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Supply chain with fuzzy analytic hierarchy process (AHP): A case study

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Abstract

To achieve real competition in the global market requires the manufacturers to have the ability to meet the needs and demands of their customers, which comes from the optimal planning of the supply chain. In this paper, consideration is given to the supply chain with multi-providers of raw materials, multi-manufacturing locations, multi- centres of selling products to customers in multiple, with instability (fuzzy) of customer demands, holding costs, costs of appointment, retire and training of workforce After building a mathematical model for the supply chain that aims to maximize the net profit and reduce all costs that include production costs, labour, raw materials, storage, transportation, and the cost shortage, the model was improved through a proposal that the decision-maker has a desire to prefer one manufacturing location over another, as the proposal relied on developing a pairwise comparison in the Analytic Hierarchy Process (AHP) when the degree of comparison between factory locations is a fuzzy nature. The results of the proposed model were applied to actual data taken from an industrial organization.

Keywords: Fuzzy supply chain, Aggregate production planning, graded mean integration method, pairwise comparison.
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1. Introduction

The importance of the supply chain is highlighted through production planning, starting from supplying raw materials to becoming a final product that meets the customer's need, and thus it is the

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essence of operations management. In [1] considers a production supply chain operates in an uncertain environment. While in [2] develop a new fuzzy supply chain model given decision-makers to express their risk and analyze the comparison between customer satisfaction and product storage. In [8] addressed the improvement of two-level, multi-period supply chains under uncertainty in demand. In [11], author proposed a multi-period, multi-product, multi-manager, supply chain network design model under the fuzzy and used a simulation of a hybrid genetic algorithm, in [3] use integrated fuzzy AHP and fuzzy multi-criteria linear programming method. Fuzzy AHP used goodness, lead time, cost, power use, trash minimization, and social participation for developing linear programming where demand is fuzzy in that model.

2. Preliminaries

2.1. Definition (1): Fuzzy numbers

Fuzzy number is a component of F(N) which the membership function M : N $\rightarrow [0,1]$, achieve the normality and fuzzy convexity, [5]:

1- There are $x \in$ N such that $M(x) = 1$.

2- If $x_1, x_2 \in$ N and $\lambda \in [0,1]$,then $M(\lambda x_1 + (1-\lambda)x_2) \geq \min\{M(x_1), M(x_2)\}$.

In general, the membership function of a fuzzy number $\tilde{A}$ is:

$$\begin{aligned} M_{\tilde{A}}(x) &= \underline{L}(x), x > a \\ &= 1, a \leq x \leq b \\ &= \bar{U}(x), x < b \end{aligned} \tag{2.1}$$

Where $\bar{U}(x)$ is continuous robustly increase of right side $x < a$ and $\underline{L}(x)$ is continuous robustly decrease of left side $x > a$.

2.2. Graded mean integration method

The importance of fuzzy logic is highlighted in modeling and analyzing problems with one or more fuzzy features to obtain the final results in decision-making; all fuzzy data must be converted into crisp data; this process is known as defuzzification. One of these methods is (Graded mean integration).

This method depending on the period value of ζ grade of universal fuzzy number to defuzzification. Let $\underline{L}^{-1}, \bar{U}^{-1}$ refer to the inverse function of $\underline{L}, \bar{U}$. The graded mean ζ level value of fuzzy number is $\frac{1}{2}\left(\zeta\left(\underline{L}^{-1}(\zeta) + \bar{U}^{-1}(\zeta)\right)\right.$ the graded mean of any fuzzy number ($\approx$), perform as:

$$\bar{G}(\ddot{\ }) = \frac{1}{\int_0^1 \zeta d\zeta} \int_0^1 \left(\frac{\underline{L}^{-1}(\zeta) + \bar{U}^{-1}(\zeta)}{2}\right) \zeta d\zeta \tag{2.2}$$

Theorem (1)
The graded mean of trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is represented as:

$$\bar{G}(\tilde{A}) = \frac{a + 2b + 2c + d}{6} \tag{2.3}$$

Proof . $\underline{L}^{-1}(\zeta) = a + (b - a)$ and $\bar{U}^{-1}(\zeta) = (d(d - c)\zeta$, by the formula (2), the graded mean integration of trapezoidal fuzzy number is:

$$\bar{G}(\tilde{A}) = \frac{1}{\int_0^1 \zeta d\zeta} \int_0^1 \left(\frac{a + d + b - a - d + c}{2} \right) \zeta d\zeta, \bar{G}(\tilde{A}) = \frac{a + 2b + 2c + d}{6}$$

□

2.3. Analytic Hierarchy Process

To make the decision-making process more organized, set up a pairwise comparison matrix, and each component in the upper scale is using to compare with other components from the lower scales. When implementing these comparisons, we need a scale of numbers to determine the importance of one component relative to another component for all components in the matrix. The value of comparisons determines according to the directions in table (1), [10].

Table 1: the values of pairwise comparison (scale of influence) According to [9]

Strength of influence	Definition
1	Similar of influence
2	Weakly
3	Mild
4	Mild major
5	Strong influence
6	Strong major
7	Very strong
8	Very ,very strong
9	Extreme influence

To construct the matrix $\coprod = \|\varpi_{ij}\|, i, j = 1, 2, \ldots, n$, let $\Phi = \{\varphi_i \geq 0, i = 1, 2, \ldots, n\}, \varpi_{ij}$ represented the scale of influence of component φ_i in compare with component φ_j, for more correspondence $\sigma_{ji} = \frac{1}{\varpi_{ij}}, \varpi_{ii} = 1$, the components of the relative importance obtained from estimates of the scales in table 1 .

3. Presentation of model

The companies deal with the traditional concept of aggregate production planning, which would consider determining the amount of production, inventory, and workforce levels to meet the diverse demand within a specific time period. The firms can treat with fluctuations in demand, in addition to the costs involved, such as:

- The ability of the manpower to change by employing or ending the work of a number of employees and workers, as well as training a number of them to ensure increased capacity of production.
- Production rates vary through different production times, including regular time, overtime, and contracting outside the company.

The costs related to the supply chain are:

- The cost of required raw materials and cost of performance of jobs such as salary, training workers.

- The holding cost of final product and raw materials and the logistic cost like transport raw materials from provider to plant, and final product from plant to consumers.

Suppose there are J locations, P providers, S region of selling the final product, each location manufacture I product from different providers of M raw materials, It has a specific capacity for storing raw materials and the final product, and limitation of production time, The problem can be identified by:

- Determine the quantity of production of product I that manufactured at site J to meet the fluctuating demand in region S in period T by worker type (Λ).

- The quantity of raw materials type M which sent from the supplier P in the period T, taking into consideration the lead times to achieve the variety of demands.

- The quantity of each raw materials type M and final products I, that must be store in location J.

4. Parameters of model

Φ_{ist} = The demand of product i in region s in duration $(t), i = 1, 2, \ldots I, s = 1, 2 \ldots S, t = 1, 2, \ldots T$.
$\Pi_{\beta j}$ = Manufacture cost (in hour), by ordinary time $\beta = 1$, by extra time $\beta = 2$ and by Contracting with an external provider $\beta = 3$ at location $j, j = 1, 2, \ldots, J$.
Γ_{ist} = Selling worth per unit (product) i in region s in duration $(t), i = 1, 2, \ldots I, s = 1, 2 \ldots S$, $t = 1, 2, \ldots T$.
$SLR_{\lambda jt}$ = The salary of worker type λ in location j in duration $(t), \lambda = 1, 2, \ldots, \Lambda, j = 1, 2 \ldots J, t = 1, 2 \ldots T$.
ξ_{ij} = Manufacturing time of product i in location $j, i = 1, 2, \ldots I, \quad j = 1, 2 \ldots J$.
$\Psi_{\lambda jt}$ = The retired cost of worker type λ in location j in duration $(t), \lambda = 1, 2, \ldots, \Lambda, j = 1, 2 \ldots J, t = 1, 2 \ldots T$.
$\Omega_{\lambda jt}$ = The cost of appoint worker type λ in location j in duration $(t), \lambda = 1, 2, \ldots, \Lambda, j = 1, 2 \ldots J, t = 1, 2 \ldots T$.
$G_{\lambda jt}$ = The guidance cost of worker type λ in location j in duration $(t), \lambda = 1, 2, \ldots, \Lambda, j = 1, 2 \ldots J, t = 1, 2 \ldots T$.
$HCR_{\mu jt}$ = The holding cost of raw material μ in location j in duration $(t), \mu = 1, 2, \ldots, M, j = 1, 2 \ldots J, t = 1, 2 \ldots T$.
HCC_{ijt} = The holding cost of commodity (product) i in location j in duration $(t), i = 1, 2, \ldots, I, \quad j = 1, 2 \ldots J, \ t = 1, 2 \ldots T$.
$NCP_{\mu\rho jt}$ = The cost of transport raw material μ from provider ρ to location j in duration $(t), \rho = 1, 2 \ldots, P, \mu = 1, 2, \ldots, M \ j = 1, 2 \ldots J, t = 1, 2 \ldots T$.
NCC_{ijst} = The cost of transport commodity i from location j to region s in duration $(t), i = 1, 2, \ldots, I, j = 1, 2 \ldots J \ s = 1, 2, \ldots, S, t = 1, 2 \ldots T$.
$CR_{\mu\rho t}$ = The cost of raw material μ which obtain from provider ρ in duration $(t), \rho = 1, 2 \ldots, P, \mu = 1, 2, \ldots, M, j = 1, 2 \ldots J, \ t = 1, 2 \ldots T$.
$\text{TLIM}_{\beta jt}$ = The limitation of ordinary time $\beta = 1$, extra time $\beta = 2$, and contracting with an external provider $\beta = 3$, at location $j, j = 1, 2 \ldots J, t = 1, 2 \ldots T$.

RLIM_j = The limitation of warehouse to save raw material at location $j, j = 1, 2 \ldots J$.
CLIM_j = The limitation of warehouse to save commodity at location $j, j = 1, 2 \ldots J$.
ARLIM $_{\mu\rho t}$ = The maximum amount of raw material μ can provider ρ supply in duration $(t), \rho = 1, 2 \ldots, P, \mu = 1, 2, \ldots, M\ j = 1, 2 \ldots J, t = 1, 2 \ldots T$.
Led $TP_{\rho j}$ = Lead time of provider ρ to location $j, \rho = 1, 2 \ldots, P, j = 1, 2 \ldots J$.
LedTC_{js} = Lead time of commodity that transport from location j to region $s, s = 1, 2, \ldots, S, j = 1, 2 \ldots J$.
CSHO_{ist} = The cost of shortage per unit (product) i in region s in duration $(t), i = 1, 2, \ldots I, \quad s = 1, 2 \ldots S, t = 1, 2, \ldots T$.
$\delta_{i\mu}$ = The amount of raw material μ that need commodity i to produce, $i = 1, 2, \ldots I, \mu = 1, 2, \ldots, M$.

5. Variables of model

$x_{ij\beta t}$ = Amount of commodity i that produce in location j by type time β in duration $(t), i = 1, 2, \ldots I, \quad j = 1, 2, \ldots, J,\ \beta = 1, 2, \ldots, B, t = 1, 2, \ldots T$.
$NW_{\lambda jt}$ = No. of worker type λ in location j in duration $(t), \lambda = 1, 2, \ldots, \Lambda, j = 1, 2 \ldots J, t = 1, 2 \ldots T$.
$NWR_{\lambda jt}$ = No. of worker type λ that retired in location j in duration $(t), \lambda = 1, 2, \ldots, \Lambda, j = 1, 2 \ldots J, t = 1, 2 \ldots T$.
$NWA_{\lambda jt}$ = No. of worker type λ that appoint in location j in duration $(t), \lambda = 1, 2, \ldots, \Lambda, j = 1, 2 \ldots J, t = 1, 2 \ldots T$.
$NWT_{\lambda jt}$ = No. of worker type λ that trained in location j in duration $(t), \lambda = 1, 2, \ldots, \Lambda, j = 1, 2 \ldots J, t = 1, 2 \ldots T$.
$INLR_{\mu jt}$ = The inventory level of raw material μ in location j in duration $(t), \mu = 1, 2, \ldots, M, j = 1, 2 \ldots J, t = 1, 2 \ldots T$.
$INLC_{ijt}$ = The inventory level of commodity i in location j in duration $(t), i = 1, 2, \ldots I, \quad j = 1, 2, \ldots, J, , t = 1, 2, \ldots T$.
$NRT_{\mu\rho jt}$ = Amount of raw material μ that transport from provider ρ in location j in duration $(t), \mu = 1, 2, \ldots, M\ \rho = 1, 2, \ldots, P, j = 1, 2, \ldots, J, t = 1, 2, \ldots T$.
NC_{ijst} = Amount of commodity i that provided from location j to region s in duration $(t), i = 1, 2, \ldots I, \quad j = 1, 2, \ldots, J,\ s = 1, 2, \ldots, S, t = 1, 2, \ldots T$.
SHC_{ist} = Amount of shortage of commodity i in region s in $(t), i = 1, 2, \ldots I, s = 1, 2, \ldots, S, t = 1, 2, \ldots T$.
$w_\lambda = 1$, if the worker type λ is training, 0 otherwise, $\lambda = 1, 2, \ldots, \Lambda$.

6. Formulation of model

The objective function is maximization of the net profit as following

$$\sum_{i}^{I}\sum_{j}^{J}\sum_{s}^{S}\sum_{t}^{T}\Gamma_{ist} * NC_{ijst} - \sum_{i}^{I}\sum_{j}^{J}\sum_{\beta}^{B}\sum_{t}^{T}\xi_{ij} * \Pi_{\beta j} * x_{ij\beta t} -$$
$$\sum_{j}^{J}\sum_{\mu}^{M}\sum_{\rho}^{P}\sum_{t}^{T} CR_{\mu\rho t} * NRT_{\mu\rho jt} - \sum_{\lambda}^{\Lambda}\sum_{j}^{J}\sum_{t}^{T} SLR_{\lambda jt} * NW_{\lambda jt} -$$
$$\sum_{\lambda}^{\Lambda}\sum_{j}^{J}\sum_{t}^{T}\Omega_{\lambda jt} * NWA_{\lambda jt} - \sum_{\lambda}^{\Lambda}\sum_{j}^{J}\sum_{t}^{T}\Psi_{\lambda jt} * NWR_{\lambda jt} -$$
$$\sum_{\lambda}^{\Lambda}\sum_{j}^{J}\sum_{t}^{T} G_{\lambda jt} * NWT_{\lambda jt} - \sum_{\mu}^{M}\sum_{j}^{J}\sum_{t}^{T} HCR_{\mu jt} * INLR_{\mu jt} -$$

$$\sum_{i}^{I}\sum_{j}^{J}\sum_{t}^{T} HCC_{ijt} * INLC_{ijt} - \sum_{\mu}^{M}\sum_{\rho}^{P}\sum_{j}^{J}\sum_{t}^{T} NCP_{\mu\rho jt} * NRT_{\mu\rho jt} -$$
$$\sum_{i}^{I}\sum_{j}^{J}\sum_{s}^{S}\sum_{t}^{T} NCC_{ijst} * NC_{ijst} - \sum_{i}^{I}\sum_{j}^{J}\sum_{s}^{S}\sum_{t}^{T} CSHO_{ist} * SHC_{ist} \tag{6.1}$$

Subject to

$$NC_{ijst} = NC_{ijs(t-1)} + \sum_{\beta} x_{ij\beta t} - \sum_{s} NC_{ijst} \tag{6.2}$$

$$INLR_{\mu jt} = INLR_{\mu j(t-1)} + \sum_{s}^{S} NRT_{\mu\rho j(t-\mathrm{LedTP}_{\rho j})} - \sum_{i}^{I}\sum_{\beta}^{B} x_{ij\beta t} * \delta_{i\mu} \tag{6.3}$$

$$\sum_{j}^{J} NRT_{\mu\rho jt} - ARLIM_{\mu\rho t} \leq 0, \tag{6.4}$$

$$mo8NWT_{\lambda jt} - w_{\lambda} \leq 0 \tag{6.5}$$

$$\sum_{\lambda}^{\Lambda} NWT_{\lambda jt} * NWR_{\lambda jt} = 0, \tag{6.6}$$

$$\sum_{\lambda}^{\Lambda} NWT_{\lambda jt} - NW_{\lambda j(t-1)} + NWR_{\lambda jt} \leq 0 \tag{6.7}$$

$$\sum_{\lambda}^{\Lambda} NWR_{\lambda jt} + NWA_{\lambda jt} - NW_{\lambda j(t-1)} \leq 0 \tag{6.8}$$

$$\sum_{i}^{I} INLC_{ijt} - CLIM_j \leq 0 \tag{6.9}$$

$$\sum_{i}^{I} INLR_{ijt} - RLIM_j \leq 0 \tag{6.10}$$

$$-\sum_{j}^{J} NC_{ijs(t-\mathrm{LedTP}_{\rho j})} SHC_{is(t-1)} + \Phi_{ist} = SHC_{ist} \tag{6.11}$$

$$\sum_{i}^{I} \xi_{ij} * x_{ij3t} - TLIM_{3jt} \leq 0 \tag{6.12}$$

$$\sum_{i}^{I} \sum_{\beta=1,2} \xi_{ij} * x_{ij\beta t} \leq \sum_{\beta=1,2} \sum_{\lambda}^{\Lambda} NWR_{\lambda jt} * TLIM_{\beta jt} \tag{6.13}$$

Non negative constraints

$x_{ij\beta t}, NW_{\lambda jt}, NWR_{\lambda jt}, NWA_{\lambda jt}, NWT_{\lambda jt}, INLR_{\mu jt}, INLC_{\mu jt}, NRT_{\mu\rho jt}, NC_{ijst}, SHC_{ist}, \geq 0,$
$w_\lambda = \{0, 1\}.$

Equation (6.1) represents the objective function by which the company wants to maximize the net profit resulting from selling its products minus production costs, holding cost, raw materials, transportation, shortage, salaries, and workers training. Constraints (6.2,6.3) of the model represent an equilibrium equation of the final product and raw materials at location J, respectively, constraint (6.4) determine the production time available to the limits of manpower regularly and overtime, taking into account their production constraint (**??**) reducing the quantity of products that manufactured by the sub-contractor, constraint (6.6) it is an equilibrium form to the shortage in the point of consumption (demand), constraints (6.7,6.8) determine the levels of stock of raw materials and finished products with the capacities of store, constraint (6.9) ensures that the variation in the level of the manpower cannot exceed the share of workers in the previous time, constraint (6.10) denoted to the number of workers type Λ who left work or under training in the current time should not exceed the available number of the manpower for the previous time, constraint (6.11) refer to the worker who training in period t cannot be out of working in the same period, constraint (6.12) verify the worker under training is done, constraint (6.13) guarantee the transport quantity from provider P does not exceed to the ability of this provider.

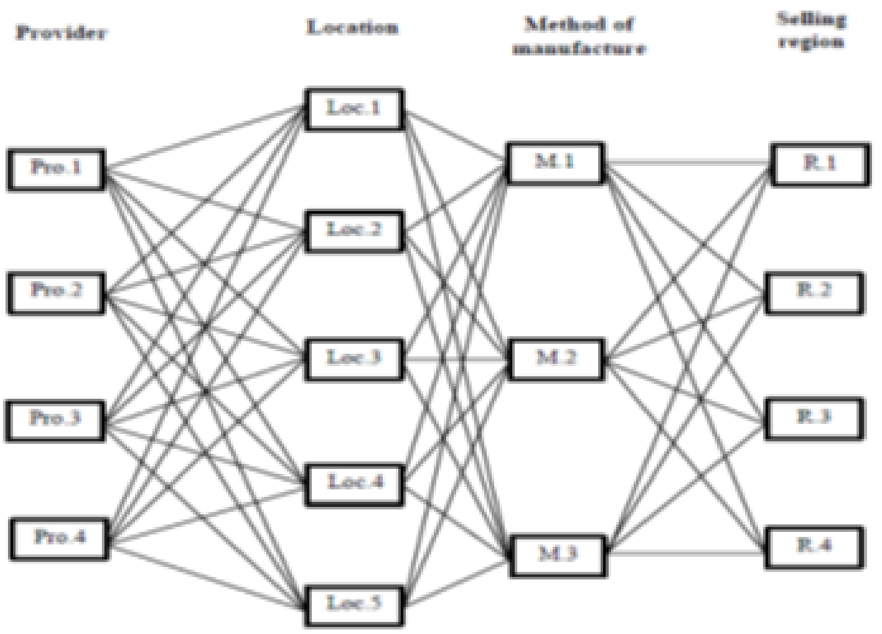


Figure 1: illustrate simplified of supply chain

7. Description data of model

The attached data of the model in Appendix (A) was adopted from [6], noting that these data in the tables are unfluctuating as trapezoidal fuzzy numbers. It will be processed in this paper to obtain crisp data before building the mathematical model. for more clarification, Table (1) in Appendix (A) illustrated manufacture time for each commodity type in each location, the cost of maintaining the stock (holding cost) and the initial stock of raw materials and the final commodity for each location, the workforce costs for any location are defined in Tables 2, 3. The expected market demand is shown in Table 4. In Table 5, the initial workforce type and limit of warehouses are specific in each location. Obtainable regular time, overtime, and subcontracting are including in Table 6. The average use of raw materials is explaining in Table 7. Tables 8 and 9 refer to the cost of shipping and lead time between providers and locations and between locations and selling Reagins. The limitations and price of raw materials supplied by undertakers are identifying in Table 10. At last, Table 11 shows the values of shortages and the price of each commodity sales for each customer region.

Before embarking on the construction of the mathematical model, it is necessary to remove the fluctuation present in the data and convert it into crisp data. By using, $\bar{G}(\tilde{A}) = \frac{a+2b+2c+d}{6}$, we obtain to the crisp data in the tables below.

Table 2: the crisp holding cost of commodity and raw materials

site	Commodity ($/unit)					Raw material ($/unit)									
	1	2	3	4	5	1	2	3	4	5	6	7	8	9	10
1	6.5	8.5	10.5	12.5	14.5	5.5	5.5	5.5	5.5	5.5	6.5	6.5	6.5	7.5	7.5
2	9.5	11.3	13.5	15.5	17.5	6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.5
3	10.5	10.5	10.5	10.5	10.5	6.5	7.5	8.5	9.5	9.5	10.3	10.5	8.5	8.5	7.5

Table 3: Labor cost (10$/manpower)

site	cost of appoint of worker type λ (10$/manpower)					cost of retired of worker type λ (10$/manpower)					salary of worker type λ (10$/manpower)				
	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
1	5.5	5.5	5.5	5.5	5.5	8.5	9.5	10.5	11.3	12.5	19.5	21.5	23.5	25.5	27.5
2	5.5	6.5	7.5	8.5	9.5	9.5	9.5	10.5	11.3	13.5	22.5	24.5	26.5	29.3	31.3
3	5.5	5.5	6.5	6.5	6.5	5.1	6.1	6.5	6.8	10.5	17.5	18.5	21.5	22.6	24.7

Table 4: Market demands for region (1)

commodity	Period (t)											
	1	2	3	4	5	6	7	8	9	10	11	12
1	115	288	403	345	115	230	287	0	115	173	115	115
2	230	287	345	403	230	230	230	403	460	516	575	403
3	173	230	287	345	115	58	0	115	230	287	345	460
4	287	115	345	287	230	115	230	345	460	460	460	345
5	173	230	230	460	345	403	115	115	173	115	115	115

Table 5: Market demands for region (2)

commodity	Period (t)											
	1	2	3	4	5	6	7	8	9	10	11	12
1	217	403	598	678	138	368	437	230	207	217	150	127
2	322	380	368	654	426	380	334	794	771	748	1093	495
3	242	426	564	460	173	81	115	184	380	437	460	713
4	345	207	426	472	357	150	311	529	886	897	598	678
5	334	460	253	793	483	437	191	217	217	138	196	161

Table 6: Market demands for region (3)

commodity	Period (t)											
	1	2	3	4	5	6	7	8	9	10	11	12
1	103	218	35	92	46	345	161	115	150	58	69	23
2	69	287	610	161	173	92	184	218	380	334	644	518
3	103	81	161	460	12	69	92	115	184	299	230	702
4	218	150	265	46	184	23	115	207	621	587	345	23
5	92	196	173	334	322	345	92	23	276	58	138	127

8. Improvement and extension of model (Proposal case)

Assuming that the decision-maker has the desire to prefer and give importance to the manufacturing site from another site to the three locations, here, in this case, the principle of Analytic Hierarchy Process in paragraph (2.3) will be taken advantage of and developed towards that the degree of pairwise comparison and importance between the sites is a fuzzy nature as follows:

1- In the beginning, calculate the eigenvector $\mathrm{Y} = (v_1, v_2 \ldots, v_m)$ which is identical to supreme eigenvalue $\alpha(\Theta)$ of matrix Θ. The value of vector $\mathrm{Y} \geq 0$, and make as levels of membership of the components of matrix Θ to fuzzy set.

Table 7: Market demands for region (4)

commodity	Period (t)											
	1	2	3	4	5	6	7	8	9	10	11	12
1	196	667	863	1012	334	403	644	0	265	356	288	380
2	529	713	540	817	782	621	656	1058	954	759	1449	932
3	230	575	345	932	184	103	0	161	713	621	632	978
4	817	276	610	932	713	207	299	598	1127	529	932	817
5	460	357	564	690	725	1276	368	230	195	207	287	218

Table 8: Cost of raw material M supplied by provider P ($)

Provider	Raw materials									
	1	2	3	4	5	6	7	8	9	10
1	1.15	2.3	1.15	3.45	2.3	1.15	2.3	1.15	2.3	1.15
2	1.15	2.3	1.15	3.45	2.3	1.15	2.3	1.15	2.3	1.15
3	1.73	1.15	1.15	2.3	1.67	2.3	1.67	1.15	1.67	1.15
4	1.73	1.73	1.15	2.3	2.3	1.15	1.15	1.15	1.67	2.3

Table 9: Shortage cost($/period,unit), sales price($/unit)

Region	Shortage cost of commodity ($/period,unit)					Selling price of commodity($/unit)				
	1	2	3	4	5	1	2	3	4	5
1	2.37	2.37	2.37	3.37	1.37	27.83	39.83	48.83	30.83	35.83
2	3.37	4.37	4.37	4.37	2.37	32.83	42.83	52.83	32.83	37.83
3	2.37	2.37	2.37	2.37	2.37	28.83	39.83	47.83	31.83	35.83
4	2.37	2.37	3.37	2.37	2.37	27.83	40.83	50.83	32.83	37.83

Table 10: production cost ($/min)

	location1	location 2	location 3
Regular time	0.575	0.625	0.475
Over time	0.975	0.75	1.075
Subcontruct	1.325	1.375	1.275

Table 11: transportation cost ($/unit)

location	Cost of shipping form provider to location ($/unit)				Cost of shipping form location to region ($/unit)			
	1	2	3	4	1	2	3	4
1	0.019	0.037	0.102	0.13	0.047	0.075	0.093	0.084
2	0.037	0.019	0.14	0.113	0.06	0.056	0.10	0.047
3	0.168	0.186	0.065	0.093	0.121	0.149	0.093	0.195

2- By comparing the relative importance of locations in the model that determine the space of results of the fuzzy linear programming model, consider the importance of location. Let specify matrix Θ, matrix of paired comparison, we must find an eigenvector Y = $(v_1, v_2 \ldots, v_m)$, for which the state $\|\varpi_{ij}\| * (v_1, v_2 \ldots, v_m) = \alpha^* (v_1, v_2 \ldots, v_m)$ where $\Theta = \|\sigma_{ij}\|$, σ_{ij} represented the scale of influence of component φ_i in compare with component φ_j in matrix Θ, where α denoted to an eigenvalue. Once more calculate $(\Theta - \alpha \coprod)\mathrm{Y} = 0$, where U denoted to the identity matrix $, \alpha = [0, 1], [1]$.

3- Here, when the decision-maker, consider location 1, 2 strongly influences to location 3, the supplement of the mathematical model as follows:

$$(\Theta - \alpha \text{ U })Y = \begin{pmatrix} 1 & 1 & 5 \\ 1 & 1 & 5 \\ 1/5 & 1/5 & 1 \end{pmatrix} - \alpha \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

$$= \begin{pmatrix} 1-\alpha & 1 & 5 \\ 1 & 1-\alpha & 5 \\ 1/5 & 1/5 & 1-\alpha \end{pmatrix} \text{, with calculating the value of } \alpha, \alpha = (0, 0, 3)$$

$$= \begin{pmatrix} -2 & 1 & 5 \\ 1 & -2 & 5 \\ 1/5 & 1/5 & -2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

4- Finally, reformulate the mathematical model with add the new constraints and put the parameter α in the objective function to maximize its value. $0.455 \le v_1, 0.455 \le v_2, 0.091 \le v_3, v_1 \ge \alpha, v_2 \ge \alpha, , v_3 \ge \alpha$

9. Solving the model

The proposed mathematical model was solved by using LINGO software [7], and the results as tables below:

Table 12: production plan of each product in location (1)

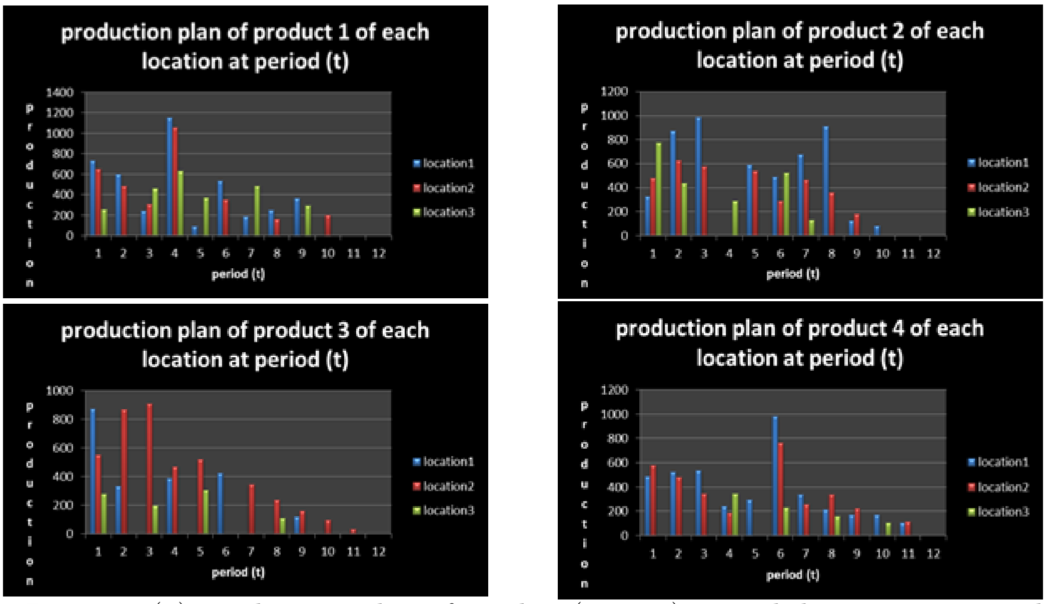
Product	method of production	Period											
(i)	β	1	2	3	4	5	6	7	8	9	10	11	12
1	1	734	601	243	1154	90	540	190	248	365	0	0	0
2	1	325	869	984	0	590	488	674	908	124	80	0	0
3	1	875	335	0	389	0	425	0	0	120	0	0	0
4	2	489	328	538	0	296	284	339	215	170	169	108	0
5	1	945	382	779	885	0	653	251	119	86	0	32	0

Table 13: production plan of each product in location (2)

Product	method of production	Period (t)											
(i)	β	1	2	3	4	5	6	7	8	9	10	11	12
1	1	650	488	310	1056	0	352	0	163	0	200	0	0
2	1	480	625	575	0	540	290	465	358	178	0	0	0
3	1	550	869	910	470	521	0	348	240	159	98	35	0
4	1	992	482	345	890	0	763	259	341	224	0	115	0
5	3	675	135	0	309	0	325	0	0	120	0	0	0

Table 14: production plan of each product in location (3)

Product	method of production	Period											
(i)	β	1	2	3	4	5	6	7	8	9	10	11	12
1	1	258	0	462	632	372	0	485	0	295	0	0	0
2	1	775	435	0	289	0	525	128	0	0	0	0	0
3	1	279	0	196	0	305	0	0	110	0	0	0	0
4	2	0	0	0	346	0	230	0	160	0	105	0	0
5	1	245	0	98	0	124	0	0	0	84	0	0	0



Digrams (1) production plan of product(1,2,3,4) in each location at period(t)

Table 15: workforce plan of each worker in location (1)

worker type λ	Period											
	1		2		3		4		5		6	
state	W	N	W	N	W	N	W	N	W	N	W	N
1	7	1	3	2	4	0	2	0	3	0	2	1
2	5	0	3	0	4	2	3	0	1	1	1	0
3	4	2	8	1	11	1		0	3	3	26	0
4	6	1	18	4	21	0	28	1	31	1	2	2
5	12	3	15	1	17	3	20	1	19	0	2	1

worker type λ	Period											
	7		8		9		10		11		12	
state	W	N	W	N	W	N	W	N	W	N	W	N
1	4	0	5	1	7	2	0	2	0	0	0	0
2	2	1	3	0	6	1	8	1	0	0	0	0
3	2	4	13	0	0	1	3	7	2	0	0	0
4	3	2	11	2	8	6	9	2	0	0	0	0
5	8	1	7	5	13	0	22	1	1	0	0	0

Table 16: workforce plan of each worker in location (2)

worker type λ	Period											
	1		2		3		4		5		6	
state	W	N	W	N	W	N	W	N	W	N	W	N
1	5	5	1	2	0	1	4	0	8	6	1	1
2	4	0	2	0	5	2	3	1	7	9	1	0
3	7	1	6	1	7	0	0	2	3	8	5	0
4	2	13	12	3	12	0	20	0	11	0	0	2
5	10	1	9	2	17	1	14	3	8	1	0	1

worker type λ	Period											
	7		8		9		10		11		12	
state	W	N	W	N	W	N	W	N	W	N	W	N
1	4	0	5	1	7	2	0	2	0	0	0	0
2	2	1	3	1	3	2	1	3	2	1	0	0
3	2	4	13	0	3	0	0	3	0	0	1	0
4	3	2	11	2	8	1	2	2	1	2	0	1
5	8	1	7	1	8	4	1	5	4	1	0	0

Table 17: workforce plan of each worker in location (3)

worker type λ	Period											
	1		2		3		4		5		6	
state	W	N	W	N	W	N	W	N	W	N	W	N
1	2	1	4	0	3	1	7	9	1	2	1	0
2	1	0	2	1	0	2	3	8	5	0	0	1
3	26	0	2	4	20	0	11	0	0	1	2	0
4	2	2	3	2	14	3	8	1	0	4	1	0
5	2	1	8	1	3	1	7	9	1	2	1	0
worker type λ	Period											
	7		8		9		10		11		12	
state	W	N	W	N	W	N	W	N	W	N	W	N
1	0	2	0	5	2	1	0	0	0	1	7	1
2	0	0	1	7	0	0	1	0	5	2	5	0
3	1	1	3	12	0	2	0	1	7	0	4	2
4	0	4	2	17	1	1	0	0	12	0	6	1
5	0	2	0	5	2	1	0	0	17	1	12	3

Table 18: The avearage of raw materials from provider to location, and commodity to regions of demands

Provider	Loc.1	Loc.2	Loc.3	location	Reg.1	Reg.2	Reg.3	Reg.4
1	1089	712	1446	Loc.1	205	315	187	126
2	203	2608	1277	Loc.2	67	152	0	496
3	1715	1387	1008	Loc.3	156	219	104	305
4	658	2119	486					

10. Conclosions

In this paper, a case study is presented of data which is unstable nature in a fuzzy environment, when there is fluctuation in the parameters of the mathematical model of the supply chain, demands of sales centers, costs of transportation, costs of production, and holding of raw materials and final products, labor costs, shortage costs multi-period. A proposal improved the model that the decision-maker has a desire to prefer one manufacturing location over another, as the proposal relied on developing a pairwise comparison in the Analytic Hierarchy Process (AHP). The study results indicate that the proposed model can be applied not only in the supply chain but also by using it in other fields and studies that require a comparison between two or more variables under a fuzzy environment.

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Table 1
Production time- raw material and end product inventory cost.

Site j	Scenario ξ	End product inventory holding cost ($/unit period)					Raw material inventory holding cost ($/unit period)										Production time (min)				
		Product i					Raw material m										Product i				
		1	2	3	4	5	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5
1	1	5	7	9	11	13	4	4	4	4	4	5	5	5	6	6	35	48	40	45	62
	2	6	8	10	12	14	5	5	5	5	5	6	6	6	7	7					
	3	7	9	11	13	15	6	6	6	6	6	7	7	7	8	8					
	4	8	10	12	14	16	7	7	7	7	7	8	8	8	9	9					
2	1	8	10	12	14	16	5	5	5	5	5	5	5	5	5	5	36	55	45	35	72
	2	9	11	13	15	17	6	6	6	6	6	6	6	6	6	6					
	3	10	12	14	16	18	7	7	7	7	7	7	7	7	7	7					
	4	11	13	15	17	19	8	8	8	8	8	8	8	8	8	8					
3	1	9	9	9	9	9	5	6	7	8	8	9	9	7	7	6	33	45	37	47	82
	2	10	10	10	10	10	6	7	8	9	9	10	10	8	8	7					
	3	11	11	11	11	11	7	8	9	10	10	11	11	9	9	8					
	4	12	12	12	12	12	8	9	10	11	11	12	12	10	10	9					
		Initial end product inventory							Initial raw material inventory												
		Product i							Raw material m												
		1	2	3	4	5	1	2	3	4	5	6	7	8	9	10					
1		2	1	5	10	2	10	15	20	12	15	20	20	20	15	20					
2		2	2	0	0	1	20	20	20	0	0	15	15	15	0	0					
3		0	0	20	1	10	10	0	0	10	0	0	0	10	0	20					

Table 2
Labor cost.

Site j	Scenario ζ	Hiring cost (10 $/manpower)					Firing cost (10 $/manpower)					Salary cost (10 $/manpower)				
		Worker type k					Worker type k					Worker type k				
		1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
1	1	4	4	4	4	4	7	8	9	10	11	18	20	22	24	26
	2	5	5	5	5	5	8	9	10	11	12	19	21	23	25	27
	3	6	6	6	6	6	9	10	11	12	13	20	22	24	26	28
	4	7	7	7	7	7	10	11	12	13	14	21	23	25	27	29
2	1	4	5	6	7	8	8	8	9	10	12	21	23	25	27	29
	2	5	6	7	8	9	9	9	10	11	13	22	24	26	28	30
	3	6	7	8	9	10	10	10	11	12	14	23	25	27	30	32
	4	7	8	9	10	11	11	11	12	13	15	24	26	28	32	34
3	1	4	4	5	5	5	5	6	7	8	9	16	17	19	20	22
	2	5	5	6	6	6	4	5	5	5	10	17	18	21	22	24
	3	6	6	7	7	7	5	6	6	6	11	18	19	22	24	26
	4	7	7	8	8	8	6	7	7	7	12	19	20	24	25	27

Table 3
Training cost at site 1 (10 $/manpower).

Scenario ζ	Worker type k	Worker type k				
		1	2	3	4	5
1	1	-	10	15	20	25
	2	-	-	10	15	20
	3	-	-	-	10	15
	4	-	-	-	-	10
2	1	-	11	16	21	26
	2	-	-	11	16	21
	3	-	-	-	11	16
	4	-	-	-	-	11
3	1	-	12	17	22	27
	2	-	-	12	17	22
	3	-	-	-	12	17
	4	-	-	-	-	12
4	1	-	13	18	23	28
	2	-	-	13	18	23
	3	-	-	-	13	18
	4	-	-	-	-	13
Workers' productivity		0.65	0.70	0.75	0.85	0.95

Table 4
Market demands for Scenario 1.

Customer's zone c	Product i	Period t											
		1	2	3	4	5	6	7	8	9	10	11	12
1	1	100	250	350	300	100	200	250	0	100	150	100	100
	2	200	250	300	350	200	200	200	350	400	450	500	350
	3	150	200	250	300	100	50	0	100	200	250	300	400
	4	250	100	300	250	200	100	200	300	400	400	400	300
	5	150	200	200	400	300	350	100	100	150	100	100	100
2	1	190	350	540	590	120	320	380	200	180	190	130	110
	2	280	330	320	570	370	330	290	690	670	650	950	430
	3	210	370	490	400	150	70	100	160	330	380	400	620
	4	300	180	370	410	310	130	270	460	770	780	520	590
	5	290	400	220	690	420	380	170	190	190	120	170	140
3	1	90	190	30	80	40	300	140	100	130	50	60	20
	2	60	250	530	140	150	80	160	190	330	290	560	450
	3	90	70	140	400	10	60	80	100	160	260	200	610
	4	190	130	230	40	160	20	100	180	540	510	300	20
	5	80	170	150	290	280	300	80	20	240	50	120	110
4	1	170	580	750	880	290	350	560	0	230	310	250	330
	2	460	620	470	710	680	540	570	920	830	660	1260	810
	3	200	500	300	830	160	90	0	140	620	540	550	850
	4	710	240	530	810	620	180	260	520	980	460	810	710
	5	400	310	490	600	630	1110	320	200	170	180	250	190

For scenarios 2, 3 and 4, the estimations are multiplied by 1.1, 1.2 and 1.3, respectively.

Table 5
Sites' data.

Site j	Storage capacity		Initial workforce					Workforce change rate
	Raw material m	End product i	Worker type k					
			1	2	3	4	5	
1	10000	15000	6	6	6	6	6	0.2
2	12000	10000	5	10	15	5	10	0.2
3	10000	10000	10	20	20	0	0	0.2

Table 6
Available time.

Period t	Regular time (hour/period)			Overtime (hour/period)			Subcontracting (hour/period)		
	Site 1	Site 2	Site 3	Site 1	Site 2	Site 3	Site 1	Site 2	Site 3
1	144	144	144	50	50	50	200	200	200
2	160	160	160	50	50	50	220	220	220
3	168	168	168	50	50	50	230	230	230
4	176	176	176	60	60	60	240	240	240
5	120	200	200	40	60	60	170	280	280
6	192	120	120	60	40	40	270	170	170
7	200	200	200	60	60	60	280	280	280
8	200	200	200	60	60	60	280	280	280
9	192	192	200	60	60	60	270	270	280
10	176	176	176	60	60	60	240	240	240
11	184	184	184	60	60	60	260	260	260
12	160	152	152	50	50	50	220	210	210
Production cost ($/min) in scenario ζ									
1	0.5	0.55	0.4	0.9	0.95	1	1.25	1.3	1.20
2	0.55	0.60	0.45	0.95	0.1	1.05	1.30	1.35	1.25
3	0.6	0.65	0.50	1	1.05	1.10	1.35	1.40	1.30
4	0.65	0.70	0.55	1.05	1.10	1.15	1.40	1.45	1.35

Table 7
Consumption rate.

Product i	Raw material m									
	1	2	3	4	5	6	7	8	9	10
1	2	3	0	4	0	0	1	2	3	0
2	2	3	1	2	2	2	0	0	0	0
3	1	0	1	2	0	0	1	0	0	2
4	0	0	0	0	2	3	2	3	2	3
5	0	1	2	0	1	0	0	0	1	2

Table 8
Transportation cost ($/unit).

Scenario ξ	Site j	Supplier s				Customer's zone c			
		1	2	3	4	1	2	3	4
1	1	0.014	0.029	0.079	0.101	0.036	0.058	0.072	0.065
	2	0.029	0.014	0.108	0.086	0.065	0.043	0.086	0.036
	3	0.13	0.144	0.05	0.072	0.094	0.115	0.072	0.151
2	1	0.016	0.032	0.088	0.112	0.04	0.064	0.08	0.072
	2	0.032	0.016	0.12	0.096	0.072	0.048	0.096	0.04
	3	0.144	0.16	0.056	0.08	0.104	0.128	0.08	0.168
3	1	0.02	0.04	0.11	0.14	0.05	0.08	0.10	0.09
	2	0.04	0.02	0.15	0.12	0.09	0.06	0.12	0.05
	3	0.18	0.20	0.07	0.10	0.13	0.16	0.10	0.21
4	1	0.024	0.048	0.132	0.168	0.06	0.096	0.12	0.108
	2	0.048	0.024	0.18	0.144	0.108	0.072	0.144	0.06
	3	0.216	0.24	0.084	0.12	0.156	0.192	0.12	0.252

Table 9
Lead time (period).

Site j	Supplier s				Customer's zone c			
	1	2	3	4	1	2	3	4
1	0	0	1	2	0	0	0	1
2	0	0	2	1	1	0	0	0
3	2	3	0	1	0	0	0	1

Table 10
Cost and capacity of raw material m provided by supplier s in period 1 in scenario ξ

Scenario ξ	Supplier s	Purchasing cost ($) Raw material m									
		1	2	3	4	5	6	7	8	9	10
1	1	1	2	1	3	2	1	2	1	2	1
	2	1	2	1	3	2	1	2	1	2	1
	3	1.5	1	1	2	1.5	2	1.5	1	1.5	1
	4	1.5	1.5	1	2	2	1	1	1	1.5	2
2	1	1.1	2.2	1.1	3.3	2.2	1.1	2.2	1.1	2.2	1.1
	2	1.1	2.2	1.1	3.3	2.2	1.1	2.2	1.1	2.2	1.1
	3	1.65	1.1	1.1	2.2	1.65	2.2	1.65	1.1	1.65	1.1
	4	1.65	1.65	1.1	2.2	2.2	1.1	1.1	1.1	1.65	2.2
3	1	1.2	2.4	1.2	3.6	2.4	1.2	2.4	1.2	2.4	1.2
	2	1.2	2.4	1.2	3.6	2.4	1.2	2.4	1.2	2.4	1.2
	3	1.8	1.2	1.2	2.4	1.8	2.4	1.8	1.2	1.8	1.2
	4	1.8	1.8	1.2	2.4	2.4	1.2	1.2	1.2	1.8	2.4
4	1	1.3	2.6	1.3	3.9	2.6	1.3	2.6	1.3	2.6	1.3
	2	1.3	2.6	1.3	3.9	2.6	1.3	2.6	1.3	2.6	1.3
	3	1.95	1.3	1.3	2.6	1.95	2.6	1.95	1.3	1.95	1.3
	4	1.95	1.95	1.3	2.6	2.6	1.3	1.3	1.3	1.95	2.6

	Production capacity Raw material m									
Supplier s	1	2	3	4	5	6	7	8	9	10
1	3500	3500	3500	3500	3500	2500	4000	3500	3000	3500
2	3000	3000	3000	3500	3000	3000	3500	3500	3500	3500
3	3500	3000	4500	4000	4000	3500	3500	4500	3500	3000
4	3000	3500	3500	3000	3000	3500	3500	3500	3500	3500

It is assumed that their capacity remains fixed in all periods.

Table 11
Shortage cost, sales price.

Scenario ζ	Customer's zone c	Shortage cost ($/period, unit)					Sales price ($/unit)				
		Product i					Product i				
		1	2	3	4	5	1	2	3	4	5
1	1	2	2	2	3	1	25	37	46	28	33
	2	3	4	4	4	2	30	40	50	30	35
	3	2	2	2	2	2	26	37	45	29	33
	4	2	2	3	2	2	25	38	48	30	35
2	1	2.25	2.25	2.25	3.25	1.25	26	38	47	29	34
	2	3.25	4.25	4.25	4.25	2.25	31	41	51	31	36
	3	2.25	2.25	2.25	2.25	2.25	27	38	46	30	34
	4	2.25	2.25	3.25	2.25	2.25	26	39	49	31	36
3	1	2.5	2.5	2.5	3.5	1.5	26.5	38.5	47.5	29.5	34.5
	2	3.5	4.5	4.5	4.5	2.5	31.5	41.5	51.5	31.5	36.5
	3	2.5	2.5	2.5	2.5	2.5	27.5	38.5	46.5	30.5	34.5
	4	2.5	2.5	3.5	2.5	2.5	26.5	39.5	49.5	31.5	36.5
4	1	2.75	2.75	2.75	3.75	1.75	37	49	58	40	45
	2	3.75	4.75	4.75	4.75	2.75	42	52	62	42	47
	3	2.75	2.75	2.75	2.75	2.75	38	49	57	41	45
	4	2.75	2.75	3.75	2.75	2.75	37	50	60	42	47