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On soft b^*-closed sets in soft topological space

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Abstract

In this paper, we introduce and study a new class of soft sets, called soft b^*-closed and soft b^*-open sets. We study several characterizations and properties of these classes of sets.

Keywords: soft $b-$open set, soft b^*-closed set and soft b^*-open set.

1. Introduction and Preliminaries

In 1999, Molodtsov [8], instigated The concept of soft set as a new Mathematical tool to deal with uncertainties problems in different fields of science. Kannan [7] defined soft generalized closed and open sets in soft topological spaces. I. Arockiarani and A. Arokialancy [10] defined soft $\beta-$ open sets and continued to study weak forms of soft open sets in soft topological space.
Later, Akdag and Ozkan [1] defined soft$\alpha-$open, while the soft b-open are studied by Metin and Alkan [2]. The b^*-closed sets were studied by S. Muthuvel, R. Parimelazhagan [9]. In this work we introduce the soft version of b^*-open sets and b^*-closed sets, and study some properties of these sets and give some new result in this filed.

Definition 1.1. *[8] Let Z be an initial universe set, $P(Z)$ the power set of Z, and A a set of parameters. A pair (F, A), where F is a map from A to $P(Z)$, is called a soft set over Z. In what follows we denote by $SS(Z, A)$ the family of all soft sets over$Z$.*

Definition 1.2. *[8] The soft set $(F, A) \in SS(Z, A)$, where $F(p) = \phi$, for every $p \in A$ is called A-null soft set of $SS(Z, A)$ and denoted by $\tilde{\phi}$. The soft set $(F, A) \in SS(Z, A)$, where $F(p) = Z$, for every$p \in A$ is called the A-absolute soft set of $SS(Z, A)$ and denoted by $\tilde{Z}$.*

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Definition 1.3. *[8] Let τ be a collection of soft open sets over Z, then τ is said to be soft topological space if*

1. *(1) $\tilde{\phi}$ and $\tilde{Z}$ belong to τ.*
2. *(2) The union of any subcollection of soft sets of τ belongs to τ.*
3. *(3) the intersection of any two soft sets in τ belongs to τ.*

Definition 1.4. *[10] Let (Z, τ, A) be a soft topological space and $(F, A) \in SS(Z, A)$. Then*

(1) The soft closure of (F, A) is the soft set $cl(F, A) = \cap\{(S, A) : (S, A) \in \tau^c, (F, A) \subseteq (S, A)\}$.

(2) The soft interior of (F, A) is the soft set $int(F, A) = \cup\{(S, A) : (S, A) \in \tau, (S, A) \subseteq (F, A)\}$.

Definition 1.5. *A soft set (F, A) of a soft topological space (Z, τ, A) is said to be*

(1) Soft α- open [2] if $(F, A) \subset int(cl(int((F, A))))$,

(2) Soft preopen [4] if $(F, A) \subset int(cl((F, A)))$,

(3) Soft semi - open [1] if $(F, A) \subset cl(int((F, A)))$,

(4) Soft β-open [4] if $(F, A) \subset cl(int(cl((F, A))))$.

Definition 1.6. *[2] A set $(P, A) \in SS(Z, A)$ is called Soft b-open [Soft b-closed] iff $(P, A) \subset int(cl((P, A))) \cup cl(int((P, A))))[(P, A) \supset int(cl((P, A))) \cap cl(int((P, A)))]$, We denote it by sb-open (sb-closed). We will denoted the family of all soft b-open sets by $SbO(Z)$.*

Definition 1.7. *[6] A set$(P, A) \in SS(Z, A)$ is called soft bc-open (sbc-open) if for any $x \in (P, A) \in SbO(Z)$, there is a soft closed set (S, A) such that $x \in (S, A) \subset (P, A)$.*

Definition 1.8. *[7] Let (Z, τ, A) be a soft topological space. A subset (S, A) of Z is said to be soft generalized closed in Z if $cl(S, A) \subseteq (L, B)$ whenever $(S, A) \subseteq (L, B)$ where (L, B) is soft open set in Z. we denote it by $sg - closed$.*

Definition 1.9. *Let (P, A) be a soft set of a soft topological space (Z, τ, A), then*

(1) [10] Soft pre-intirior of (P, A) in Z is defined by
$sPint((P, A)) = \cup\{(L, A) : (L, A)$ is a soft preopen set and $(L, A) \subset (P, A)\}$.

(2) Soft pre-closure of (P,A) in $Z$is defined by
$sPcl((P, A)) = \cap\{(H, A) : (H, A)$is a soft preclosed set and $(P, A) \subset (H, A)\}$.

(3) [2] Soft b-interior of (P,A) in Z is defined by
$sbint((P, A)) = \cup\{(L, A) : (L, A)$is a soft b-open set and$(L, A) \subset (P, A)\}$.

(4) Soft b-closure of a soft set (P, A) in Z is defined by
$sbcl((P, A)) = \cap\{(H, E) : (H, E)$ is a soft b-closed set and $(P, A) \subset (H, E)\}$.

Clearly $sbcl((P, A))$ (resp., $sPcl((P, A)))$ is the smallest soft b-closed (resp. soft pre-closed) set over Z which contains (P, A) and $sbint((N, A))$ (resp. $sPint((P, A)))$ is the largest soft b-open (resp. soft pre-open) set over Z which is contained in $(P, A))$.

Definition 1.10. *[3] Let (Z, τ, A) be a soft topological space. A soft set (S, A) of Z is said to be Soft generalized b-closed (briefly soft gb-closed) if $sbcl(S, A) \subseteq (P, B)$ whenever $(S, A) \subseteq (P, B)$ and $(P, B) \in \tau$.*

The main results In this part we go to introduce the concepts of:
soft b^*-closed, soft b^*-open sets and give some properties of these two concepts, moreover, we study the relation between these new concepts.
Now we give the main part of this work,

2. Soft b^*-closed and some properties

Definition 2.1. *A soft set (P, A) of a soft topological space (Z, τ, A) is called a Soft b^*-closed (briefly sb^*-closed) if $int(cl(P, A)) \subseteq (S, A)$, whenever $(P, A) \subset (S, A)$ and (S, A) is soft $b-$open.*

Theorem 2.2. *If a soft subset (S, A) of a soft topological space Z is soft b-closed then it is Soft b^*-closed.*
Proof . *Suppose (S, A) is a soft b-closed, let (L, A) be an open set containing (S, A) in Z, then $cl(S, A) \subset (L, A)$. Now $int(cl(S, A) \subset cl(S, A) \subset (L, A)$. Thus (S, A) is Soft b^*-closed.* □

Remark 2.3. *The following example shows that the converse of the theorem 2.2 need not true in general.*

Example 2.4. *Let $Z = \{h_1, h_2, h_3, h_4\}, A = \{e_1, e_2, e_3\}$and $\tau = \{\tilde{\phi}, \tilde{Z}, (P_1, A), (P_2, A), \ldots, (P_{15}, A)\}$ where $(P_1, A), (P_2, A), \ldots, (P_{15}, A)$ are soft set over Z, define as follows:*

$$\begin{aligned}
(P_1, A) &= \{(e_1, \{h_1\}), (e_2, \{h_2, h_3\}), (e_3, \{h_1, h_4\})\}, \\
(P_2, A) &= \{(e_1, \{h_2, h_4\}), (e_2, \{h_1, h_3, h_4\}), (e_3, \{h_1, h_2, h_4\})\}, \\
(P_3, A) &= \{(e_2, \{h_3\}), (e_3, \{h_1\})\}, \\
(P_4, A) &= \{(e_1, \{h_1, h_2, h_4\}), (e_2, \tilde{Z}), (e_3, \tilde{Z})\}, \\
(P_5, A) &= \{(e_1, \{h_1, h_3\}), (e_2, \{h_2, h_4\}), (e_3, \{h_2\})\}, \\
(P_6, A) &= \{(e_1, \{h_1\}), (e_2, \{h_2\})\}, \\
(P_7, A) &= \{(e_1, \{h_1, h_3\}), (e_2, \{h_2, h_3, h_4\}), (e_3, \{h_1, h_2, h_4\})\}, \\
(P_8, A) &= \{(e_2, \{h_4\}), (e_3, \{h_2\})\}, \\
(P_9, A) &= \{(e_1, \tilde{Z}), (e_2, \tilde{Z}), (e_3, \{h_1, h_2, h_3\})\}, \\
(P_{10}, A) &= \{(e_1, \{h_1, h_3\}), (e_2, \{h_2, h_3, h_4\}), (e_3, \{h_1, h_2\})\}, \\
(P_{11}, A) &= \{(e_1, \{h_2, h_3, h_4\}), (e_2, \tilde{Z}), (e_3, \{h_1, h_2, h_3\})\}, \\
(P_{12}, A) &= \{(e_1, \{h_1\}), (e_2, \{h_2, h_3, h_4\}), (e_3, \{h_1, h_2, h_4\})\}, \\
(P_{13}, A) &= \{(e_1, \{h_1\}), (e_2, \{h_2, h_4\}), (e_3, \{h_2\})\}, \\
(P_{14}, A) &= \{(e_1, \{h_3, h_4\}), (e_2, \{h_1, h_2\})\}, \\
(P_{15}, A) &= \{(e_1, \{h_1\}), (e_3, \{h_2, h_3\}), (e_3, \{h_1\})\}.
\end{aligned}$$

Then τ is a soft topology on Z, and soft closed sets are $\tilde{Z}, \tilde{\phi}, (P_1, A)^c, (P_2, A)^c, (P_3, A)^c, \ldots, (P_{15}, A)^c$. Let us take $(K, A) = \{(e_1, \{h_2, h_4\}), (e_2, \{h_1, h_3\}), (e_3, \{h_1, h_3, h_4\})\}$ is sb-open and take $(M, A) = \{(e_1, \{h_2\}), (e_2, \{h_1\}), (e_3, \{h_1, h_3\})\}$ is a soft set where $(M, A) \subset (K, A)$ then (M, A) is sb^*-closed but not sb-closed.

Theorem 2.5. *If a soft subset (S, A) of space Z is both soft open and sb^*-closed then it is soft closed.*
Proof . *Suppose a subset (S, A) of Z is both soft open and soft sb^*-closed. Now $int(cl(S, A)) \subseteq cl(S, A) \subseteq (S, A)$. Then $cl(S, A) \subseteq (S, A)$. Therefore (S, A) is closed.* □

Theorem 2.6. *A soft set (P, A) is sb^*-closed if and only if $int(cl(P, A)) - (P, A)$ contains no non-empty soft closed set.*
Proof . *Suppose (S, A) is a non-empty soft closed subset of $int(cl(P, A))$. Now $int(cl(P, A)) - (P, A) \subseteq (P, A)$ implies $int(cl(P, A)) \cap (P, A)^c \subseteq (S, A)$, since $int(cl(P, A)) - (P, A) = int(cl(P, A)) \cap (P, A)^c$. Thus $int(cl(P, A)) \subseteq (S, A)$. Now $(P, A)^c \subseteq (S, A)$ implies $(S, A)^c \subseteq (P, A)$. Here $(S, A)^c$ is soft open and (P, A) is sb^*-closed, we have $(S, A)^c \subseteq int(cl(P, A))$. Thus $(S, A) \subseteq [int(cl(P, A))]^c$. Hence $int(cl(P, A)) \cap [int(cl(P, A))]^c \subseteq (S, A) = \phi$. i.e. $(S, A) = \phi$ implies $int(cl(P, A)) - (P, A)$ contains no non empty soft closed set. Conversely, Let $(K, A) \subseteq (P, A)$ is $sb-$open. Suppose that $int(cl(P, A))$ is contained in (K, A), then $int(cl(P, A)) \cap (K, A)^c$ is a non-empty soft closed set of $int(cl(P, A)) - (P, A)$ which is contradiction. Therefore $(K, A) \subseteq int(cl(P, A))$ and hence (P, A) is sb^*-closed.* □

Corollary 2.7. *Let (F, A) be a sgb-closed set then (P, A) is sb^*-closed if and only if $int(cl(P, A)) - (P, A)$ is soft closed.*
Proof . *Let (P, A) be $sgb-$closed set. If (P, A) is sb^*-closed, then we have $int(cl(P, A)) - (F, A) = \phi$ which is soft closed set. Conversely, let $int(cl(P, A)) - (P, A)$ be soft closed. Then by 2.6 $int(cl(P, A)) - (P, A)$ doesn't contain a non-empty soft closed subset and since $int(cl(P, A))$ is soft closed subset of itself.*
Then $int(cl(P, A)) - (P, A) = \phi$. Thus implies that $(P, A) = int(cl(P, A))$ and so (P, A) is sb^*-closed.
□

Theorem 2.8. *Let $(S, A) \subseteq (P, A) \subseteq Z, (S, A)$is sb^*-closed set relative to (P, A) and (P, A) is both sb-open and sb^*-closed subset of Z, then (S, A) is sb^*-closed set relative to Z.*
Proof . *Let $(K, A) \subseteq (S, A)$and (K, A) be a sb-open set in Z. But given that $(S, A) \subseteq (P, A) \subseteq Z$, therefore $(S, A) \subseteq (P, A)$ and $(K, A) \subseteq (S, A)$. This implies $(P, A) \cap (K, A) = (S, A)$. Since (S, A) is sb^*-closed set relative to $(P, A), (P, A) \cap (K, A) \subseteq int(cl(P, A))$. i.e. $(P, A) \cap (K, A) \subseteq (P, A) \cap int(cl(P, A))$ implies $(K, A) \subseteq (P, A) \cap int(cl(P, A))$.*
Thus $(K, A) \cup [int(cl(S, A))]^c \subseteq (P, A) \cap int(cl(S, A)) \cup [int(cl(S, A))]^c$ implies $(K, A) \cup [int(cl(S, A))]^c \subseteq (P, A) \cup [int(cl(S, A))]^c$. Since (P, A) is sb^*-closed in Z, we have $(K, A) \cup [int(cl(S, A))]^c \subseteq int(cl(P, A))$. Also $(S, A) \subseteq (P, A)$ implies $int(cl(P, A)) \subseteq int(cl(S, A))$.
Thus $(K, A) \cup [int(cl(S, A))]^c \subseteq int(cl(P, A)) \subseteq int(cl(S, A))$. Therefore $(K, A) \subseteq int(cl(S, A))$, since $int(cl(S, A))$ is not contained in $[int(cl(S, A)]^c$. Thus (S, A) is sb^*-closed relative to Z. □

Theorem 2.9. *Let $(P, A) \subseteq Y \subseteq Z$ and supposed that (P, A) is sb^*-closed in Z then (P, A) is sb^*-closed in Y.*
Proof . *Given that $(P, A) \subseteq Y \subseteq Z$ and (P, A) is sb^*-closed in Z. To show that (P, A) is sb^*-closed relative to Y. Let $Y \cap (S, A) \subseteq (P, A)$ where $(S, A)$is $sb-open$ in$Z$. Since (P, A) is sb^*-closed in $Z, (S, A) \subseteq (P, A)$ implies $(S, A) \subseteq int(cl(P, A))$ i.e. $Y \cap (S, A) \subseteq Y \cap int(cl(P, A))$ where $Y \cap int(cl(P, A))$ is interior of closure of (P, A) in Y. This (P, A) is sb^*-closed in Y.* □

Theorem 2.10. *If a soft subset (P, A) of a soft topological space (Z, τ, A) is soft preclosed then it is sb^*-closed.*

Proof . *Suppose (P, A) is soft preclosed, (S, A) be a sb-open set containing (P, A). All (P, A) is preclosed $(P, A) \subseteq int(cl(P, A))$. Thus (P, A) is sb^*-closed in Z.* □

Remark 2.11. *The following example shows that the converse of the theorem 2.10 need not true in general.*

Example 2.12. *Let $Z = \{h_1, h_2, h_3, h_4\}, A = \{e_1, e_2, e_3\}$ and let (Z, τ, A) be soft topological space. consider the soft topology τ on Z given in example 2.4.*
Then, let us take soft set $(S, A) = \{(e_1, \{h_2, h_4\}), (e_2, \{h_3\}), (e_3, \{h_1\})\}$, then $int(cl(S, A)) = \{(e_3, \{h_1\}), (e_2, \{h_3\}) \subseteq (K, A)$ whenever $(S, A) \subseteq (K, A)$ and (K, A) is sb-open. Therefore, (S, A) is sb^*-closed set but not soft preclosed set.

Theorem 2.13. *Every soft α-closed set is soft b^*-closed.*

Proof . *Suppose (P, A) be a soft α-closed set in Z. Let (S, A) be a soft open set in Z such that $(P, A) \subseteq (S, A)$. Since (P, A) is soft α-closed set. Then $s\alpha cl(P, A) \subseteq (S, A)$.*
Now $\alpha cl(P, A) \subseteq cl(int(P, A)) \subseteq (S, A)$. Since every soft open is soft b-open.
Therefore, (P, A) is soft b^*-closed set in Z. □

3. Soft b^*-open sets

Definition 3.1. *A soft set (P, A) is called Soft b^*-open set (briefly sb^*-open) if it's complement $(P, A)^c$ is soft b^*-closed. The family of all sets of sb^*-open denoted by $Sb^*O(Z)$.*

Theorem 3.2. *If a set (P, A) of a soft topological space Z is $sg-$open, then it is sb^*-open but not conversely.*

Proof . *Let (P, A) be a $sg-$open set in space Z. Then $(P, A)^c$ is sb^*-closed. Therefore (P, A) is sb^*-open in Z.* □

Remark 3.3. *The following example shows that the converse of the Theorem 3.2 need not true in general.*

Example 3.4. *Let $Z = \{h_1, h_2, h_3, h_4\}, A = \{e_1, e_2, e_3\}$ and let (Z, τ, A) be soft topological space over Z. consider the soft topology τ on Z given in example 2.4.*
Then, let us take soft set $(M, A) = \{(e_1, \{h_1, h_3\}), (e_2, \{h_1, h_2, h_4\}), (e_3, \{h_2, h_3, h_4\})\}$ is soft sb^*-open but not soft $g-$open.

Theorem 3.5. *A set (S, A) of space Z is sb^*-open if and only if $(P, A) \subseteq cl(int(S, A))$ whenever (P, A) is soft closed and $(P, A) \subseteq (S, A))$.*

Proof . *We have (S, A) is sb^*-open. Then $(S, A)^c$ is sb^*-closed. Let (P, A) be a soft closed set in Z contained in (S, A), then $(P, A)^c$ is an open set in Z containing $(S, A)^c$. Since $(S, A)^c$ is sb^*-closed, $int(cl(S, A)^c) \subseteq (P, A)^c$ taking complement on both sides, then $(P, A) \subseteq cl(int(S, A))^c$. Conversely, we have $(P, A)^c$ is contained in $cl(int(S, A))$ whenever (P, A) is contained in (S, A) and (P, A) is soft closed in Z. Let (K, A) be a soft open set containing $(P, A)^c$, then $(K, A)^c \subseteq cl(int(S, A)^c)$ taking complement on both side we get $int(cl(S, A)^c) \subseteq (K, A)$. Hence $(S, A)^c$ is sb^*-closed. Therefore (S, A) is sb^*-open.* □

Theorem 3.6. *The following are true in general.*

(1) Every soft open is soft b^*-open.

(2) Every soft $\alpha-$open is soft b^*-open.

(3) Every soft b^*-open set is soft $b-$open.

Proof . *The proof is Obvious.* □

Definition 3.7. *Let (Z, τ, A) be a soft topological space. A subset $(F, A) \subseteq Z$ is called a sb^*-neighbourhood (briefly sb^*-nbd) of a point $x \in Z$ if there exists an sb^*-open set (P, A) such that $x \in (P, A) \subseteq (F, A)$.*

Definition 3.8. *Let (Z, τ, A) be a soft topological space. A subset $(F, A) \subseteq Z$ is called a sb^*-neighbourhood of $(S, A) \subseteq Z$ if there exists an sb^*-open set (P, A) such that $(S, A) \in (P, A) \subseteq (F, A)$.*

Remark 3.9. *The family of all sb^*-neighbourhood of a point $x \in Z$ is a sb^*-neighbourhood system of x and it denoted by $sb^*N(x)$.*

Theorem 3.10. *Let (Z, τ, A) be a soft topological space and for each $x \in Z$, then we have the following result:*

(1) For every $x \in Z$, $sb^*N(x) \neq \phi$.

(2) $(N, A) \in sb^*N(x) \Longrightarrow x \in (N, A)$.

(3) $(N, A) \in sb^*N(x), (N, A) \subseteq (M, A) \Longrightarrow (M, A) \in sb^*N(x)$.

(4) $(N, A) \in sb^*N(x), (N, A) \Longrightarrow$ there exists$(M, A) \in sb^*N(x)$such that$(M, A) \subseteq (N, A)$and $(M, A) \in sb^*N(y)$for every $y \in (M, A)$.

Proof .

(1) Since Z is a sb^*-open set, it is an sb^*-neighbourhood for every$x \in Z$. Hence $sb^*N(x) \neq \phi$ for every $x \in Z$.

(2) If $(N, A) \in sb^*N(x)$, then (N, A) is an sb^*-neighbourhood of x. By definition of sb^*-neighbourhood, $x \in (N, A)$.

(3) Let $(N, A) \in sb^*N(x)$ and $(N, A) \subseteq (M, A)$. Then there is an sb^*-open set (P, A) such that $x \in (P, A) \subseteq (N, A)$, since $(N, A) \subseteq (M, A), x \in (P, A) \subseteq (M, A)$. Therefore, (M, A) is an sb^*-neighbourhood of x. Hence $(M, A) \in sb^*N(x)$.

(4) If $(N, A) \in sb^*N(x)$, then $x \in (M, A) \subseteq (N, A)$, where (M, A) is an sb^*-open set, then it is an sb^*-neighbourhood of each its points. Therefore, $(M, A) \in sb^*N(y)$for every $y \in (M, A)$.

□

Definition 3.11. *Let (P, A) be a soft subset of Z. Then*
$sb^*int(P, A) = \cup\{(L, A) : (L, A)$*is a soft* $b^* -$ *open set and* $(L, A) \subset (P, A)\}$.

Definition 3.12. *Let (P, A) be a soft subset of Z. A point $x \in Z$ is said to be an sb^*int point of (P, A) if (P, A) is an sb^*-neighbourhood of x.*

Proposition 3.13. *Let (P, A) be a soft subset of Z, then* $sb^*int(P, A) = \cup\{x : x \text{ is an interior point of } (P, A)\}$.

Proof . *Let (P, A) be a soft subset of Z, then*
$x \in sb^*int(P, A) \Longleftrightarrow x \in \cup\{(L, A) : (L, A) \text{ is a soft } b^* - \text{open set and } (L, A) \subset (P, A)\}$.
$\Longleftrightarrow$ *there exists an sb^*-open set (L, A) such that $x \in (L, A) \subseteq (P, A)$.*
$\Longleftrightarrow$ *(P, A) is an sb^*nbd of the point x*
$\Longleftrightarrow$ *x is an sb^*int point of (P, A).*
Hence $sb^*int(P, A) = \cup\{x : x \text{ is an interior point of } (P, A)\}$. □

Theorem 3.14. *In a soft topological space Z the following hold for sb^*int.*

(1) $sb^*int(Z) = Z$ *and* $sb^*int(\phi) = \phi$.

(2) $sb^*int(P, A) \subseteq (P, A)$.

(3) *If (S, A) is any sb^*int-open set contained in (P, A), then $(S, A) \subseteq sb^*int(P, A)$.*

(4) *If $(P, A) \subseteq (S, A)$, then $sb^*int(P, A) \subseteq sb^*int(S, A)$.*

(5) $sb^*int(sb^*int(P, A)) = sb^*int(P, A)$.

(6) $sb^*int(Z - (P, A)) \subseteq Z - (sb^*int(P, A))$.

(7) $sb^*int((P, A) - (S, A)) \subseteq sb^*int(P, A) - sb^*int(S, A)$.

Proof . *The proof is Obvious.* □

Theorem 3.15. *If a soft subset (P, A) of Z is sb^*-open, then $sb^*int(P, A) = (P, A)$.*

Proof . *Let (P, A) be an sb^*-open set of Z. we know that $sb^*int(P, A) \subseteq (P, A)$. Since (P, A) is an sb^*-open set contained in (P, A). By Theorem 3.14 (3), $(P, A) \subseteq sb^*int(P, A)$ implying $sb^*int(P, A) = (P, A)$.* □

Theorem 3.16. *If (P, A) and (S, A) are soft subsets of Z, then* $sb^*int(P, A) \cup sb^*int(S, A) \subseteq sb^*int((P, A) \cup (S, A))$.

Proof . *We know that $(P, A) \subseteq (P, A) \cup (S, A)$ and $(S, A) \subseteq (P, A) \cup (S, A)$. So $sb^*int(P, A) \subseteq sb^*int(P, A) \cup (S, A))$ and $sb^*int(S, A) \subseteq sb^*int((P, A) \cup (S, A))$. This implies that* $sb^*int(P, A) \cup sb^*int(S, A) \subseteq sb^*int((P, A) \cup (S, A))$. □

Definition 3.17. *Let (P, A) be a soft subset of a soft space Z. Then the soft b^*-closure of (P, A) is defined as the intersection of all soft b^*-closed set containing (P, A), that is* $sb^*cl(P, A) = \cap\{(H, E) : (H, E) \text{ is a soft } b^* - \text{closed set and } (P, A) \subset (H, E)\}$.

Theorem 3.18. *If (P, A) and (S, A) are soft subset of a space Z, then*

(1) $sb^*cl(Z) = Z$ *and* $sb^*cl(\phi) = \phi$.

(2) $(P, A) \subseteq sb^*cl(P, A)$.

(3) *If (S, A) is any sb^*-closed set containing (P, A), then $sb^*cl(P, A) \subseteq (S, A)$.*

(4) *If $(P, A) \subseteq (S, A)$, then $sb^*cl(P, A) \subseteq sb^*cl(S, A)$.*

(5) $sb^*cl(P,A) = sb^*cl(sb^*cl(P,A))$.

Proof . *The proof is Obvious.* □

Theorem 3.19. *If a soft subset* (P,A) *of* Z *is* sb^*-*closed, then* $sb^*cl(P,A) = (P,A)$.
Proof . *Let* (P,A) *be an* sb^*-*closed set of* Z*. Since* $(P,A) \subseteq Z$ *and* (P,A) *is an* sb^*-*closed set* $sb^*cl(P,A) \subseteq (P,A)$*, also* $(P,A) \subseteq sb^*cl(P,A)$*. Hence* $sb^*cl(P,A) = (P,A)$. □

Theorem 3.20. *If* (P,A) *and* (S,A) *are soft subsets of* Z*, then* $sb^*cl((P,A) \cap (S,A)) \subseteq sb^*cl(P,A) \cap sb^*cl(S,A)$.
Proof . *Let*(P,A) *and* (S,A) *is a soft subset of* Z*. Clearly* $(P,A) \cap (S,A) \subseteq (P,A)$ *and* $(P,A) \cap (S,A) \subseteq (S,A)$*, then* $sb^*cl((P,A) \cap (S,A)) \subseteq sb^*cl(P,A)$ *and* $sb^*cl((P,A) \cap (S,A)) \subseteq sb^*cl(S,A)$*. Hence* $sb^*cl((P,A) \cap (S,A)) \subseteq sb^*cl(P,A) \cap sb^*cl(S,A)$. □

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