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On the complex SEE change and systems of ordinary differential equations

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Abstract

In this paper, we present the properties and meaning of the new indispensable change, called complex SEE the change. Farther, we use the complex SEE integral change to solve systems of ordinary differential equations (ODEs).

Keywords: Complex SEE change, arrangement of differential conditions.
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1. Introduction

In the writing there are various basic changes and generally utilized cosmology just as in designing. The fundamental change techniques is an effective strategy to address the arrangement of common differential conditions [2-5]. Sadiq A. Mehdi, Emad A Kuffi and Eman A. Mansour presented another change and named as perplexing SEE vital change which is characterized as follows, [1]:

$$S^c[f(t)] = \frac{1}{v^n}\int_{t=0}^{\infty} f(t)e^{-ivt}dt = T(iv)$$
$$t \geq 0, v \in (l_1, l_2), n \in \mathbb{Z} \quad i \text{ is a complex number} \tag{1.1}$$

Furthermore, applied this new fundamental change to the arrangement of arrangement of customary differential conditions.
The complex SEE integral change of frequently used functions [1],

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S.N.	$f(t)$	$S^c[f(t)] = T(iv)$
1.	1	$\frac{i}{v^{n+1}}$
2.	t	$-\frac{1}{v^{n+2}}$
3.	t^2	$\frac{(2!)i}{v^{n+3}}$
4.	t^3	$\frac{3!}{v^{n+4}}$
5.	e^{at}	$\frac{-1}{v^n}\left[\frac{a}{a^2+v^2}+i\frac{v}{a^2+v^2}\right]$, a constant
6 .	$\sin(at)$	$\frac{-a}{v^n(v^2-a^2)}$
7 .	$\cos(at)$	$\frac{-iv}{v^n(v^2-a^2)}$
8 .	$\sinh(at)$	$\frac{-a}{v^n(v^2+a^2)}$
9 .	$\cosh(at)$	$\frac{-iv}{v^n(v^2+a^2)}$

Theorem 1.1. [1] *Let $T(iv)$ is complex SEE change of $[S^c[f(t)] = T(iv)]$ then:*

(i) $S^c\left[f'^{(t)}\right] = \frac{-f(0)}{v^n} + ivT(iv)$.

(ii) $S^c\left[f''(t)\right] = \frac{-f'(0)}{v^n} - \frac{if(0)}{v^{n-1}} - v^2T(iv)$.

(iii) In general case:

$$S^c\left[f^{(m)}(t)\right] = \frac{1}{v^n}\left[-f^{(m-1)}(0) - ivf^{(m-2)}(0) - (iv)^2f^{(m-3)}(0) - \cdots - (iv)^{m-1}f(0)\right] + (iv)^mT(iv).$$

Arrangement of Ordinary Differential Equations (ODEs)

The complex SEE integral change method is compelling for the arrangement of the response of a direct system represented by an standard differential condition to the underlying conditions and (or) to an external disturbance. More precisely, we seek the arrangement of a linear framework for its state at subsequent time $t > 0$ due to the initial state $t = 0$ and (or) to the unsettling influence applied fort > 0.

Complex SEE Change System O.D.Es

Example 1.2. *(Arrangement of first request conventional differential conditions)*
Think about the accompanying framework:

$$\left.\begin{aligned} \tfrac{dx_1}{dt} &= a_{11}x_1 + a_{12}x_2 + b_1(t) \\ \tfrac{dx_2}{dt} &= a_{21}x_1 + a_{22}x_2 + b_2(t) \end{aligned}\right\} \tag{1.2}$$

With the underlying information $x_1(0) = x_{10}$ and $x_2(0) = x_{20}$ where $a_{11}, a_{12}, a_{21}, a_{22}$ are constants.

Introducing the matrices

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \frac{dx}{dt} = \begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{pmatrix}, A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$b(t) = \begin{pmatrix} b_1(t) \\ b_2(t) \end{pmatrix} \text{ and } x_0 = \begin{pmatrix} x_{10} \\ x_{20} \end{pmatrix},$$

We can compose the above framework in network differential framework as

$$\frac{dx}{dt} = Ax + b(t), x(0) = x_0 \tag{1.3}$$

We take complex SEE integral change of the framework with beginning data, we get:

$$\begin{cases} (iv - a_{11})\,\overline{x_1} - a_{12}\overline{x_2} = \overline{b_1(v)} + \frac{1}{v^n}x_{10} \\ -a_{21}\overline{x_1} + (iv - a_{22})\,\overline{x_2} = \overline{b_2(v)} + \frac{1}{v^n}x_{20} \end{cases}$$

Where $\overline{x_1}, \overline{x_2}, \overline{b_1}, \overline{b_2}$ *are complex SEE integral change of* x_1, x_2, b_1, b_2 *are respectively.*

The solution of the algebraic system is:

$$\overline{x_1}(v) = \frac{\begin{vmatrix} \overline{b_1}(v) + \frac{x_{10}}{v^n} & -a_{12} \\ \overline{b_2}(v) + \frac{x_{20}}{v^n} & iv - a_{22} \end{vmatrix}}{\begin{vmatrix} iv - a_{11} & -a_{12} \\ -a_{21} & iv - a_{22} \end{vmatrix}}$$

And

$$\overline{x_2}(v) = \frac{\begin{vmatrix} iv - a_{11} & \overline{b_1}(v) + \frac{x_{10}}{v^n} \\ -a_{21} & \overline{b_2}(v) + \frac{x_{20}}{v^n} \end{vmatrix}}{\begin{vmatrix} iv - a_{11} & -a_{12} \\ -a_{21} & iv - a_{22} \end{vmatrix}}$$

Expanding these determinates, results for $\overline{x_1}(v)$ *and* $\overline{x_2}(v)$ *can promptly be transformed and the arrangements x_1 (t) and x_2 (t) can be found in shut structures.*

Example 1.3. *Settle the arrangement of the standard differential conditions:*

$$\begin{matrix} \frac{dx}{dt} = 2x - 3y \\ \frac{dy}{dt} = y - 2x \end{matrix}, t > 0$$

With the underlying conditions: $x(0) = 8$ *and* $y(0) = 3$. *Take complex SEE change of the framework with the underlying information, to get:*

$$\overline{x_1}(v) = \frac{\begin{vmatrix} \frac{8}{v^n} & 3 \\ \frac{3}{v^n} & iv - 1 \end{vmatrix}}{\begin{vmatrix} iv - 2 & 3 \\ 2 & iv - 1 \end{vmatrix}} = \frac{\frac{8}{v^n}(iv - 1) - \frac{9}{v^n}}{(iv - 2)(iv - 1) - 6}$$

$$\begin{aligned} \overline{x_1}(v) &= \frac{1}{v^n}\left[\frac{8iv - 17}{(iv - 2)(iv - 1) - 6}\right] = \frac{1}{v^n}\left[\frac{8iv - 17}{i^2v^2 - 3iv - 4}\right], \\ &= \frac{1}{v^n}\left[\frac{8iv - 17}{(iv - 4)(iv + 1)}\right], \end{aligned}$$

$$\overline{x_1}(v)=\frac{1}{v^n}\left[\frac{A}{iv-4}+\frac{B}{iv+1}\right]$$

After simple computations, we have
$A=3,B=5$, *then:*

$$\overline{x_1}(v)=\frac{1}{v^n}\left[\frac{3}{iv-4}\right]+\frac{1}{v^n}\left[\frac{5}{iv+1}\right]=\frac{3}{v^n}\left[\frac{4}{v^2+16}+\frac{iv}{v^2+16}\right]+\frac{5}{v^n}\left[\frac{-1}{v^2+1}+\frac{iv}{v^2+1}\right]$$

By taking inverse SEE change, we have

$$x_1(t)=3e^{4t}+5e^{-t}.$$

Also

$$\bar{y}=\frac{\begin{vmatrix} iv-2 & \frac{8}{v^n} \\ 2 & \frac{3}{v^n}\end{vmatrix}}{\begin{vmatrix} iv-2 & 3 \\ 2 & iv-1\end{vmatrix}}=\frac{\frac{3(iv-2)}{v^n}-\frac{16}{v^n}}{(iv-2)(iv-1)-6}$$

$$\bar{y}=\frac{1}{v^n}\frac{(3iv-6-16)}{i^2v^2-3iv+2-6}=\frac{1}{v^n}\left[\frac{3iv-22}{(iv-4)(iv+1)}\right]$$

We have $A=-2,B=5$, *then:*

$$\bar{y}=\frac{1}{v^n}\left[\frac{-2}{iv-4}\right]+\frac{1}{v^n}\left[\frac{5}{iv+1}\right]$$

$$\bar{y}=\frac{-2}{v^n}\left[\frac{4}{v^2+16}+iv\frac{1}{v^2+16}\right]+\frac{5}{v^n}\left[\frac{-1}{v^2+1}+\frac{iv}{v^2+1}\right]$$

Take inverse of complex SEE change, we obtain:

$$y(t)=-2e^{4t}+5e^{-t}.$$

Example 1.4. *consider a system of linear ordinary differential equation*

$$\left.\begin{array}{c}\frac{dx}{dt}+y=2\cos t\\ x+\frac{dy}{dt}=0\end{array}\right\}\tag{1.4}$$

$$\text{with } x(0)=0, y(0)=1\tag{1.5}$$

taking complex SEE change of equation (1.4), we have

$$\left.\begin{array}{c}S^c\left[\frac{dx}{dt}\right]+S^c[y]=2S^c[\cos t]\\ S^c[x]+S^c\left[\frac{dy}{dt}\right]=0\end{array}\right\}\tag{1.6}$$

Now, using the property, complex SEE change of the derivative of the function in equation (1.6), we get:

$$\frac{-1}{v^n}x(0)+iv\bar{x}+\bar{y}=\frac{-2iv}{v^n\left(v^2-1\right)}$$

$$\bar{x}+\left[\frac{-1}{v^n}y(0)+iv\bar{y}\right]=0$$

Now, using equation (1.5), we have:

$$\left.\begin{array}{r} iv\bar{x}+\bar{y}=\frac{-2iv}{v^n(v^2-1)} \\ \bar{x}+iv\bar{y}=\frac{1}{v^n} \end{array}\right\} \tag{1.7}$$

Solving the equation (1.7) for $\bar{x}(v)$ and $\bar{y}(v)$, we have:

$$\bar{y}=\frac{\begin{vmatrix} \frac{-2iv}{v^n(v^2-1)} & 1 \\ \frac{1}{v^n} & iv \end{vmatrix}}{\begin{vmatrix} iv & 1 \\ 1 & iv \end{vmatrix}}=\frac{\frac{-2i^2v^2}{v^n(v^2-1)}-\frac{1}{v^n}}{-v^2-1}$$

$$\begin{aligned}\bar{x}(v)&=\frac{1}{v^n}\left[\frac{\frac{2v^2-\left(v^2-1\right)}{v^2-1}}{-\left(v^2+1\right)}\right]\\ &=\frac{1}{v^n}\left[\frac{\frac{\eta^2+1}{v^2-1}}{-\left(v^2+1\right)}\right]\\ &=\frac{-1}{v^n}\left[\frac{1}{v^2-1}\right]\end{aligned}$$

The solution is:

$$x(t)=S^c\left[\frac{-1}{v^n}\left(\frac{1}{v^2-1}\right)\right]=\sin(t)$$

$$\bar{y}(v)=\frac{\begin{vmatrix} iv & \frac{-2iv}{v^n(v^2-1)} \\ 1 & \frac{1}{v^n} \end{vmatrix}}{\frac{iv}{-(v^2+1)}}=\frac{\frac{iv}{v^n}+\frac{2iv}{v^n(v^2-1)}}{-\left(v^2+1\right)}$$

$$\begin{aligned}\bar{y}(v)&=\frac{1}{v^n}\left[\frac{\frac{iv\left(v^2-1\right)+2iv}{v^2-1}}{-\left(v^2+1\right)}\right]\\ &=\frac{1}{v^n}\left[\frac{\frac{iv^3-iv+2iv}{v^2-1}}{-\left(v^2+1\right)}\right]=\frac{1}{v^n}\left[\frac{iv\left(v^2+1\right)}{-\left(v^2+1\right)\left(v^2-1\right)}\right]\\ &=\frac{-1}{v^n}\left[\frac{iv}{v^2-1}\right]\end{aligned}$$

The solution is

$$y(t)=\cos(t)$$

Problem of System Pharmacokinetics

Solution of the problem for the system pharmacokinetics

$$\frac{d}{dt}C_1(t) + \lambda C_1(t) = \frac{\gamma}{VOL}C_2(t) \text{ with } C_1(0) = C_2(0) = 1 \text{ and } t > 0$$
$$\frac{d}{dt}C_2(t) + 3\lambda C_2(t) = \frac{1}{VOL}C_1(t)$$

Here $C_1(t)$ and $C_2(t)$: at any time t, the medications concentration in the blood t
λ : consistent speed of end.
γ : the proportion of implantation (in mg/min).
VOL : volume in which drug is distributed.
Taking complex SEE change of the above system, we have

$$S^c\left[C_1'(t)\right] + \lambda S^c\left[C_1(t)\right] = \frac{\gamma}{VOL}S^c\left[C_2(t)\right]$$

$$S^c\left[C_2'(t)\right] + 3\lambda S^c\left[C_2(t)\right] = \frac{1}{VOL}S^c\left[C_1(t)\right]$$

Now, using the property of the function in above equation, we get

$$\frac{-1}{v^n}C_1(0) + iv\overline{C_1} + \lambda\overline{C_1} = \frac{\gamma}{VOL}\overline{C_2}$$

$$\frac{-1}{v^n}C_2(0) + iv\overline{C_2} + 3\lambda\overline{C_2} = \frac{1}{VOL}\overline{C_1}$$

$$(iv+\lambda)\overline{C_1} - \frac{\gamma}{VOL}\overline{C_2} = \frac{1}{v^n}$$

$$\frac{-1}{VOL} + (iv+3\lambda)\overline{C_2} = \frac{1}{v^n}$$

$$\overline{C_1}(v) = \frac{\begin{vmatrix} \frac{1}{v^n} & (iv+\lambda) \\ \frac{1}{v^n} & \frac{-1}{VoL} \end{vmatrix}}{\begin{vmatrix} iv+\lambda & \frac{-v}{VoL} \\ \frac{-1}{VoL} & (iv+3\lambda) \end{vmatrix}}(iv+\lambda)(iv+3\lambda) - \frac{-1}{v^n(VoL)} - \frac{(iv+\lambda)}{v^n}(ivoL)^2$$

After simple computations and take inverse we get $\overline{C_1}(t)$.
Similarly, we get $\overline{C_2}(t)$

Open problems

- Use the complex SEE integral transform in solving systems of partial differential equations and their engineering, physical and biological applications.
- Use the complex SEE integral transform in solving systems of algebraic-differential equations and their applications.
- Use the complex SEE integral transform in solving nonlinear systems of differential equations and their applications.
- Use the complex SEE integral transform in solving applications of nuclear physics systems and grafting models and studying their stability.

Conclusions

The complex SEE change gives incredible strategy to dissecting subsidiaries. It is intensely used to settle standard differential conditions, and arrangement of conventional differential conditions.

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