Int. J. Nonlinear Anal. Appl. 12 (2021) No. 2, 1653–1658
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/IJNAA.2021.5294

Right Γ-n-derivations in prime Γ-near-rings

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(Communicated by Madjid Eshaghi Gordji)

Abstract

The main purpose of this paper is to study and investigate some results of right Γ-n-derivation on prime Γ-near-ring G which force G to be a commutative ring.

Keywords: Prime Γ –near-ring, Γ-n- derivation

1. Introduction

Throughout this paper, a Γ- near ring is a triple $(G, +, \Gamma)$, where (i) $(G, +)$ is a (not necessarily abelian) group; (ii) Γ is a non-empty set of binary operations on G such that for each $\gamma \in \Gamma$, $(G, +, \gamma)$ is a left near-ring (iii) $s\gamma(r\mu c) = (s\gamma r)\mu c$, for all $s, r, c \in G$ and $\gamma, \mu \in \Gamma$ [5, 7, 8]. And G will denote a zero–symmetric left Γ- near ring with multiplicative center $Z(G)$. For a Γ-near-ring G, the set $G_0 = \{s \in G : 0\rho s = 0, \forall \rho \in \Gamma\}$ is called zero symmetric part of G. If $G = G_0$, then G is called zero symmetric [8, 9]. A Γ-near-ring G is said to be prime Γ-near-ring if $s\Gamma G\Gamma r = 0$ implies $s = 0$ or $r = 0$, for every $s, r \in G$ and it said to be semiprime if $s\Gamma G\Gamma s = 0$ implies $s = 0$ for every $s \in G$ [7, 8]. The other commutators are; $[s, r]_\rho = s\rho r - r\rho s$ and $(s, r) = s + r - s - r$ denote the additive-group commutator [1, 9]. Γ-near-ring G is called commutative if $(G, +)$ is abelian [2, 3].

An additive mapping $h : G \times G \times \cdots \times G \longrightarrow G$ is said to be Γ-n-derivation if the relations

$$\begin{aligned}
h(x_1\gamma x_1', x_2, \ldots, x_n) &= h(x_1, x_2, \ldots, x_n)\gamma x_1' + x_1\gamma h(x_1', x_2, \ldots, x_n)\\
h(x_1, x_2\gamma x_2', \ldots, x_n) &= h(x_1, x_2, \ldots, x_n)\gamma x_2' + x_2\gamma h(x_1, x_2', \ldots, x_n)\\
&\vdots\\
h(x_1, x_2, \ldots, x_n\gamma x_n') &= h(x_1, x_2, \ldots, x_n)\gamma x_n' + x_n\gamma h(x_1, x_2, \ldots, x_{n'})
\end{aligned}$$

Hold for all $x_1, x_1', x_2, x_2', \ldots, x_n, x_n' \in G$.

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Received: March 2021 *Accepted:* June 2021

An n-additive mapping $h : \underbrace{G \times G \times \cdots \times G}_{n-times} \longrightarrow G$ is said to be right Γ-n-derivation if the relations

$$h(x_1\gamma x_1', x_2, \ldots, x_n) = h(x_1, x_2, \ldots, x_n)\gamma x_1' + h(x_1', x_2, \ldots, x_n)\gamma x_1$$
$$h(x_1, x_2\gamma x_2', \ldots, x_n) = h(x_1, x_2, \ldots, x_n)\gamma x_2' + h(x_1, x_2', \ldots, x_n)\gamma x_2$$
$$\vdots$$
$$h(x_1, x_2, \ldots, x_n\gamma x_n') = h(x_1, x_2, \ldots, x_n)\gamma x_n' + h(x_1, x_2, \ldots, x_{n'})\gamma x_n$$

Hold for all $x_1, x_1', x_2, x_2', \ldots, x_n, x_n' \in G$ and $\gamma \in \Gamma$

In this work, we defined the concept Γ-n-derivation and right Γ-n-derivation. Also we investigate the commutativity of addition and multiplaction of Γ-near-rings satisfying certainidentities involving right Γ-n-derivation. And the purpose of this paper is to study and generalize some results of [1, 2, 3, 4, 5] on commutativity of prime Γ–near-ring on which admits suitably constrained right Γ-n-derivations.

2. Preliminary results

We begin with the following lemmas which are essential for developing the proofs of our main results

Lemma 2.1.[5, 8] . Let G be a prime Γ- near ring. there exists a element u of $Z(G)$ such that $u + u \in Z(G)$, then $(G, +)$ is abelian.

Lemma 2.2. Let G be a Γ-near-ring admitting right Γ- n-derivation h, then for every $s_1, s_1', \ldots, s_n, r \in G$ and $\gamma, \beta \in \Gamma$,

$\{h(s_1, s_2, \ldots, s_n)\gamma s_1' + h(s_1', s_2, \ldots, s_n)\gamma s_1\}\beta r = h(s_1, s_2, \ldots, s_n)\gamma s_1'\beta r + h(s_1', s_2, \ldots, s_n)\gamma s_1\beta r$

Proof . Assume that

$$h((s_1\gamma s_1')\beta r, s_2, \ldots, s_n) = h(s_1\gamma s_1', s_2, \ldots, s_n)\beta r + h(r, s_2, \ldots, s_n)\beta(s_1\gamma s_1')$$
$$= (h(s_1, s_2, \ldots, s_n)\gamma s_1' + h(s_1', s_2, \ldots, s_n)\gamma s_1)\beta r + h(r, s_2, \ldots, s_n)\beta(s_1\gamma s_1').$$

Also

$$h(s_1\gamma(s_1'\beta r), s_2, \ldots, s_n) = h(s_1, s_2, \ldots, s_n)\gamma s_1'\beta r + h(s_1'\beta r, s_2, \ldots, s_n)\gamma s_1$$
$$= h(s_1, s_2, \ldots, s_n)\gamma s_1'\beta r + (h(s_1', s_2, \ldots, s_n)\beta r + h(r, s_2, \ldots, s_n)\beta s_1')\gamma s_1$$
$$= h(s_1, s_2, \ldots, s_n)\gamma s_1'\beta r + h(s_1', s_2, \ldots, s_n)\beta r\gamma s_1 + h(r, s_2, \ldots, s_n)\beta s_1'\gamma s_1$$

Combining the above two relations, we get

$$(h(s_1, s_2, \ldots, s_n)\gamma s_1' + h(s_1', s_2, \ldots, s_n)\gamma s_1)\beta r = h(s_1, s_2, \ldots, s_n)\gamma s_1'\beta r + h(s_1', s_2, \ldots, s_n)\gamma s_1\beta r$$

□

Lemma 2.3.2.3 Let G be a prime Γ- near-ring admitting a nonzero right Γ-n-derivation h of G and $a \in G$. If $h(G, G, ..., G)\gamma a = \{0\}$, then $a = 0$.

Proof . Suppose that $h(x_1, x_2, \ldots, x_n)\gamma a = 0$, for all $x_1, x_2, \ldots, x_n \in G$ and $\gamma \in \Gamma$.

Putting $x_1\beta s$ instead of x_1 where $s \in G$ and $\beta \in \Gamma$ in pervious equation we get $h(x_1\beta s, x_2, \ldots, x_n)\gamma a = 0$. So we get $h(s, x_2, \ldots, x_n)\Gamma G\Gamma a = \{0\}$. Since $h \neq 0$ and G is a prime Γ-near-ring, we conclude that $a = 0$. □

Lemma 2.4. Let G be a prime Γ-near-ring and let h be a nonzero right Γ-derivation of G and $a \in G$. If $h(G)\gamma a = \{0\}$, then $a = 0$.

3. Main results

Theorem 3.1. Let G be a prime Γ-near-ring and h be a nonzero right Γ-n-derivation of G. If $h(G, G, \ldots, G) \subseteq Z$, then G is a commutative ring.

Proof . Since$h(G, G, \ldots, G) \subseteq Z$and h is a nonzero right Γ-n-derivation, there exist nonzero elements $x_1, x_2, \ldots, x_n \in G$, such that $h(x_1, x_2, \ldots, x_n) \in Z \setminus \{0\}$. We have $h(x_1 + x_1, x_2, \ldots, x_n) = h(x_1, x_2, \ldots, x_n) + h(x_1, x_2, \ldots, x_n) \in Z$. By Lemma 2.1 we obtain that $(G, +)$ is abelian.

By hypothesis we get $h(y_1, y_2, \ldots, y_n)\gamma y = y\gamma h(y_1, y_2, \ldots, y_n)$, for all $y, y_1, y_2, \ldots, y_n \in G$ and $\gamma \in \Gamma$. Now replacing y_1 by $y_1\beta s$ where $s \in G$in previous equation, we get

$$(h(y_1, y_2, \ldots, y_n)\beta s + h(s, y_2, \ldots, y_n)\beta y_1)\gamma y = y\gamma(h(y_1, y_2, \ldots, y_n)\beta s + h(s, y_2, ., y_n)\beta y_1) \qquad (1)$$

By definition of h we get $h(y_1\beta y_1', y_2, \ldots, y_n) = h(y_1, y_2, \ldots, y_n)\beta y_1' + h(y_1', y_2, \ldots, y_n)\beta y_1$ (2).

Thus $h(y_1'\beta y_1, y_2, \ldots, y_n) = h(y_1', y_2, \ldots, y_n)\beta y_1 + h(y_1, y_2, \ldots, y_n)\beta y_1'$ (3).

Since $(G, +)$ is abelian, from equation (2) and (3) we conclude that

$$h(y_1\beta y_1', y_2, \ldots, y_n) = h(y_1'\beta y_1, y_2, \ldots, y_n)$$

for all $y_1, y_1', y_2, \ldots, y_n \in G$ and $\beta \in \Gamma$.

So we get $h([y_1, y_1']_\beta, y_2, \ldots, y_n) = 0$ for all $y_1, y_1', y_2, \ldots, y_n \in G$ and $\beta \in \Gamma$.

Replacing y_1' by $y_1\gamma y_1'$ in previous equation and using it again, we get $h(y_1, y_2, \ldots, y_n)\Gamma G\Gamma[y_1, y_1']_\beta = \{0\}$ for all $y_1, y_1', y_2, \ldots, y_n \in G$.

Primeness of G implies that for each $y_1 \in G$. either $h(y_1, y_2, \ldots, y_n) = 0$ for all $y_2, \ldots, y_n \in G$ or $y_1 \in Z$. If $h(y_1, y_2, \ldots, y_n) = 0$, then equation (1) takes the form $h(y_1', y_2, \ldots, y_n)\Gamma G\Gamma[y, y_1]\beta = \{0\}$. Since $h \neq 0$, primeness of G implies that $y_1 \in Z$. Hence we find that $G = Z$, we conclude that G is a commutative ring. □

Corollary 3.2. Let G be a prime Γ- near-ring and h be a nonzero right Γ-derivation of G. If $h(G) \subseteq Z$, then G is a commutative ring.

Theorem 3.3. Let G be a prime Γ-near-ring then G admit no nonzero right Γ-n-derivation h such that $x_1\gamma h(y_1, y_2, \ldots, y_n) = h(x_1, x_2, \ldots, x_n)\gamma y_1$, for all $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n \in G$ and $\gamma \in \Gamma$, then $h = 0$.

Proof . Assume that there is a nonzero right Γ-n-derivation h of G such that $x_1\gamma h(y_1, y_2, \ldots, y_n) = h(x_1, x_2, \ldots, x_n)\gamma y_1$, for all $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n \in G$ and $\gamma \in \Gamma$ (4).

Substituting $y_1\beta z_1$ for y_1, where $z_1 \in G$ in equation (4), we get

$$x_1\gamma h(y_1\beta z_1, y_2, \ldots, y_n) = h(x_1, x_2, \ldots, x_n)\gamma y_1\beta z_1.$$

Thus, $x_1\gamma h(y_1, y_2, \ldots, y_n)\beta z_1 + x_1\gamma h(z_1, y_2, \ldots, y_n)\beta y_1 = h(x_1, x_2, \ldots, x_n)\gamma y_1\beta z_1$.

Using equation (4) in previous equation we get $x_1\gamma h(z_1, y_2, \ldots, y_n)\beta y_1 = 0$.

By primeness of G implies that $h(z_1, y_2, \ldots, y_n)\beta y_1 = 0$. Now replacing y_1 by $y_1\gamma h(z_1, y_2, \ldots, y_n)$ in previous equation we get $h(z_1, y_2, \ldots, yn)\Gamma G\Gamma h(z_1, y_2, \ldots, y_n) = \{0\}$.Since G is prime Γ-near-ring implies that $h = 0$. □

Corollary 3.4. Let G be a prime Γ-near-ring and h be a right Γ-derivation such that $x\gamma h(y) = h(x)\gamma y$ for all $x, y \in G$ and $\gamma \in \Gamma$, then $h = 0$.

Theorem 3.5. Let G be a prime Γ- near-ring admitting a nonzero right Γ-n-derivation h on G. If $h([x, y]_\gamma, x_2, \ldots, x_n) = 0$ for all $x, y, x_2, \ldots, x_n \in G$ and $\gamma \in \Gamma$ then G is a commutative ring.

Proof . By hypothesis, we have $h([x, y]_\gamma, x_2, \ldots, x_n) = 0$ for all $x, y, x_2, \ldots, x_n \in G$ and $\gamma \in \Gamma$. Replace y by $x\beta y$ in previous equation and using it again we get $h(x, x_2, \ldots, x_n)\beta[x, y]_\gamma = 0$. Replacing y by $y\mu z$ in pervious equation , we get $h(x, x_2, \ldots, x_n)\mu[x, z]_\gamma = 0$ Hence we get

$h(x, x_2, \ldots, x_n)\Gamma G\Gamma[x,z]_\gamma = \{0\}$. For each fixed $x \in G$, primeness of G yields either $x \in Z$ or $h(x, x_2, \ldots, x_n) = 0$for all $x_2, \ldots, x_n G$ (5).

If first case holds then

$h(x\gamma t, x_2, \ldots, x_n) = h(t\gamma x, x_2, \ldots, x_n)$, for all $t, x_2, \ldots, x_n \in G$ and $\gamma \in \Gamma$.

$h(x, x_2, \ldots, xn)\gamma t + h(t, x_2, \ldots, x_n)\gamma x = h(t, x_2, \ldots, x_n)\gamma x + h(x, x_2, \ldots, x_n)\gamma t$.

Its mean $h(x, x_2, \ldots, x_n) \in Z$. And second case implies $h(x, x_2, ..., x_n) = 0$ that is $h(x, x_2, \ldots, x_n) = 0 \in Z$. Including both the cases we get $h(x, x_2, \ldots, x_n) \in Z$ for all $x, x_2, \ldots, x_n \in G$. That is $h(G, G, \ldots, G) \subseteq Z$, Hence, by Theorem 3.1 then G is a commutative ring. □

Corollary 3.6. Let G be a prime Γ-near-ring admitting a right Γ-derivations h, If $h([x,y]_\Gamma) = 0$ for all $x, y \in G$, then G is a commutative ring.

Theorem 3.7. Let G be a prime Γ-near-ring and h be a no nonzero right Γ-n-derivation on G such that $h((x \circ y)_\gamma, x_2, \ldots, x_n) = 0$ for all $x, y, x_2, \ldots, x_n \in G$ and $\gamma \in \Gamma$ then G is commutative ring.

Proof . Assume that $h((x \circ y)_\gamma, x_2, \ldots, x_n) = 0$ for all $x, y, x_2, \ldots, x_n \in G$ and $\gamma \in \Gamma$ (6).

Replace y by $x\beta y$ in equation (6) we get $h((x \circ (x\gamma y))_\gamma, x_2, \ldots, x_n) = 0$ Which implies that $h(x, x_2, \ldots, x_n)\beta(x \circ y)\ \gamma$ + h((x $\circ y)_\gamma, x_2, \ldots, x_n)\beta x = 0$.

Using equation (6) in previous equation we get $h(x, x_2, \ldots, x_n)\beta(x \circ y)_\gamma = 0$.

$$h(x, x_2, \ldots, x_n)\beta y\gamma x = -h(x, x_2, \ldots, x_n)\beta x\gamma y \qquad (7)$$

Replacing y by $y\mu z$, where $z \in G$, we get $h(x, x_2, \ldots, x_n)\beta y\mu z\gamma x = -h(x, x_2, \ldots, x_n)\beta x\gamma y\mu z$.

Now substituting the values from equation (7) in the preceding relation we get

$$h(x, x_2, \ldots, x_n)\beta y\mu z\gamma x = -h(x, x_2, \ldots, x_n)\beta y\gamma y x\mu z$$

Hence we get $h(x, x_2, ..., x_n)\Gamma G\Gamma[x,z]_\gamma = \{0\}$. Since G is a prime Γ-near-ring we get either $x \in Z$ or $h(x, x_2, \ldots, x_n) = 0$ for all $x_2, \ldots, x_n \in G$, for each fixed $x \in G$.

Which is identical with the equation (5) in Theorem 3.5 Now arguing in the same way in the Theorem 3.5 .We conclude that G is a commutative ring. □

Corollary 3.8. Let G be a prime Γ-near-ring and let h be a no nonzero right Γ-derivation on G such that $h(x \circ y)_\gamma = 0$ for all $x, y \in G$ and $\gamma \in \Gamma$ then G is a commutative ring.

Theorem 3.9. Let G be a prime Γ-near-ring admitting a right Γ-n-derivation h of G. If

$[h(x, x_2, \ldots, x_n), y]_\gamma \in Z$ for all $x, y, x_2, \ldots, x_n G$ and $\gamma \in \Gamma$ and $c\gamma x\beta y = c\beta x\gamma y$ for all $c, x, y \in G$ and $\gamma, \beta \in \Gamma$, then G is a commutative ring.

Proof . Assume that $[h(x, x_2, \ldots, x_n), y]_\gamma \in Z$ for all $x, y, x_2, \ldots, x_n \in G$ and $\gamma \in \Gamma$ (8).

Therefore, $[[h(x, x_2, \ldots, x_n), y]_\gamma, t]_\beta = 0$ for all $x, y, t, x_2, \ldots, x_n \in G$ and $\gamma, \beta \in \Gamma$ (9).

Replacing y by $h(x, x_2, \ldots, x_n)\mu y$ in equation (9) , we get

$$[h(x, x_2, \ldots, x_n)\mu[h(x, x_2, \ldots, x_n), y]_\gamma, t]_\beta = 0 \quad (10)$$

In view of equation (8), equation (10) assures that

$$[h(x, x_2, \ldots, x_n), y]\gamma\Gamma G\Gamma[h(x, x_2, \ldots, x_n), t]_\beta = \{0\}$$

Primeness of G implies that $[h(x, x_2, \ldots, x_n), y]_\gamma = 0$ for all $x, y, x_2, \ldots, x_n \in G$.

Hence $h(G, G, \ldots, G) \subseteq Z$and application of Theorem 3.1 assures that G is a commutative ring. □

Corollary 3.10. Let G be a prime Γ-near-ring and let h be a right Γ-n-derivation of G. If $[h(x), y]_\gamma \in Z$ for all $x, y \in G$, then G is a commutative ring.

Theorem 3.11. Let G be a prime Γ-near-ring, h_1 and h_2 be any two nonzero right Γ-n-derivations. If $[h_1(G, G, \ldots, G), h_2(G, G, \ldots, G)]_\gamma = \{0\}$ then $(G, +)$ is abelian.

Proof . Assume that $[h_1(G, G, \ldots, G), h_2(G, G, \ldots, G)]_\gamma = \{0\}$.

If both z and $z + z$ commute element wise with $h_2(G, G, \ldots, G)$, then

$$z\gamma h_2(x_1, x_2, \ldots, x_n) = h_2(x_1, x_2, \ldots, x_n)\gamma z \quad (11)$$

And $(z+z)\gamma h_2(x_1, x_2, \ldots, x_n) = h_2(x_1, x_2, \ldots, x_n)\gamma(z+z)$ (12).

Substituting $x_1 + x_1^{'}$ instead of x_1 in equation (12), we get

$$(z + z)\ \gamma h_2(x_1 + x_1^{'}, x_2, \ldots, x_n) = h_2(x_1 + x_1^{'}, x_2, \ldots, x_n\gamma(z+z)$$

From equation (11) and (12) the previous equation can be reduced to

$$z\gamma h_2(x_1 + x_1^{'} - x_1 - x_1^{'}, x_2, \ldots, x_n) = 0.\ (i.e.)\ z\gamma h_2((x_1, x_1^{'}), x_2, \ldots, x_n) = 0$$

Putting $z = h_1(y_1, y_2, \ldots, y_n)$, we get $h_1(y_1, y_2, \ldots, y_n)\gamma h_2((x_1, x_1^{'}), x_2, \ldots, x_n) = 0$.

By Lemma 2.3 we conclude that $h_2((x_1, x_1^{'}), x_2, \ldots, x_n) = 0$ (13).

Since we know that for each $w \in G$,

$$w\gamma(x_1, x_1^{'}) = w\gamma(x_1 + x_1^{'} - x_1 - x_1^{'}) = w\gamma x_1 + w\gamma x_1^{'} - w\gamma x_1 - w\gamma x_1^{'} = (w\gamma x_1, w\gamma x_1^{'})$$

Which is again an additive commutator. Putting $w\gamma(x_1, x_1^{'})$ instead of $(x_1, x_1^{'})$ in equation (13) we get $h_2(w\gamma(x1, x_1^{'}), x_2, \ldots, x_n) = 0$, for all $w, x_1, x_1^{'}, x_2, \ldots, x_n \in G$ and $\gamma \in \Gamma$. i.e.;

$$h_2(w, x_2, \ldots, x_n)\gamma(x_1, x_1^{'}) + h_2((x_1, x_1^{'}), x_2, \ldots, x_n)\gamma w = 0$$

Using equation (13) in previous equation yields $h_2(w, x_2, \ldots, x_n)\gamma(x_1, x_1^{'}) = 0$.

Using Lemma 2.3 we conclude that $(x_1, x_1^{'}) = 0$. Hence $(G, +)$ is abelain. □

Corollary 3.12. Let G be a prime Γ-near-ring and h_1, h_2 be any two nonzero right Γ-derivations. If $[h_1(G), h_2(G)]_\gamma = \{0\}$ then $(G, +)$ is abelian.

Theorem 3.13. Let G be a prime Γ-near-ring and h_1 and h_2 be any two nonzero right Γ-n-derivations. If $h_1(x_1, x_2, \ldots, x_n)\gamma h_2(y_1, y_2, \ldots, y_n) + h_2(x_1, x_2, \ldots, x_n)\gamma h_1(y_1, y_2, \ldots, y_n) = 0$ for all $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n \in G$ and $\gamma \in \Gamma$, then $(G, +)$ is abelian.

Proof . By our hypothesis we have

$$h_1(x_1, x_2, \ldots, x_n)\gamma h_2(y_1, y_2, \ldots, y_n) + h_2(x_1, x_2, \ldots, x_n)\gamma h_1(y_1, y_2, \ldots, y_n) = 0 \quad (14)$$

Substituting $y_1 + y_1^{'}$ instead of y_1 in equation (14) we get

$$h_1(x_1, x_2, \ldots, x_n)\gamma h_2(y_1 + y_1^{'}, y_2, \ldots, y_n) + h_2(x_1, x_2, \ldots, x_n)\gamma h_1(y_1 + y_1^{'}, y_2, \ldots, y_n) = 0, \text{ for all } x_1, x_2, \ldots, x_n, y_1, y_1^{'}, y_2, \ldots, y_n \in G \text{ and } \gamma \in \Gamma$$

.Therefore

$$h_1(x_1, x_2, \ldots, x_n)\gamma h_2(y_1, y_2, \ldots, y_n) + h_1(x_1, x_2, \ldots, x_n)\gamma h_2(y_1^{'}, y_2, \ldots, y_n) + h_2(x_1, x_2, \ldots, x_n)\gamma h_1(y_1, y_2, \ldots, y_n) + h_2(x_1, x_2, \ldots, x_n)\gamma h_1(y_1^{'}, y_2, \ldots, y_n) = 0$$

Using equation (14) again in preceding equation, we get

$$h_1(x_1, x_2, \ldots, x_n)\gamma h_2(y_1, y_2, \ldots, y_n) + h_1(x_1, x_2, \ldots, x_n)\gamma h_2(y_1^{'}, y_2, \ldots, y_n) + h_1(x_1, x_2, \ldots, x_n)\gamma h_2(-y_1, y_2, \ldots, y_n) + h_1(x_1, x_2, \ldots, x_n)\gamma h_2(-y_1^{'}, y_2, \ldots, y_n) = 0$$

Which means that $h_1(x_1, x_2, \ldots, x_n)\gamma h_2((y_1, y_1^{'}), y_2, \ldots, y_n) = 0$.

By Lemma 2.3 we obtain $h_2((y_1, y_1^{'}), y_2, ..., yn) = 0$, for all $y_1, y_1^{'}, y_2, \ldots, y_n \in G$ and $\gamma \in \Gamma$. Now putting $w\gamma(y_1, y_1^{'})$ instead of $(y_1, y_1^{'})$, where $w \in G$ in previous equation and using it again, we get $h_2(w, y_2, \ldots, y_n)\gamma(y_1, y_1^{'}) = 0$, for all $w, y_1, y_1^{'}, y_2, \ldots, y_n \in G$ and $\gamma \in \Gamma$. Using Lemma 2.3 as used in the Theorem 3.11 we conclude that $(G, +)$ is abelain. □

Corollary 3.14. Let G be a prime Γ-near-ring and h_1, h_2 be any two nonzero right Γ-derivations. If $h_1(x)\gamma h_2(y) + h_2(x)\gamma h_1(y) = 0$,for all $x, y \in G$, then $(G, +)$ is abelian.

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