag, S.K. Samanta, Finite dimensional fuzzy normed linear spaces. J. Fuzzy Math. 11, (2003), 687-705.

[4] M. Eshaghi-Gordji, and S. Abbaszadeh. Stability of Cauchy-Jensen inequalities in fuzzy Banach spaces. Applied and Computational Mathematics, 11, (2012), 27–36.

[5] M. Eshaghi Gordji, G. Askari, N. Ansari, G. A. Anastassiou and C. Park, Stability and hyperstability of generalized orthogonally quadratic ternary homomorphisms in non-Archimedean ternary Banach algebras: a fixed point approach, J. Computational Analysis And Applications, 21, (2016), 1–6.

[6] M. Eshaghi Gordji, M. Ramezani, M. De La Sen, and Y.J. Cho, On orthogonal sets and Banach fixed point theorem. Fixed Point Theory, 18, (2017), 569–578.

[7] C. Felbin, Finite-dimensional fuzzy normed linear space. Fuzzy Sets Syst. 48, (1992), 239-248.

[8] P. Gavruta and L. Gavruta, A new method for the generalized Hyers-Ulam-Rassias stability, Int. J. Nonlinear Anal. Appl. 1 (2010), 11–18.

[9] R. Gholami, G. AskariI and M. Eshaghi Gordji, Stability and hyperstability of orthogonally ring $*$-n-derivations and orthogonally ring $*$-n-homomorphisms on C^* -algebras, J. Linear Topol. Algebra, 7, (2018), 109–119.

[10] D.H. Hyers, On the stability of the linear functional equation, Proc. Nat. Acad. Sci. 27, (1941), 222–224.

[11] B. Margolis and J.B. Diaz, A fixed point theorem of the alternative for contractions on the generalized complete metric space, Bull. Amer. Math. Soc. 126 (1968), 305–309.

[12] D. Mihet, V. Radu, On the stability of the additive Cauchy functional equation in random normed spaces. J. Math. Anal. Appl. 343, (2008) , 567-572.

[13] A.K. Mirmostafaee, M. Mirzavaziri, M.S. Moslehian, Fuzzy stability of the Jensen functional equation. Fuzzy Sets Syst. 159, (2008), 730–738

[14] A.K. Mirmostafaee, M.S. Moslehian, Fuzzy approximately cubic mappings. Inf. Sci. 178, (2008) , 3791–3798.

[15] C. Park, Lie $*$-homomorphism between Lie C^*-algebra and Lie $*$-derrivation on Lie C^*-algebra, J. Math. Anal. Appl. 15, (2004), 419–434.

[16] V. Radu, The fixed point alternative and the stability of functional equations, Fixed Point Theory, 4 (2003), 91–96.

[17] T.M. Rassias, On the stability of the linear mapping in Banach spaces. Proc. Am. Math. Soc. 72, (1978), 297–300.

[18] D. Shin, C. Park and Sh. Farhadabadi, On the superstability of ternary Jordan C$*$-homomorphisms, J. Comput. Anal. Appl. 16 (2014), 964–973.

[19] R. Thakur and S.K. Samanta, Fuzzy Banach algebra with Felbin's type fuzzy norm, J. Fuzzy Math. 18, (2011), 943–954.

[20] S.M. Ulam, Problems in Modern Mathematics, Chapter VI, science Editions., Wiley, New York, (1964).