Right $\Gamma$-$n$-derivations in prime $\Gamma$-near-rings

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Abstract

The main purpose of this paper is to study and investigate some results of right $\Gamma$-$n$-derivation on prime $\Gamma$-near-ring $G$ which force $G$ to be a commutative ring.

Keywords: Prime $\Gamma$-near-ring, $\Gamma$-$n$-derivation

1. Introduction

Throughout this paper, a $\Gamma$-near ring is a triple $(G,+,\Gamma)$, where (i) $(G,+)$ is a (not necessarily abelian) group; (ii) $\Gamma$ is a non-empty set of binary operations on $G$ such that for each $\gamma \in \Gamma$, $(G,+,\gamma)$ is a left near-ring (iii) $s\gamma(r\mu c) = (s\gamma r)\mu c$, for all $s,r,c \in G$ and $\gamma,\mu \in \Gamma$ [5, 7, 8]. And $G$ will denote a zero–symmetric left $\Gamma$-near ring with multiplicative center $Z(G)$. For a $\Gamma$-near-ring $G$, the set $G_0 = \{s \in G : 0ps = 0, \forall p \in \Gamma\}$ is called zero symmetric part of $G$. If $G = G_0$, then $G$ is called zero symmetric [8, 9]. A $\Gamma$-near-ring $G$ is said to be prime $\Gamma$-near-ring if $s\Gamma Gr = 0$ implies $s = 0$ or $r = 0$, for every $s,r \in G$ and it said to be semiprime if $s\Gamma Gs = 0$ implies $s = 0$ for every $s \in G$ [7, 8]. The other commutators are; $[s,r]_\rho = s\rho r – r\rho s$ and $(s,r) = s + r – r$s $r$ denote the additive-group commutator [1, 2]. $\Gamma$-near-ring $G$ is called commutative if $(G,+)$ is abelian [2, 3].

An additive mapping $h : G \times G \times \cdots \times G \longrightarrow G$ is said to be $\Gamma$-$n$-derivation if the relations

\[
h(x_1, x_2, \ldots, x_n) = h(x_1, x_2, \ldots, x_n)\gamma x_n' + \sum_{\eta = 1}^{n} x_\eta \gamma h(x_1, x_2, \ldots, x_n')
\]

\[
h(x_1, x_2, \ldots, x_n) = h(x_1, x_2, \ldots, x_n)\gamma x_2' + \sum_{\eta = 2}^{n} x_\eta \gamma h(x_1, x_2, \ldots, x_n')
\]

\[
\vdots
\]

\[
h(x_1, x_2, \ldots, x_n) = h(x_1, x_2, \ldots, x_n)\gamma x_n' + \sum_{\eta = n}^{1} x_\eta \gamma h(x_1, x_2, \ldots, x_n')
\]

Hold for all $x_1, x_2, x_3, \ldots, x_n, x_n' \in G$.

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An $n$-additive mapping $h : \underbrace{G \times G \times \cdots \times G}_{n\text{-times}} \to G$ is said to be right $\Gamma$-$n$-derivation if the relations

\[
h(x_1 x_1', x_2, \ldots, x_n) = h(x_1, x_2, \ldots, x_n) \gamma x_1' + h(x_1, x_2, \ldots, x_n) \gamma x_1
\]

\[
h(x_1, x_2 x_2', \ldots, x_n) = h(x_1, x_2, \ldots, x_n) \gamma x_2' + h(x_1, x_2, \ldots, x_n) \gamma x_2
\]

\[\vdots\]

\[
h(x_1, x_2, \ldots, x_n \gamma x_n') = h(x_1, x_2, \ldots, x_n) \gamma x_n' + h(x_1, x_2, \ldots, x_n) \gamma x_n
\]

Hold for all $x_1, x_1', x_2, x_2', x_n, x_n' \in G$ and $\gamma \in \Gamma$.

In this work, we defined the concept $\Gamma$-$n$-derivation and right $\Gamma$-$n$-derivation. Also we investigate the commutativity of addition and multiplication of $\Gamma$-near-rings satisfying certain identities involving $\Gamma$-$[1, 2, 3, 4, 5]$ on commutativity of prime $\Gamma$-near-ring on which admits suitably constrained right $\Gamma$-$n$-derivations. The purpose of this paper is to study and generalize some results of $\underline{1} \underline{2} \underline{3} \underline{4} \underline{5}$ on commutativity of prime $\Gamma$-near-ring on which admits suitably constrained right $\Gamma$-$n$-derivations.

2. Preliminary results

We begin with the following lemmas which are essential for developing the proofs of our main results.

**Lemma 2.1.** Let $G$ be a prime $\Gamma$-near-ring. There exists an element $u$ of $Z(G)$ such that $u + u \in Z(G)$, then $(G, +)$ is abelian.

**Lemma 2.2.** Let $G$ be a $\Gamma$-near-ring admitting right $\Gamma$-$n$-derivation $h$, then for every $s_1, s_1', \ldots, s_n, r \in G$ and $\gamma, \beta \in \Gamma$,

\[
\{h(s_1, s_2, \ldots, s_n) \gamma s_1' + h(s_1, s_2, \ldots, s_n) \gamma s_1\} \beta r = h(s_1, s_2, \ldots, s_n) \gamma s_1' \beta r + h(s_1, s_2, \ldots, s_n) \gamma s_1 \beta r
\]

**Proof.** Assume that

\[
h((s_1 \gamma s_1') \beta r, s_2, \ldots, s_n) = h(s_1 \gamma s_1, s_2, \ldots, s_n) \beta r + h(r, s_2, \ldots, s_n) \beta (s_1 \gamma s_1')
\]

\[
= (h(s_1, s_2, \ldots, s_n) \gamma s_1' + h(s_1, s_2, \ldots, s_n) \gamma s_1) \beta r + h(r, s_2, \ldots, s_n) \beta (s_1 \gamma s_1').
\]

Also

\[
h(s_1 \gamma (s_1' \beta r), s_2, \ldots, s_n) = h(s_1, s_2, \ldots, s_n) \gamma s_1' \beta r + h(s_1' \beta r, s_2, \ldots, s_n) \gamma s_1
\]

\[
= h(s_1, s_2, \ldots, s_n) \gamma s_1' \beta r + h(s_1', s_2, \ldots, s_n) \beta r \gamma s_1 + h(r, s_2, \ldots, s_n) \beta (s_1 \gamma s_1')
\]

Combining the above two relations, we get

\[
(h(s_1, s_2, \ldots, s_n) \gamma s_1' + h(s_1, s_2, \ldots, s_n) \gamma s_1) \beta r = h(s_1, s_2, \ldots, s_n) \gamma s_1' \beta r + h(s_1, s_2, \ldots, s_n) \gamma s_1 \beta r
\]

\[\square\]

**Lemma 2.3.** Let $G$ be a prime $\Gamma$-near-ring admitting a nonzero right $\Gamma$-$n$-derivation $h$ of $G$ and $a \in G$. If $h(G, G', \ldots, G') \gamma a = \{0\}$, then $a = 0$.

**Proof.** Suppose that $h(x_1, x_2, \ldots, x_n) \gamma a = 0$, for all $x_1, x_2, \ldots, x_n \in G$ and $\gamma \in \Gamma$.

Putting $x_1 \beta s$ instead of $x_1$ where $s \in G$ and $\beta \in \Gamma$ in pervious equation we get $h(x_1 \beta s, x_2, \ldots, x_n) \gamma a = 0$. So we get $h(s, x_2, \ldots, x_n) \Gamma G a = \{0\}$. Since $h \neq 0$ and $G$ is a prime $\Gamma$-near-ring, we conclude that $a = 0$. \[\square\]

**Lemma 2.4.** Let $G$ be a prime $\Gamma$-near-ring and let $h$ be a nonzero right $\Gamma$-derivation of $G$ and $a \in G$. If $h(G) \gamma a = \{0\}$, then $a = 0$. 


3. Main results

**Theorem 3.1.** Let $G$ be a prime $\Gamma$-near-ring and $h$ be a nonzero right $\Gamma$-derivation of $G$. If $h(G, G, \ldots, G) \subseteq Z$ and $h$ is a nonzero right $\Gamma$-derivation, there exist nonzero elements $x_1, x_2, \ldots, x_n \in G$, such that \( (x_1, x_2, \ldots, x_n) \in Z \setminus \{0\} \). We have \( (x_1 + x_2, x_2, \ldots, x_n) = (x_1, x_2, \ldots, x_n) + h(x_1, x_2, \ldots, x_n) \in Z \). By Lemma 2.1 we obtain that $(G, +)$ is abelian.

By hypothesis we get \( h(y_1, y_2, \ldots, y_n) \gamma y = y \gamma h(y_1, y_2, \ldots, y_n) \), for all $y, y_1, y_2, \ldots, y_n \in G$ and $\gamma \in \Gamma$. Now replacing $y_1$ by $y_1 \beta s$ where $s \in G$ in previous equation, we get

\[
(h(y_1, y_2, \ldots, y_n) \beta s + h(s, y_2, \ldots, y_n) \beta y_1) \gamma y = y \gamma (h(y_1, y_2, \ldots, y_n) \beta s + h(s, y_2, \ldots, y_n) \beta y_1)
\]  

(1)

By definition of $h$ we get \( h(y_1, y_2, \ldots, y_n) \beta y_1 = h(y_1, y_2, \ldots, y_n) \beta y_1 + h(y_1, y_2, \ldots, y_n) \beta y_1 \) (2).

Thus \( h(y_1, y_2, \ldots, y_n) = h(y_1, y_2, \ldots, y_n) \beta y_1 + h(y_1, y_2, \ldots, y_n) \beta y_1 \) (3).

Since $(G, +)$ is abelian, from equation (2) and (3) we conclude that

\[ h(y_1, y_2, \ldots, y_n) = h(y_1, y_2, \ldots, y_n) \beta y_1, \beta y_2, \ldots, y_n \]  

for all $y_1, y_2, \ldots, y_n \in G$ and $\beta \in \Gamma$.

So we get $h(y_1, y_1', y_2, \ldots, y_n) = 0$ for all $y_1, y_1', y_2, \ldots, y_n \in G$ and $\beta \in \Gamma$.

Replacing $y_1$ by $y_1 \gamma y_1$ in previous equation and using it again, we get $h(y_1, y_2, \ldots, y_n) \gamma G \Gamma[y_1, y_1'] = \{0\}$ for all $y_1, y_1', y_2, \ldots, y_n \in G$.

Primeness of $G$ implies that for each $y_1 \in G$. either $h(y_1, y_2, \ldots, y_n) = 0$ for all $y_2, \ldots, y_n \in G$ or $y_1 \in Z$. If $h(y_1, y_2, \ldots, y_n) = 0$, then equation (1) takes the form $h(y_1, y_2, \ldots, y_n) \gamma G \Gamma[y_1, y_1'] = \{0\}$.

Since $h \neq 0$, primeness of $G$ implies that $y_1 \in Z$. Hence we find that $G = Z$, we conclude that $G$ is a commutative ring.

**Corollary 3.2.** Let $G$ be a prime $\Gamma$-near-ring and $h$ be a nonzero right $\Gamma$-derivation of $G$. If $h(G) \subseteq Z$, then $G$ is a commutative ring.

**Theorem 3.3.** Let $G$ be a prime $\Gamma$-near-ring then $G$ admit no nonzero right $\Gamma$-derivation $h$ such that \( x_1 \gamma h(y_1, y_2, \ldots, y_n) = h(x_1, x_2, \ldots, x_n) \gamma y_1 \), for all $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n \in G$ and $\gamma \in \Gamma$, then $h = 0$.

**Proof.** Assume that there is a nonzero right $\Gamma$-derivation $h$ of $G$ such that \( x_1 \gamma h(y_1, y_2, \ldots, y_n) = h(x_1, x_2, \ldots, x_n) \gamma y_1 \), for all $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n \in G$ and $\gamma \in \Gamma$. (4).

Substituting $y_1 \beta z_1$ for $y_1$, where $z_1 \in G$ in equation (4), we get

\[ x_1 \gamma h(y_1, y_2, \ldots, y_n) = h(x_1, x_2, \ldots, x_n) \gamma y_1 \beta z_1. \]

Thus, \( x_1 \gamma h(y_1, y_2, \ldots, y_n) \beta z_1 + x_1 \gamma h(z_1, y_2, \ldots, y_n) \beta y_1 = h(x_1, x_2, \ldots, x_n) \gamma y_1 \beta z_1. \)

Using equation (4) in previous equation we get \( x_1 \gamma h(z_1, y_2, \ldots, y_n) \beta y_1 = 0. \)

By primeness of $G$ implies that $h(z_1, y_2, \ldots, y_n) \beta y_1 = 0$. Now replacing $y_1$ by $y_1 \gamma h(z_1, y_2, \ldots, y_n)$ in previous equation we get $h(z_1, y_2, \ldots, y_n) \gamma G \Gamma h(z_1, y_2, \ldots, y_n) = \{0\}$. Since $G$ is prime $\Gamma$-near-ring implies that $h = 0$. □

**Corollary 3.4.** Let $G$ be a prime $\Gamma$-near-ring and $h$ be a right $\Gamma$-derivation such that $x \gamma h(y) = h(x) \gamma y$ for all $x, y \in G$ and $\gamma \in \Gamma$, then $h = 0$.

**Theorem 3.5.** Let $G$ be a prime $\Gamma$-near-ring admitting a nonzero right $\Gamma$-derivation $h$ on $G$. If \( h([x, y], x_2, \ldots, x_n) = 0 \) for all $x, y, x_2, \ldots, x_n \in G$ and $\gamma \in \Gamma$ then $G$ is a commutative ring.

**Proof.** By hypothesis, we have $h([x, y], x_2, \ldots, x_n) = 0$ for all $x, y, x_2, \ldots, x_n \in G$ and $\gamma \in \Gamma$. Replace $y$ by $x \beta y$ in previous equation and using it again we get $h([x, x_2, \ldots, x_n] \beta [x, y], \gamma) = 0$. Replacing $y$ by $y \beta z$ in previous equation, we get $h(x, x_2, \ldots, x_n) \mu [x, z] = 0$ Hence we get
Theorem 3.7. Let \( x, y \in G \), primeness of \( G \) yields either \( x \in Z \) or \( h(x, x_2, \ldots, x_n) = 0 \) for all \( x_2, \ldots, x_n \). (5)

If first case holds then

\[
h(x\gamma t, x_2, \ldots, x_n) = h(t\gamma x, x_2, \ldots, x_n), \quad \forall t, x_2, \ldots, x_n \in G \text{ and } \gamma \in \Gamma.
\]

Its mean \( h(x, x_2, \ldots, x_n) \in Z \). And second case implies \( h(x, x_2, \ldots, x_n) = 0 \) that is \( h(x, x_2, \ldots, x_n) = 0 \) for all \( x, x_2, \ldots, x_n \in G \). That is \( h(G, G, \ldots, G) \subseteq Z \).

Hence we get \( h(x, x_2, \ldots, x_n) \in Z \) for all \( x, x_2, \ldots, x_n \in G \). That is \( h(G, G, \ldots, G) \subseteq Z \).

□

Corollary 3.6. Let \( G \) be a prime \( \Gamma \)-near-ring admitting a right \( \Gamma \)-derivations \( h \), If \( h([x, y]_\Gamma) = 0 \) for all \( x, y \in G \), then \( G \) is a commutative ring.

Theorem 3.7. Let \( G \) be a prime \( \Gamma \)-near-ring and \( h \) be a nonzero right \( \Gamma \)-derivation on \( G \) such that \( h((x \circ y)_\gamma, x_2, \ldots, x_n) = 0 \) for all \( x, y, x_2, \ldots, x_n \in G \) and \( \gamma \in \Gamma \) then \( G \) is a commutative ring.

Proof. Assume that \( h((x \circ y)_\gamma, x_2, \ldots, x_n) = 0 \) for all \( x, y, x_2, \ldots, x_n \in G \) and \( \gamma \in \Gamma \).

Replace \( y \) by \( x\beta y \) in equation (6) we get \( h((x \circ (x\beta y))_\gamma, x_2, \ldots, x_n) = 0 \) Which implies that \( h(x, x_2, \ldots, x_n)\beta(x \circ y)_\gamma = h((x \circ y)_\gamma, x_2, \ldots, x_n)\beta x = 0 \).

Using equation (6) in the preceding equation we get \( h(x, x_2, \ldots, x_n)\beta(x \circ y)_\gamma = 0 \).

Replacing \( y \) by \( x\gamma y \), where \( z \in G \), we get \( h(x, x_2, \ldots, x_n)\beta x\gamma y = -h(x, x_2, \ldots, x_n)\beta x\gamma yz \).

Now substituting the values from equation (7) in the preceding relation we get

\[
h(x, x_2, \ldots, x_n)\beta(y\gamma z) = -h(x, x_2, \ldots, x_n)\beta y\gamma yz \gamma
\]

Hence we get \( h(x, x_2, \ldots, x_n)\Gamma GT[x, z]_\gamma = \{0\} \). Since \( G \) is a prime \( \Gamma \)-near-ring we get either \( x \in Z \) or \( h(x, x_2, \ldots, x_n) = 0 \) for all \( x_2, \ldots, x_n \in G \), for each fixed \( x \in G \).

Which is identical with the equation (5) in Theorem 3.5. Now arguing in the same way in the Theorem 3.5. We conclude that \( G \) is a commutative ring. □

Corollary 3.8. Let \( G \) be a prime \( \Gamma \)-near-ring and let \( h \) be a nonzero right \( \Gamma \)-derivation on \( G \) such that \( h((x \circ y)_\gamma) = 0 \) for all \( x, y \in G \) and \( \gamma \in \Gamma \) then \( G \) is a commutative ring.

Theorem 3.9. Let \( G \) be a prime \( \Gamma \)-near-ring admitting a right \( \Gamma \)-derivation \( h \) of \( G \). If \( [h(x, x_2, \ldots, x_n), y]_\gamma \in Z \) for all \( x, y, x_2, \ldots, x_n \in G \) and \( \gamma \in \Gamma \) and \( c\beta x\gamma y = c\beta x\gamma y \) for all \( c, x, y \in G \) and \( \gamma, \beta \in \Gamma \), then \( G \) is a commutative ring.

Proof. Assume that \( [h(x, x_2, \ldots, x_n), y]_\gamma \in Z \) for all \( x, y, x_2, \ldots, x_n \in G \) and \( \gamma \in \Gamma \).

Therefore, \( [h(x, x_2, \ldots, x_n), y]_\gamma, t]_\beta = 0 \) for all \( x, y, t, x_2, \ldots, x_n \in G \) and \( \gamma, \beta \in \Gamma \).

Replacing \( y \) by \( h(x, x_2, \ldots, x_n)\mu y \) in equation (9) , we get

\[
[h(x, x_2, \ldots, x_n)\mu [h(x, x_2, \ldots, x_n), y]_\gamma, t]_\beta = 0
\]

In view of equation (8), equation (10) assures that

\[
[h(x, x_2, \ldots, x_n), y]_\gamma \Gamma GT[h(x, x_2, \ldots, x_n), t]_\beta = \{0\}
\]

Primeness of \( G \) implies that \( [h(x, x_2, \ldots, x_n), y]_\gamma = 0 \) for all \( x, y, x_2, \ldots, x_n \in G \).

Hence \( h(G, G, \ldots, G) \subseteq Z \). And application of Theorem 3.1 assures that \( G \) is a commutative ring.

□

Corollary 3.10. Let \( G \) be a prime \( \Gamma \)-near-ring and let \( h \) be a right \( \Gamma \)-derivation of \( G \). If \( [h(x), y]_\gamma \in Z \) for all \( x, y \in G \), then \( G \) is a commutative ring.

Theorem 3.11. Let \( G \) be a prime \( \Gamma \)-near-ring, \( h_1 \) and \( h_2 \) be any two nonzero right \( \Gamma \)-derivations. If \( [h_1(G, G, \ldots, G), h_2(G, G, \ldots, G)]_\gamma = \{0\} \) then \( (G, +) \) is abelian.

Proof. Assume that \( [h_1(G, G, \ldots, G), h_2(G, G, \ldots, G)]_\gamma = \{0\} \).

If both \( z \) and \( z + z \) commute element wise with \( h_2(G, G, \ldots, G) \), then
From equation (11) and (12) the previous equation can be reduced to
\[ z\gamma h_2(x_1, x_2, \ldots, x_n) = h_2(x_1, x_2, \ldots, x_n)\gamma z \quad (11) \]
And \((z + z)\gamma h_2(x_1, x_2, \ldots, x_n) = h_2(x_1, x_2, \ldots, x_n)\gamma(z + z) \quad (12)\).
Substituting \(x_1 + x_1\) instead of \(x_1\) in equation (12), we get
\[ (z + z)\gamma h_2(x_1 + x'_1, x_2, \ldots, x_n) = h_2(x_1 + x'_1, x_2, \ldots, x_n)\gamma(z + z) \]
From equation (11) and (12) the previous equation can be reduced to
\[ z\gamma h_2(x_1 + x'_1 - x_1 - x'_1, x_2, \ldots, x_n) = 0. \quad (i.e.) \]
Putting \(z = h_1(y_1, y_2, \ldots, y_n)\), we get \(h_1(y_1, y_2, \ldots, y_n)\gamma h_2((x_1, x'_1), x_2, \ldots, x_n) = 0\).
By Lemma 2.3 we conclude that \(h_2((x_1, x'_1), x_2, \ldots, x_n) = 0 \quad (13)\).
Since we know that for each \(w \in G\),
\[ w\gamma(x_1, x'_1) = w\gamma(x_1 + x'_1 - x_1 - x'_1) = w\gamma x_1 + w\gamma x'_1 - w\gamma x_1 - w\gamma x'_1 = (w\gamma x_1, w\gamma x'_1) \]
Which is again an additive commutator. Putting \(w\gamma(x_1, x'_1)\) instead of \((x_1, x'_1)\) in equation (13) we get \(h_2(w\gamma(x_1, x'_1), x_2, \ldots, x_n) = 0\), for all \(w, x_1, x'_1, x_2, \ldots, x_n \in G\) and \(\gamma \in \Gamma\). i.e.;
\[ h_2(w, x_2, \ldots, x_n)\gamma(x_1, x'_1) + h_2((x_1, x'_1), x_2, \ldots, x_n)\gamma w = 0 \]
Using equation (13) in previous equation yields \(h_2(w, x_2, \ldots, x_n)\gamma(x_1, x'_1) = 0\).
Using Lemma 2.3 we conclude that \((x_1, x'_1) = 0\). Hence \((G, +)\) is abelian.

**Corollary 3.12.** Let \(G\) be a prime \(\Gamma\)-near-ring and \(h_1, h_2\) be any two nonzero right \(\Gamma\)-derivations. If \([h_1(G), h_2(G)]_{\gamma} = \{0\}\) then \((G, +)\) is abelian.

**Theorem 3.13.** Let \(G\) be a prime \(\Gamma\)-near-ring and \(h_1\) and \(h_2\) be any two nonzero right \(\Gamma\)-derivations. If \(h_1(x_1, x_2, \ldots, x_n)\gamma h_2(y_1, y_2, \ldots, y_n) + h_2(x_1, x_2, \ldots, x_n)\gamma h_1(y_1, y_2, \ldots, y_n) = 0\) for all \(x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n \in G\) and \(\gamma \in \Gamma\), then \((G, +)\) is abelian.

**Proof.** By our hypothesis we have
\[ h_1(x_1, x_2, \ldots, x_n)\gamma h_2(y_1, y_2, \ldots, y_n) + h_2(x_1, x_2, \ldots, x_n)\gamma h_1(y_1, y_2, \ldots, y_n) = 0 \quad (14) \]
Substituting \(y_1 + y'_1\) instead of \(y_1\) in equation (14) we get
\[ h_1(x_1, x_2, \ldots, x_n)\gamma h_2(y_1 + y'_1, y_2, \ldots, y_n) + h_2(x_1, x_2, \ldots, x_n)\gamma h_1(y_1 + y'_1, y_2, \ldots, y_n) = 0 \]
for all \(x_1, x_2, \ldots, x_n, y_1, y_1 + y'_1, y_2, \ldots, y_n \in G\) and \(\gamma \in \Gamma\).
Therefore
\[ h_1(x_1, x_2, \ldots, x_n)\gamma h_2(y_1, y_2, \ldots, y_n) + h_1(x_1, x_2, \ldots, x_n)\gamma h_2(y'_1, y_2, \ldots, y_n) + h_2(x_1, x_2, \ldots, x_n)\gamma h_1(y_1, y_2, \ldots, y_n) + h_2(x_1, x_2, \ldots, x_n)\gamma h_1(y'_1, y_2, \ldots, y_n) = 0 \]
Using equation (14) again in preceding equation, we get
\[ h_1(x_1, x_2, \ldots, x_n)\gamma h_2(y_1, y_2, \ldots, y_n) + h_1(x_1, x_2, \ldots, x_n)\gamma h_2(y'_1, y_2, \ldots, y_n) + h_1(x_1, x_2, \ldots, x_n)\gamma h_2(-y_1, y_2, \ldots, y_n) + h_1(x_1, x_2, \ldots, x_n)\gamma h_2(-y'_1, y_2, \ldots, y_n) = 0 \]
Which means that \(h_1(x_1, x_2, \ldots, x_n)\gamma h_2((y_1, y'_1), y_2, \ldots, y_n) = 0\).
By Lemma 2.3 we obtain \(h_2((y_1, y'_1), y_2, \ldots, y_n) = 0\), for all \(y_1, y'_1, y_2, \ldots, y_n \in G\) and \(\gamma \in \Gamma\). Now putting \(w\gamma(y_1, y'_1)\) instead of \((y_1, y'_1)\), where \(w \in G\) in previous equation and using it again, we get \(h_2(w, y_2, \ldots, y_n)\gamma(y_1, y'_1) = 0\), for all \(w, y_1, y'_1, y_2, \ldots, y_n \in G\) and \(\gamma \in \Gamma\). Using Lemma 2.3 as used in the Theorem 3.11 we conclude that \((G, +)\) is abelian.

**Corollary 3.14.** Let \(G\) be a prime \(\Gamma\)-near-ring and \(h_1, h_2\) be any two nonzero right \(\Gamma\)-derivations. If \(h_1(x)\gamma h_2(y) + h_2(x)\gamma h_1(y) = 0\), for all \(x, y \in G\), then \((G, +)\) is abelian.
References