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Estimation of the survival function based on the Log-Logistic distribution

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Abstract

This paper proposes a new method by hybrid Simplex Downhill Algorithm with Moment Method (SMOM) to estimate the parameters of Log-Logistic distribution based on Survival functions. Simulation is used to compare the suggested methods with two classical methods (Maximum Likelihood Estimator and with Moment Method). The results demonstrate that SMOM was efficient than the maximum likelihood Estimator and Moment method based on Mean Square Error (MSE).

Keywords: hybrid Simplex Downhill Algorithm, Log-Logistic distribution, Mean Square Error

1. Introduction

Recently, Survival Analysis (SA) is one of the widely used techniques in medical statistics, physics, medicine, epidemiology engineering, economics, biology, and public health [9, 4]. Estimating survival functions has interested statisticians for numerous years. Chung et al. [5] described the statistical methods of survival analysis and their implementation in criminology for predicting the time until recidivism. Recently, Cruz and Wishart [2] and Kourou et al. [3] discussed applications in cancer prediction and used several survival analysis methods.

The log-logistic distribution has its own standing as a life testing model, it is viewed as a weighted exponential distribution. Due to the importance of this distribution in reliability, it has been used to estimate the estimators to find parameters. This distribution and for the adoption process to assess the estimators of those two parameters have been estimated the survival of this distribution.

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The probability density function (p.d.f) and the cumulative distribution function (c.d.f) of Log Logistic distribution are expressed, respectively as:

$$f(x,a,\beta)=\frac{\frac{\beta}{a}\left(\frac{x}{a}\right)^{\beta-1}}{\left(1+\left(\frac{x}{a}\right)^{\beta}\right)^{2}} \tag{1.1}$$

$$F\left(x,a,\beta\right)=\frac{1}{1+\left(\frac{x}{a}\right)^{-\beta}}, \tag{1.2}$$

where x is a value of random variable, α and β are scale and shape parameters, respectively, and $\alpha,\beta>0$. The survival function $S(x)$ for the Log-Logistic distribution as follows:

$$S\left(x\right)=1-\ F\left(x,\alpha,\beta\right)=1-\ \frac{1}{1+\left(\frac{x}{\alpha}\right)^{-\beta}}\ =\ \frac{1}{1+\left(\frac{x}{a}\right)^{\beta}} \tag{1.3}$$

The nonlinearity model of log-logistic distribution makes the estimation of parameter and the statistical analysis of parameter estimates more difficult and challenging [1]. Therefore, simplex downhill algorithm (SDA) was adopted to estimate the parameters of Log-Logistic distribution based on Survival functions. Since, SDA was a good choice for many practitioners in the fields of physical, statistics, medical sciences, and engineering. Since, it is very easy to use and code [11, 12, 13].

The organized paper as follows: Section 2 offers some information of Log Logistic distribution. Section 3. clarifying Maximum Likelihood Estimation method. Section 4. clarifying Moments Method. Section 5. Describe the proposed (SMOM). Section 6. Simulation study. Section 6, demonstrates the effectiveness of the proposed method through numerical results. Finally, in Section 6, a conclusion is provided.

2. Maximum Likelihood Estimation Method (MLE)

Let $x_1,x_2,\ldots,x_n$ be order random sample of sized (n) from a distribution with $p.d.f$ $\mathrm{f}\left(x,a,\beta\right)$ such that (a,β) are the parameters, then the likelihood function $L\left(a,\beta\right)$ is the joint $p.d.f$ of the random samples is [7, 8, 10]:

$$L\left(x_1,x_2,\ldots,x_n,a,\beta\right)=\ \prod_{i=1}^{n}f(x_i)=\prod_{i=1}^{n}\left(\frac{\frac{\beta}{a}\left(\frac{x_i}{a}\right)^{\beta-1}}{\left(1+\left(\frac{x_i}{a}\right)^{\beta}\right)^{2}}\right)=\frac{\beta^n}{a^n}\prod_{i=1}^{n}\left(\frac{\left(\frac{x_i}{a}\right)^{\beta-1}}{\left(1+\left(\frac{x_i}{a}\right)^{\beta}\right)^{2}}\right) \tag{2.1}$$

Taking the natural logarithm for both sides:

$$\mathrm{Ln}L\left(x_i,a,\beta\right)=n\mathrm{Ln}\beta-n\mathrm{Ln}\alpha+\ \beta\sum_{i=1}^{n}\mathrm{Ln}x_i-\ \beta\sum_{i=1}^{n}\mathrm{Ln}\alpha-\sum_{i=1}^{n}\mathrm{Ln}\ x_i+\sum_{i=1}^{n}\mathrm{Ln}\alpha-2\sum_{i=1}^{n}\mathrm{Ln}\left(1+\left(\frac{x_i}{a}\right)^{\beta}\right)$$

The partial derivative for the equation (2.1) with respect to the unknown parameters (α,β) (α,β), respectively:

$$\frac{\partial\ \mathrm{Ln}L\ \left(x_i,a,\beta\right)}{\partial a}=\frac{2\beta}{\alpha}\ \sum_{i=1}^{n}\frac{\left(\frac{\mathrm{xi})}{\alpha}\right)^{\beta}}{1+\left(\frac{x_i}{a}\right)^{\beta}}-\frac{n\beta}{\alpha}$$
$$2\sum_{i=1}^{n}\frac{\left(\frac{x_i}{\alpha}\right)^{\beta}}{1+\left(\frac{x_i}{\alpha}\right)^{\beta}}-n=0 \tag{2.2}$$

And,

$$\begin{aligned}\frac{\partial\ \mathrm{Ln}L\ (x_),a,\beta)}{\partial\beta} &= \frac{n}{\beta}+\ \sum_{i=1}^{n}\mathrm{Ln}\left(\frac{x_i}{\alpha}\right)-2\sum_{i=1}^{n}\frac{\left(\frac{x_i}{\alpha}\right)^{\beta}\mathrm{Ln}\left(\frac{x_i}{\alpha}\right)}{1+\ \left(\frac{x_i}{\alpha}\right)^{\beta}}\\ \frac{n}{\beta}+\sum_{i=1}^{n}\mathrm{Ln}\left(\frac{x_i}{\alpha}\right)-2\sum_{i=1}^{n}\frac{\mathrm{Ln}\ \left(\frac{x_i}{\alpha}\right)}{1+\left(\frac{x_i}{a}\right)^{-\beta}} &= 0\end{aligned} \tag{2.3}$$

Since the two-nonlinear equations are complicated to be solved, Newton-Raphson method was used to estimate the parameters (α,β). From equation (2.2), Let

$$f\left(\alpha,\beta\right)=2\sum_{i=1}^{n}\frac{\left(\frac{x_i}{\alpha}\right)^{\beta}}{1+\left(\frac{x_i}{\alpha}\right)^{\beta}}-n \tag{2.4}$$

And,

$$g\left(a,\beta\right)=\frac{n}{\beta}+\sum_{i=1}^{n}\mathrm{Ln}\left(\frac{x_i}{\alpha}\right)-2\sum_{i=1}^{n}\frac{\mathrm{Ln}\ \left(\frac{x_i}{\alpha}\right)}{1+\left(\frac{x_i}{a}\right)^{-\beta}} \tag{2.5}$$

Now, we find the formulas of Jacobean matrix as follows:

$$J=\begin{vmatrix}\frac{-2\beta}{\alpha}\ \sum_{i=1}^{n}\frac{1}{\left(1+\left(\frac{x_i}{\alpha}\right)^{-\beta}\right)\left(1+\left(\frac{x_i}{\alpha}\right)^{\beta}\right)} & \frac{2\beta}{\alpha}\sum_{i=1}^{n}\frac{\mathrm{Ln}\left(\frac{x}{\alpha}\right)}{\left(1+\left(\frac{x_i}{\alpha}\right)^{-\beta}\right)\left(1+\left(\frac{x_i}{\alpha}\right)^{\beta}\right)}\\ \frac{n}{\alpha}+\frac{2}{\alpha}\sum_{i=1}^{n}\frac{1}{1+\left(\frac{x_i}{\alpha}\right)^{-\beta}}+\frac{2\beta}{\alpha}\ \sum_{i=1}^{n}\frac{\mathrm{Ln}\left(\frac{x_i}{\alpha}\right)}{\left(1+\left(\frac{x_i}{\alpha}\right)^{-\beta}\right)\left(1+\left(\frac{x_i}{\alpha}\right)^{\beta}\right)} & \frac{n}{\beta^2}-2\sum_{i=1}^{n}\frac{\mathrm{Ln}\left(\frac{x_i}{\alpha}\right)^2}{\left(1+\left(\frac{x_i}{\alpha}\right)^{-\beta}\right)\left(1+\left(\frac{x_i}{\alpha}\right)^{\beta}\right)}\end{vmatrix}$$

Thus, the following equations matrixes are applied to estimate the parameters for Log-Logistic distribution by using Newton-Raphson method.

$$\left(\frac{\widehat{\alpha}_{\mathrm{MLE}}}{\widehat{\beta}_{\mathrm{MLE}}}\right)=\left(\frac{\alpha_0}{\beta_0}\right)-j^{-1}\left(\frac{2\sum_{i=1}^{n}\frac{\left(\frac{x_i}{\alpha}\right)^{\beta}}{1+\left(\frac{x_i}{\alpha}\right)^{\beta}}-n}{\frac{n}{\beta}+\sum_{i=1}^{n}\mathrm{Ln}\left(\frac{x_i}{\alpha}\right)-2\sum_{i=1}^{n}\frac{\mathrm{Ln}\ \left(\frac{x_i}{\alpha}\right)}{1+\left(\frac{x_i}{a}\right)^{-\beta}}}\right)$$

$$v_1=\frac{\partial f\left(\alpha,\beta\right)}{\partial\alpha}=-2\frac{\beta}{\alpha}\ \sum_{i=1}^{n}\frac{1}{\left(1+\left(\frac{x_i}{\alpha}\right)^{-\beta}\right)\left(1+\left(\frac{x_i}{\alpha}\right)^{\beta}\right)}$$

$$v_2=\frac{\partial f\left(\alpha,\beta\right)}{\partial\beta}=2\frac{\beta}{\alpha}\ \sum_{i=1}^{n}\frac{\mathrm{Ln}\left(\frac{x_i}{\alpha}\right)}{\left(1+\left(\frac{x_i}{\alpha}\right)^{-\beta}\right)\left(1+\left(\frac{x_i}{\alpha}\right)^{\beta}\right)}$$

$$v_3=\ f\left(\alpha,\beta\right)=\ \ 2\sum_{i=1}^{n}\frac{\left(\frac{x_i}{\alpha}\right)^{\beta}}{1+\left(\frac{x_i}{\alpha}\right)^{\beta}}-n$$

$$v_4=\frac{\partial g\left(\alpha,\beta\right)}{\partial\alpha}=\ \frac{n}{\alpha}+\frac{2}{\alpha}\sum_{i=1}^{n}\frac{1}{1+\left(\frac{x_i}{\alpha}\right)^{-\beta}}+\frac{2\beta}{\alpha}\sum_{i=1}^{n}\frac{\mathrm{Ln}\left(\frac{x_i}{\alpha}\right)}{\left(1+\left(\frac{x_i}{\alpha}\right)^{-\beta}\right)\left(1+\left(\frac{x_i}{\alpha}\right)^{\beta}\right)}$$

$$v_5=\frac{\partial g\left(\alpha,\beta\right)}{\partial\beta}=\frac{n}{\beta^2}-2\sum_{i=1}^{n}\frac{\mathrm{Ln}\left(\frac{x_i}{\alpha}\right)^2}{\left(1+\left(\frac{x_i}{\alpha}\right)^{-\beta}\right)\left(1+\left(\frac{x_i}{\alpha}\right)^{\beta}\right)}$$

$$v_6=\ g(\alpha,\beta)=\frac{n}{\beta}+\sum_{i=1}^{n}\mathrm{Ln}\left(\frac{x_i}{\alpha}\right)-2\sum_{i=1}^{n}\frac{\mathrm{Ln}\ \left(\frac{x_i}{\alpha}\right)}{1+\left(\frac{x_i}{a}\right)^{-\beta}}$$

$$\widehat{\alpha}_{\mathrm{MLE}} = \alpha_0 + h \tag{2.6}$$

$$\widehat{\beta}_{\mathrm{MLE}} = \beta_0 + k \tag{2.7}$$

Where: $h = \frac{v_4 v_6 - v_3 v_5}{v_1 v_5 - v_2 v_4}$, and $k = \frac{-v_3 - v_1 h_1}{v_4}$. So, to estimator the survival analyses $\widehat{S}(x)$, we substitute equations (2.6) and (2.7) in equation (1.3).

$$\widehat{S}\,(x)\,MLE = \frac{1}{1 + (\frac{x}{\widehat{\alpha}_{\mathrm{MLE}}})^{\widehat{\beta}_{\mathrm{MLE}}}} \tag{2.8}$$

3. Moments Method (MOM)

In this section, Moment estimation method will be used to estimate the parameters α, and β for LOG LGD.

$$E\left(x^j\right) = \frac{1}{n}\sum_{i=1}^{n} x_i^j, \qquad \text{where } \ j = 1, 2, \ldots$$

The first and second moment of population and sample for two parameters of Log-Logistic distribution are given, respectively:

$$\begin{aligned} E\,(x) &= \frac{\alpha\pi}{\beta \sin\left(\frac{\pi}{\beta_0}\right)} \\ M1 &= E\,(x) \\ \overline{X} &= \frac{\propto \pi}{\beta \sin\left(\frac{\pi}{\beta_0}\right)} \Longrightarrow \alpha = \frac{\overline{X}\beta \sin\left(\frac{\pi}{\beta_0}\right)}{\pi} \\ E\left(x^2\right) &= \frac{2 \propto^2 \pi}{\beta \sin\left(\frac{2\pi}{\beta_0}\right)} \end{aligned} \tag{3.1}$$

And the second moment of population and sample were

$$\begin{aligned} M2 &= \frac{1}{n}\sum_{i=1}^{n} x_i^2 = \overline{x^2} \\ M2 &= E\left(x^2\right) \\ \overline{x^2} &= \frac{2\alpha^2\pi}{\beta \sin\left(\frac{2\pi}{\beta_0}\right)} \Longrightarrow \beta = \frac{2\alpha^2\pi}{\overline{x^2} \sin\left(\frac{\pi}{\beta_0}\right)} \end{aligned} \tag{3.2}$$

Then substitute equation (3.1) in to equation (3.2) to get

$$\widehat{\beta}_{\mathrm{Mom}} = \frac{\pi\overline{x^2} \sin\left(\frac{2\pi}{\beta_0}\right)}{2\overline{x}^2 \left(\sin\left(\frac{\pi}{\beta_0}\right)\right)^2} \tag{3.3}$$

and

$$\widehat{\alpha}_{\text{Mom}} = \frac{\overline{x^2}\sin\left(\frac{2\pi}{\beta_0}\right)}{2\overline{X}\ \left(\sin\left(\frac{\pi}{\beta_0}\right)\right)^2} \tag{3.4}$$

$$\widehat{S}\,(x)\,\text{Mom} = \frac{1}{1+(\frac{x}{\widehat{\alpha}_{\text{Mom}}})^{\widehat{\beta}_{\text{Mom}}}} \tag{3.5}$$

4. Simplex Downhill Algorithm and Moment Method (SMOM)

Simplex downhill algorithm (SDA) was introduced in 1962 [6]. Simplex downhill algorithm was a mathematical method that uses geometric relationships to aid in finding approximate solutions to solve complex and optimization problems. The idea of SDA generates generate $N + 1$ vertex in an N-dimensional space. then the vertices sorted by ascending order such us: $f(x_1) \leq f(x_2) \leq \ldots \leq f(x_n) \leq f(x_{n+1})$, where $f(x_{n+1})$ is worse solution and $f(x_1)$ best solution.
The objective function of the Simplex downhill algorithm based on moment method

$$M_1 = \overline{x}, \qquad \mu_1 = \frac{\alpha\pi}{\beta\sin\left(\frac{\pi}{\beta}\right)}, \qquad and\ \ M_2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2, \qquad \mu_2 = \frac{2\alpha^2\pi}{\beta\sin\left(\frac{2\pi}{\beta}\right)}$$
$$f(x) = (M_1-\mu_1)^2 + (M_2-\mu_2)^2 = \sqrt{\left(\overline{x} - \frac{\alpha\pi}{\beta\sin\left(\frac{\pi}{\beta}\right)}\right)^2 + \left(\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \frac{2\alpha^2\pi}{\beta\sin\left(\frac{2\pi}{\beta}\right)}\right)^2} \tag{4.1}$$

The algorithm iteration updates to improve the worst solution by four operations as follows:

Reflection step

compute the reflection point xr from $xr = m + \lambda\ (m - x_{n+1})$ then evaluate $f(xr,)$ where m is the centroid of the N best solution in the vertices of the simplex $m =\ mean\ (\ x\ (1\ :\ \text{n}))\ and\ \lambda\ =\ 1$. $If\ \ f\ (x_1) \leq\ f\ (xr) <\ f\ (xn)$, then put the worst solution in reflected point $x_{n+1} =\ xr$.

Expansion step

$If\ \ f(x_r)\ <\ f(x_1)$, then generate a new point x_e by expansion, from $x_e = x_r + \beta(xr\ -\ m)$, where $\beta = 2$.
- If $f(x_e) < f(x_r)$, then replace x_{n+1} with x_e.
- else x_{n+1}= x_r.

Contraction step

If $f\ (x_{n+1}) \leq\ f\ (xr)\ ,\ generate\ a\ new$ solution xc where $xc\ =\ m\ +\ \gamma\ (m\ -\ x_{n+1})$.
- If $f\ (xc) <\ f\ (xr)$, then $x_{n+1} =\ xc$
- else $x_{n+1} =\ xr$.
The step of shrinkage is used, if the three steps are fails in above.

Shrinkage Step

We keep the best one x_1 then generate the n new vertices by using $x_{sj} = x_1 + \sigma(x_{sj} - x_1), j = \{2 \ldots n+1\}$ and $\sigma = 0.5$. The next iteration consist of the simplex vertices as $x_1, xs_2, \ldots, xs_{n+1.}$

Simulation

The estimation performance of the proposed method is verified through simulation in this section. In addition, each simulation condition was generated by 1000 replications. The simulation program was written by Matlab 2016. For examining the effect of sample size, various sample sizes are tested: 15, 30, and 50. The simulation steps as follows;

Step 1: Generate random samples as $u_1, u_2, \ldots, u_n$, which are follows the continuous uniform distribution defined on the interval (0,1). Then transform it to a random samples follows Log-Logistic distribution using c.d.f. as follow

$$F\left(x, a, \beta\right) = \frac{1}{1 + (\frac{x_i}{a})^{-\beta}}, \quad U_i = \frac{1}{1 + (\frac{x_i}{a})^{-\beta}}, \; x_i = a(\frac{\text{Ui}}{1 - Ui})^{\frac{1}{\beta}}$$

Then, let G is a vector for all parameters required such as $G = (a, \beta)$ and generate $k+1$ solutions for X. where k is number of parameters required for output

Step 2: Recall the S from equation (1.3).

Step 3: Compute $\widehat{S}$ based on MLE and MOM using equations (3.3) and (3.4), and the best solution from (f) SMOM method.

Step 4: Based on $L = 1000$ trials, MSE will be calculated as follows

$$MSE = \frac{1}{L} \sum_{i=1}^{L} \left(\widehat{S}_i - \; S\right)^2$$

Result of Simulation

In order to verify the performance of the SMOM method to estimate the parameters, we made simulation by examining various samples sizes (15, 30, 50) . different values of parameters of (α, β) were considered as (1,0.5) and (0.5,1). Each table explained the result of estimate values of α and β, survival , estimate survival and MSE, respectively.

Tables 1 to 6 showed that the proposed method offered less MSE for estimate parameters based on survival function.

Table 1: $\widehat{S}$ and MSE values of S when $\alpha = 1$, $\beta = 0.5$ when $n = 15$.

S		*SMOM*	*MOM*	*MLE*
0.896786226914398	$\widehat{s}$	0.778372898745492	1	0.940666379911005
	MSE	0.010541611336268	0.015397508187505	0.009514130607448
0.877186432370071	$\widehat{s}$	0.761895628388226	1	0.925008569773291
	MSE	0.011942595236002	0.012347111099908	0.016125651512360
0.858139879071672	$\widehat{s}$	0.747281914382417	1	0.908848857030191
	MSE	0.012669069981870	0.009044961774433	0.024841883515893
0.789529820578519	$\widehat{s}$	0.722061329374219	0.999999999999928	0.843917230894206
	MSE	0.001087479333701	0.092377931164583	0.077561257912078
0.675374346644247	$\widehat{s}$	0.640386313944542	0.999999998586163	0.717613910863188
	MSE	0.002958159265159	0.015816071253952	0.235589519000635
0.638736622888212	$\widehat{s}$	0.622396400096761	0.999999978321794	0.673545145762828
	MSE	0.001306073974551	0.028655013963128	0.299079372337131
0.583248845123912	$\widehat{s}$	0.595961154454685	0.999998905094714	0.604648180722393
	MSE	0.000110004786635	0.052130205366718	0.399876831541004
0.444753369246002	$\widehat{s}$	0.530843498458440	0.987410508740043	0.428048028759794
	MSE	0.001117322057574	0.109643081751776	0.620457076237883
0.429663190289397	$\widehat{s}$	0.523579842995112	0.965548913497642	0.408934245370479
	MSE	0.001312774346608	0.114449277996574	0.608825330140380
0.310150997637136	$\widehat{s}$	0.462353945101117	0.004832065524912	0.263183020195887
	MSE	0.002104612652495	0.135531390563687	0.007988068911149
0.297278171477952	$\widehat{s}$	0.455171780063720	0.001745279530904	0.248344324858004
	MSE	0.002090563719131	0.135827360150627	0.007204421609963
0.204908886166737	$\widehat{s}$	0.397648675200812	4.27877435593693e-0	0.149032607359190
	MSE	0.001532061018727	0.125894126639783	0.001834633668412
0.204015732847010	$\widehat{s}$	0.397023929854419	3.90218390933228e-07	0.148142905013497
	MSE	0.001523816451807	0.125692063536203	0.001805635871731
0.183568418423319	$\widehat{s}$	0.382225910736243	4.33448293923888e-08	0.128197836944232
	MSE	0.001327123066748	0.120488438577123	0.001232811916653
0.166141368917058	$\widehat{s}$	0.368759974567788	5.70418268175388e-09	0.111868754329593
	MSE	0.001151280887343	0.115160293227685	0.000864784149090

Table 2: $\widehat{S}$ **and** MSE values of S when $\alpha = 1$, $\beta = 0.5$ when $n = 30$

Survival		*S/MOM*	*MOM*	*MLE*
0.940908555895633	$\widehat{s}$	0.928235115427575	1	0.958113455699663
	MSE	5.27574652714e-06	0.000860468717265	0.001800401429606
0.919938438773295	$\widehat{s}$	0.914417830133562	1	0.940249507930026
	MSE	1.63486082792e-05	0.000674093065635	0.003554409371478
0.918919533215964	$\widehat{s}$	0.913785295469396	1	0.939357425543417
	MSE	1.70979721742e-05	0.000662729097997	0.003656763510197
0.905950548178775	$\widehat{s}$	0.905981463919884	1	0.927827844243929
	MSE	2.85791437233e-05	0.000509543556017	0.005104454412001
0.892027239916652	$\widehat{s}$	0.898036619469861	1	0.915115289902766
	MSE	4.52810696232e-05	0.000342011971235	0.006967209992581
0.876971402288456	$\widehat{s}$	0.88984611199655	1	0.901017640808746
	MSE	6.89610425951e-05	0.0001801917488196	0.009356697887567
0.836902643547434	$\widehat{s}$	0.809513074480323	1.00000000000000	0.861969201491186
	MSE	0.012163419155447	0.42966523581163	0.017734517267106
0.819073018793369	$\widehat{s}$	0.860959866911548	1.00000000000000	0.843975764430408
	MSE	0.012820936354251	0.098117187323378	0.022459650849116
0.794412481004594	$\widehat{s}$	0.849481836349598	0.999999999999997	0.818552444975855
	MSE	0.000316280508958	0.000523057458895	0.030063611159331
0.582506645599609	$\widehat{s}$	0.756563322430523	0.999999999723945	0.583877503823740
	MSE	0.001605548039766	0.021756581803771	0.152832365069871
0.574775978220308	$\widehat{s}$	0.753126773958754	0.999999999604135	0.575019247365040
	MSE	0.001653331492771	0.023221049424516	0.152832365069871
0.544154342996376	$\widehat{s}$	0.739344893258942	0.999999998373761	0.539882661607562
	MSE	0.0018302175918694	0.029493390069106	0.159406767079218
0.524494284690380	$\widehat{s}$	0.730327304022940	0.999999996006571	0.517311322437432
	MSE	0.001930878819223	0.033902020320135	0.159406767079218
0.508673788986681	$\widehat{s}$	0.722956908860100	0.999999991793474	0.499159065091621
	MSE	0.002003034076088	0.037653572010577	0.186962411137779
0.481891033948011	$\widehat{s}$	0.710211386787635	0.999999972249386	0.468485261164484
	MSE	0.002104559752265	0.044387331099788	0.205932356821384
0.465423924766608	$\widehat{s}$	0.702184459322338	0.999999941194103	0.449681058327670
	MSE	0.002152707475428	0.048743544273682	0.221924248896568
0.462651436379068	$\widehat{s}$	0.700817302765123	0.999999933248606	0.446520389492648
	MSE	0.002159671742149	0.049491630706521	0.250471818807398
0.457910493661804	$\widehat{s}$	0.698468458781727	0.999999917074946	0.441119606415465
	MSE	0.002170795189589	0.050780048715625	0.268940653404630
0.347389731361929	$\widehat{s}$	0.638661735031731	0.999984201729249	0.317338550294499
	MSE	0.002124101080617	0.082744379431591	0.272118488501308
0.337764296942889	$\widehat{s}$	0.632865112443188	0.999974318043237	0.306819595370152
	MSE	0.002091354586246	0.085538914608837	0.277598053391400
0.317442770586342	$\widehat{s}$	0.620220336977525	0.999926683613906	0.284790578245603
	MSE	0.002007700394974	0.091320150597110	0.421169451975963
0.289417778463787	$\widehat{s}$	0.601755210318042	0.999668795764145	0.254848761944866
	MSE	0.001861796110376	0.098878602507986	0.435045210416623
0.280494042339499	$\widehat{s}$	0.595591977362752	0.999455181270977	0.245430989761713
	MSE	0.001808459563161	0.101144079344669	0.464985683993131
0.208290054601509	$\widehat{s}$	0.539122404735084	0.954514196043492	0.171649673581396
	MSE	0.001281639065944	0.115228325690275	0.507461424802151
0.205436724969994	$\widehat{s}$	0.536590587431351	0.945116245709014	0.168833260591681
	MSE	0.001258290196192	0.115569387769865	0.521132064205598
0.192045108886682	$\widehat{s}$	0.524319750128732	0.868726920861456	0.155728337867176
	MSE	0.00114733112043961	0.116872937801282	0.569790628903718
0.191475179701565	$\widehat{s}$	0.523782569130009	0.863885527202110	0.155174855781357
	MSE	0.00114256793896115	0.116916892868952	0.560223264430161
0.184672171855745	$\widehat{s}$	0.517269874345295	0.792723034345099	0.148595713064573
	MSE	0.00108552910377281	0.117363570777555	0.381969522923901
0.178892757722372	$\widehat{s}$	0.511584866693859	0.710876411721946	0.143047156286581
	MSE	0.001037872130053	0.117624723392074	0.294041737159300
0.034401152742265	$\widehat{s}$	0.029957603943068	0.028201012636230	0.0205079509368838
	MSE	0.000223606775090	0.000571841893278	0.0007303745377824

Table 3: $\widehat{S}$ and MSE values of S when $\alpha = 1$, $\beta = 0.5$ when $n = 50$

Survival		*S/MOM*	*MOM*	*MLE*
0.987109526266006	$\widehat{s}$	0.976061494585719	1	0.987141141201526
	MSE	7.12575374924e-05	4.83799133629253e-05	0.000288420194567
0.975057041219814	$\widehat{s}$	0.966016422500347	1	0.974965772786008
	MSE	2.155542752692e-07	3.97030227610076e-06	0.001023425089520
0.966309244648564	$\widehat{s}$	0.960119891921827	1	0.966093167811428
	MSE	9.88251467957e-04	7.68566299769550e-03	0.001819228156533
0.960598757134944	$\widehat{s}$	0.956644541391150	1	0.960290150144108
	MSE	1.900327745438e-06	3.81568168464004e-05	0.002453476386558
0.931815948217535	$\widehat{s}$	0.941775263220963	1	0.930953882314212
	MSE	1.29186413382e-04	0.000637970444826	0.006969380167010
0.899186989828239	$\widehat{s}$	0.927877438089812	1	0.897594516037847
	MSE	4.07341272060e-03	0.002375296064627	0.014607035926127
0.897218501212216	$\widehat{s}$	0.927100636326677	1	0.895579878633614
	MSE	4.296221249566e-3	0.002518472384674	0.015149608909954
0.876313510744807	$\widehat{s}$	0.919161205347843	1	0.874175580536424
	MSE	7.04310640273e-05	0.004313352207950	0.021466619032489
0.837617927519147	$\widehat{s}$	0.905572921600327	1	0.834531235805273
	MSE	0.000138492557841	0.008957694210656	0.035748287159802
0.834453395213868	$\widehat{s}$	0.904508496506164	1	0.831288648753906
	MSE	0.000144956901330	0.009412665515167	0.037060252591976
0.822453959550511	$\widehat{s}$	0.900522267556501	1	0.818993497072047
	MSE	0.000170571923181	0.011239485224629	0.042227854277034
0.818824527227505	$\widehat{s}$	0.899330862327994	1	0.815274784167323
	MSE	0.000178647873335	0.011823506028397	0.043850489213814
0.801622346642865	$\widehat{s}$	0.893761935304928	1	0.797651336081646
	MSE	0.000218818723819	0.014787744653594	0.051912403351147
0.783844663119787	$\widehat{s}$	0.888120042196754	1	0.779443144157529
	MSE	0.000263236959495	0.018185754559750	0.060877926313523
0.750391958536599	$\widehat{s}$	0.877727292970007	1	0.745200455840625
	MSE	0.000352883107621	0.025470292347295	0.079448924443919
0.671909368476300	$\widehat{s}$	0.853851868084247	1	0.665020168993325
	MSE	0.000575178114978	0.046769828057230	0.131342067986892
0.553836442582863	$\widehat{s}$	0.817057880381738	1	0.544991923536870
	MSE	0.000858915810438	0.087960447100126	0.229942676963468
0.548934340547672	$\widehat{s}$	0.815463274997903	1	0.540027706322960
	MSE	0.000867777176771	0.089855929428668	0.234543110772214
0.543510733209629	$\widehat{s}$	0.813690144697889	1	0.534537331121505
	MSE	0.000877233051436	0.091966708798010	0.239678774303083
0.534726989483103	$\widehat{s}$	0.810797746762829	1	0.525649869552304
	MSE	0.000891750117032	0.095414291114599	0.248097922922434
0.533672792239296	$\widehat{s}$	0.810448822495176	1	0.524583595192584
	MSE	0.000893424805104	0.095830387673940	0.249116794914752
0.504084124264449	$\widehat{s}$	0.800484128541390	1	0.494689224573959
	MSE	0.000934234198728	0.107691740770113	0.278446255577023
0.495631645321666	$\widehat{s}$	0.797571311118318	1	0.486161515145437
	MSE	0.000943603661274	0.111137134992127	0.287082771344008
0.492717895535410	$\widehat{s}$	0.796559758441537	1	0.483223110017111
	MSE	0.000946588772224	0.112329836918582	0.290086381601476
0.478415374640460	$\widehat{s}$	0.791536021729954	1	0.468809217585523
	MSE	0.000959384912498	0.118217255732327	0.305025725938700
0.454951118249832	$\widehat{s}$	0.783065535890417	1	0.445197834794753
	MSE	0.000973511428080	0.127968431257571	0.330235516772194
0.449237967974006	$\widehat{s}$	0.780956142263310	1	0.439455726541852
	MSE	0.000975627352347	0.130354580954701	0.336504832445763
0.447809809621478	$\widehat{s}$	0.780425787024625	1	0.438020756739558
	MSE	0.000976074135778	0.130951543341318	0.338080018262635
0.443024910698013	$\widehat{s}$	0.778639766870188	1	0.433214290960533
	MSE	0.000977330449618	0.132952705244792	0.343380806156699
0.414920133937214	$\widehat{s}$	0.767845348259452	1	0.405022593144916
	MSE	0.000977147733562	0.144713151078876	0.375237679536091
0.412643103162456	$\widehat{s}$	0.766946383716107	1	0.402741553949957
	MSE	0.000976562162182	0.145664205098714	0.377872587642685
0.400699182614421	$\widehat{s}$	0.762165759302970	1	0.390784231553522
	MSE	0.000972080379692	0.150641438273668	0.391825652652055
0.396525951811604	$\widehat{s}$	0.760468600557689	1	0.386609367641622
	MSE	0.000969955401331	0.152374672679124	0.396753078780214

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Table 3: $\widehat{S}$ **and** MSE values of S when $\alpha = 1$, $\beta = 0.5$ when $n = 50$ (cont.).

Survival		*S/MOM*	*MOM*	*MLE*
0.390868233095659	$\widehat{s}$	*0.758144574696777*	*1*	*0.380951986816492*
	MSE	*0.000966612136319*	*0.154718315664000*	*0.403476393583134*
0.363379773683705	$\widehat{s}$	*0.746443132969547*	*1*	*0.353507835428966*
	MSE	*0.000942818835820*	*0.165964730710595*	*0.436847708680208*
0.357521764321006	$\widehat{s}$	*0.743854137599770*	*1*	*0.347668613007640*
	MSE	*0.000936140773618*	*0.168321764125557*	*0.444110574706388*
0.326659427858967	$\widehat{s}$	*0.729579955017516*	*1*	*0.316961417926657*
	MSE	*0.000891826413023*	*0.180406611344417*	*0.483251651409482*
0.289355261513021	$\widehat{s}$	*0.710642535201387*	*1*	*0.279976192499609*
	MSE	*0.000818620271590*	*0.193947219200256*	*0.532537770338926*
0.279743319351908	$\widehat{s}$	*0.705405826115927*	*1*	*0.270470807960676*
	MSE	*0.000796488170546*	*0.197176955009740*	*0.545589480959053*
0.246816691689277	$\widehat{s}$	*0.686112939976919*	*1*	*0.237988712987186*
	MSE	*0.000711451062405*	*0.207154263205852*	*0.591405734092565*
0.211719342083898	$\widehat{s}$	*0.662685273811963*	*1*	*0.203507911331956*
	MSE	*0.000607117136139*	*0.215326513102924*	*0.642157984875767*
0.164426463437253	$\widehat{s}$	*0.624431306482857*	*1*	*0.157299708902155*
	MSE	*0.000450575816503*	*0.220230270591583*	*0.713781110863713*
0.164076033790682	$\widehat{s}$	*0.624110065070096*	*1*	*0.156958483117908*
	MSE	*0.000449372027768*	*0.220231461704571*	*0.714326164491235*
0.145035891435103	$\widehat{s}$	*0.605580233402466*	*1*	*0.138445954790476*
	MSE	*0.000383524956870*	*0.219296415820724*	*0.744269784159301*
0.142075740487679	$\widehat{s}$	*0.602490711883553*	*1*	*0.135572809536216*
	MSE	*0.000373245761316*	*0.218957832509277*	*0.748983954814609*
0.114652398783000	$\widehat{s}$	*0.570506699747824*	*1*	*0.109023166917448*
	MSE	*0.000278600858779*	*0.212759695405201*	*0.793439332738041*
0.103741412741301	$\widehat{s}$	*0.555691923645967*	*1*	*0.098495791985641*
	MSE	*0.000241772467633*	*0.208432800290929*	*0.811535206094306*
0.074217933294967	$\widehat{s}$	*0.506659039471483*	*0.999990644804318*	*0.070125600525011*
	MSE	*0.000147463987065*	*0.189200729331844*	*0.861747772649961*
0.017422816686607	$\widehat{s}$	*0.313714659411873*	*0*	*0.0161824300674375*
	MSE	*1.421083439617e-03*	*0.087626586638700*	*0.0003131759203073*
0.009689347619223	$\widehat{s}$	*0.249401481322111*	*0*	*0.00894548912414894*
	MSE	*0.057289230084558*	*5.248361765031e-03*	*0.00010099836529481*

Table 4: $\widehat{S}$ **and** MSE values of S when $\alpha = 0.5$, $\beta = 1$ when $n = 15$

Survival		*S/MOM*	*MOM*	*MLE*
0.896786225431860	$\widehat{s}$	0.634530785677445	0.522840916901840	0.940666379991178
	MSE	0.001454651562124	0.071785846865318	0.144110340761442
0.877186430861497	$\widehat{s}$	0.618710809390159	0.520044449844231	0.925008569844778
	MSE	0.002694926286416	0.064669697582925	0.124586927084329
0.858139877574791	$\widehat{s}$	0.605097844971683	0.517672772572350	0.908848857087058
	MSE	0.004404127413569	0.056306104331666	0.105439276450218
0.789529819351169	$\widehat{s}$	0.565093367862237	0.510847013604125	0.843917230854258
	MSE	0.014947771151388	0.024463591741862	0.044375431082624
0.675374346353008	$\widehat{s}$	0.514603945567668	0.502420802014118	0.717613910554293
	MSE	0.041109935689606	4.16392979832010e-1	0.000148926614615
0.638736622967499	$\widehat{s}$	0.526570956104155	0.500094611345458	0.503545145353552
	MSE	0.001847668410959	0.002251737412964	0.022206756841492
0.583248845791307	$\widehat{s}$	0.499414203211739	0.496752655525991	0.604648180161696
	MSE	0.010043339147904	0.012717850897953	0.016669883173863
0.444753371340072	$\widehat{s}$	0.432674741670460	0.488776690170055	0.428048027902581
	MSE	0.052008430417984	0.054268130126709	0.083554041203176
0.429663192521842	$\widehat{s}$	0.427486994098301	0.487900308191592	0.408934244493470
	MSE	0.049901739740952	0.058666013744145	0.091581282098443
0.310151000721190	$\widehat{s}$	0.384559389166910	0.480528011660051	0.263183019284942
	MSE	0.028480189072224	0.084303892650795	0.149243102791131
0.297278174619851	$\widehat{s}$	0.379599437462140	0.479659017521129	0.248344323957569
	MSE	0.026086880507154	0.085834522660685	0.154476411913049
0.204908889446771	$\widehat{s}$	0.340229367716922	0.472589868321629	0.149032606617254
	MSE	0.011249082254121	0.088444568746501	0.184690921383718
0.204015736125578	$\widehat{s}$	0.339804149803856	0.472511601522051	0.148142904273784
	MSE	0.011132903956708	0.088393821943281	0.184915856122381
0.183568421650781	$\widehat{s}$	0.329739382824304	0.470645166842548	0.128197836258802
	MSE	0.008642557800846	0.086805610078909	0.189689605614257
0.166141372072542	$\widehat{s}$	0.320587297018523	0.468923448608700	0.111868753695392
	MSE	0.006782236728186	0.084785838277825	0.193174516758054

Table 5: $\widehat{S}$ **and** MSE values of S when $\alpha = 0.5$, $\beta = 1$ when $n = 30$

Survival		*S/MOM*	*MOM*	*MLE*
0.964633187065820	$\widehat{s}$	0.920410224526841	0.852347622347063	0.977974365873251
	MLE	0.000183620874886	0.001925287387475	0.012530720110238
0.920860333923007	$\widehat{s}$	0.840269586227595	0.766564036081091	0.943209530052794
	MSE	0.001631958587863	0.003867597191185	0.018467602934043
0.901843589435134	$\widehat{s}$	0.808735813231349	0.737417002769850	0.926712127722282
	MSE	0.002835195840994	0.004131580355199	0.018386324284603
0.892443525921654	$\widehat{s}$	0.793731182938815	0.724138982421189	0.918308565348269
	MSE	0.003563204527035	0.004145103529300	0.017949214600183
0.882635613937105	$\widehat{s}$	0.710945154660024	0.909380312559527	0.842386918322452
	MSE	0.004414114820100	0.004987990061723	0.017276937234689
0.874242498029506	$\widehat{s}$	0.765657442697654	0.700128852628252	0.901617753100951
	MSE	0.002213574890363	0.003988663851137	0.016559690132297
0.808709582577343	$\widehat{s}$	0.626810502898730	0.837700146639883	0.721501781132952
	MSE	0.001328408462639	0.002303986059036	0.008966438173630
0.737469854027583	$\widehat{s}$	0.585666041574957	0.561648376540277	0.763024920823237
	MSE	0.023708121385599	0.030488716747124	0.042127479087289
0.718188982511331	$\widehat{s}$	0.694571688198524	0.782106430642893	0.578238717373725
	MSE	0.000409260962741	0.000912394728094	0.001057669777035
0.675038023037111	$\widehat{s}$	0.596747908757352	0.511997865183950	0.694439772868011
	MSE	0.031789009681362	0.050949637168551	0.098984291978181
0.630632152120797	$\widehat{s}$	0.597144666354646	0.479541601880375	0.604410067930936
	MSE	0.005961206991675	0.021335922282970	0.010698181327891
0.562952530130368	$\widehat{s}$	0.497361232667805	0.433281687388668	0.566919164785467
	MSE	0.008998280836788	0.009577716443506	0.009308732165727
0.543126831972312	$\widehat{s}$	0.519584857516443	0.420287220303609	0.544059304123737
	MSE	0.001090602728232	0.002030651229733	0.005740351966223
0.542897098464487	$\widehat{s}$	0.489381304236531	0.420137857096108	0.543794199192705
	MSE	0.001908959709821	0.002035995190208	0.005757555298993
0.522388902041971	$\widehat{s}$	0.371426589323814	0.406905983265581	0.520117730473699
	MSE	0.001881331594370	0.002516200579339	0.007334404539063
0.459984175739689	$\widehat{s}$	0.319235659458860	0.367643930229087	0.448163133210491
	MSE	0.003589133977518	0.003889783338306	0.012271410432284
0.444613895084338	$\widehat{s}$	0.396897814627587	0.358144077681057	0.430521716341317
	MSE	0.004755136567811	0.004178063960443	0.013429149723905
0.406150524087263	$\widehat{s}$	0.376829079638832	0.334547147970172	0.356637822577768
	MSE	0.011349649624902	0.044766025795007	0.026066697573833
0.396640212477622	$\widehat{s}$	0.269562141017052	0.328737305894452	0.375859943347847
	MSE	0.003404829131785	0.004877990158461	0.016645586210021
0.379089354639339	$\widehat{s}$	0.256315772485670	0.318025751750348	0.356061512790026
	MSE	0.004577583712680	0.005046604717799	0.017622413327422
0.334099489541176	$\widehat{s}$	0.299271179292162	0.290523642216117	0.305958949475486
	MSE	0.000551986164815	0.005241830251539	0.019502945908802
0.246822454750428	$\widehat{s}$	0.262390982767661	0.235970886408785	0.212361822062984
	MSE	0.003654561892724	0.004648910558355	0.020096698122270
0.233434954413674	$\widehat{s}$	0.153371350806694	0.227322275443321	0.198537911355362
	MSE	0.002248335547818	0.004455189635053	0.019795964230761
0.229164521314266	$\widehat{s}$	0.190509676923948	0.224540660792404	0.194163031186373
	MSE	0.001810574142580	0.004388426802994	0.019677408353508
0.194196822107561	$\widehat{s}$	0.127335568750601	0.201263171410802	0.159022900605514
	MSE	0.001457048071904	0.003761223734290	0.018294298926245
0.143512965863615	$\widehat{s}$	0.094422853370726	0.165329578862269	0.110557808977141
	MSE	0.002480669354657	0.002661507630538	0.015005399152838
0.142701308077088	$\widehat{s}$	0.093900827084928	0.164725616160752	0.109808719784627
	MSE	0.004426647549879	0.005642747765690	0.014940772222936
0.137911952282430	$\widehat{s}$	0.090823011657377	0.161139363978963	0.105407521400890
	MSE	0.001114431808246	0.002531643306191	0.014552028848321
0.124454290619184	$\widehat{s}$	0.082194944984357	0.150840215021616	0.093219901822771
	MSE	0.001297891626586	0.002216854106202	0.013393143627689
0.109427200320369	$\widehat{s}$	0.072587156809741	0.138893833884855	0.079943008921564
	MSE	0.002493950486220	0.003864507094939	0.011987319005499

Table 6: $\widehat{S}$ and MSE values of S when $\alpha = 0.5$, $\beta = 1$ when $n = 50$

Survival		*S/MOM*	*MOM*	*MLE*
0.962438460703666	$\widehat{s}$	0.859518627561707	0.743478734256815	0.972132075082478
	MSE	0.035720563484273	0.043534207427374	0.050484885797085
0.959884516964250	$\widehat{s}$	0.854287446981635	0.738429411092760	0.970013081239244
	MSE	0.029884076087147 1	0.044633522409124	0.051585542317909
0.958659267840027	$\widehat{s}$	0.951864692895990	0.736094149757453	0.968990596573031
	MSE	0.002111339662291	0.045123797957775	0.052072579983407
0.918781997227951	$\widehat{s}$	0.834051885119412	0.718984843882484	0.990619228553320
	MSE	0.011124104866082	0.048301808521811	0.055151594187813
0.904587925771653	$\widehat{s}$	0.876344472941791	0.664176359920788	0.950977471751193
	MSE	0.000327944476602	0.051313007421393	0.056973803478100
0.904250731767947	$\widehat{s}$	0.875989655072045	0.663841845611124	0.930663868338845
	MSE	0.000331394995219	0.051288633691239	0.056938430323694
0.880406251527569	$\widehat{s}$	0.852925678761849	0.642151375728443	0.898140398310805
	MSE	0.000631375883594	0.048438509832759	0.053297174382894
0.868618093293902	$\widehat{s}$	0.692738864104517	0.632602227603796	0.686774083518706
	MSE	0.000821418486167	0.046386303298347	0.050855406760544
0.856327663055417	$\widehat{s}$	0.792789279993994	0.623291851054715	0.874777897445697
	MSE	0.001049173885676	0.043931169992867	0.048002648501054
0.845822561388337	$\widehat{s}$	0.824749731651777	0.615779747925951	0.864415006056216
	MSE	0.001267478649871	0.041641944901442	0.045383299502379
0.833778630756783	$\widehat{s}$	0.815987968011751	0.697603121639773	0.852418803389552
	MSE	0.001543812389466	0.038863953848954	0.042240227123421
0.765724944784732	$\widehat{s}$	0.593126706139193	0.567732511137355	0.782687006007092
	MSE	0.003553345899443	0.022485095139180	0.024131914892482
0.764541313552331	$\widehat{s}$	0.572457153408468	0.567111067418734	0.881449130577983
	MSE	0.003593679526821	0.022214301053529	0.023836492481185
0.735508743168889	$\widehat{s}$	0.556630599645856	0.552431209355712	0.750864650706837
	MSE	0.004619116501285	0.015944092770872	0.017022241004115
0.686626484670359	$\widehat{s}$	0.532075532832785	0.529683917852213	0.698547759905501
	MSE	0.006416997001844	0.007459087696138	0.007877916005364
0.678452845700225	$\widehat{s}$	0.528171937696800	0.526070027815600	0.518971648369753
	MSE	0.006114728318978	0.006336326009917	0.006675372471766
0.655657217116983	$\widehat{s}$	0.617531726825703	0.516221621181395	0.564983069194771
	MSE	0.003685338701397	0.003846120220128	0.003846120220128
0.642421012909190	$\widehat{s}$	0.591501533916128	0.560641194671606	0.550559600062587
	MSE	0.001975186105992	0.002475471106396	0.002561822110683
0.637059101304571	$\widehat{s}$	0.609086343566545	0.508406281225049	0.664705269241729
	MSE	0.001152538188440	0.002054578242280	0.002116691718414
0.605496798907207	$\widehat{s}$	0.495950025175788	0.495510848190303	0.610131244527083
	MSE	0.000121553967525	0.000379249377102	0.000365326000251
0.601972132584711	$\widehat{s}$	0.593620125328249	0.494095205675233	0.606259988533404
	MSE	0.000220669556131	0.002760741385281	0.000260512464918
0.555174463509375	$\widehat{s}$	0.453694573901440	0.475654904471119	0.454730650867185
	MSE	0.000325018720633	0.000423375607524	0.00507890392108
0.481125251121829	$\widehat{s}$	0.473059186154433	0.447278348749681	0.473064815758236
	MSE	0.001114217231983	0.005687939942632	0.006342146533416
0.468468328354985	$\widehat{s}$	0.457862335951163	0.442459831839900	0.449137066028657
	MSE	0.006111970290642	0.007085488884729	0.007880617468086
0.460054378672828	$\widehat{s}$	0.449406293159892	0.439254397976593	0.447888662181107
	MSE	0.001102094707689	0.008079166414240	0.008974206418920
0.459811884088161	$\widehat{s}$	0.454306651928076	0.439161969721224	0.447622254906813
	MSE	0.007110133644113	0.008108525437364	0.009006516560131
0.438319052889429	$\widehat{s}$	0.425460661052312	0.430953398187687	0.426044874662105
	MSE	0.010278839052421	0.010856768053168	0.012031578669206
0.422932826726949	$\widehat{s}$	0.419100420284777	0.425047593037866	0.409215425859745
	MSE	0.010825148737105	0.012979826898274	0.014370307165759
0.412027989465433	$\widehat{s}$	0.414572122417988	0.420840684559673	0.397316872805838
	MSE	0.010683886477276	0.014548752689168	0.016100253055586
0.408771999581952	$\widehat{s}$	0.413216074033767	0.419580512417547	0.393769302638141
	MSE	0.010636634355903	0.015026078244102	0.016626901552467
0.374809881954454	$\widehat{s}$	0.398930765616390	0.406294059782667	0.356926038991183
	MSE	0.010011690495877	0.020181962381037	0.022328287242515
0.362812377507335	$\widehat{s}$	0.393807895168486	0.401524024670480	0.343989542390497
	MSE	0.009736983262002	0.022050639443208	0.024401784872979
0.359607048860728	$\widehat{s}$	0.392431083426916	0.400241520616343	0.340541108091852
	MSE	0.009659187981676	0.022551393515810	0.024958202114196

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Table 6: $\widehat{S}$ **and** MSE values of S when $\alpha = 0.5$, $\beta = 1$ when $n = 50$ (cont.).

Survival		*S/MOM*	*MOM*	*MLE*
0.351375436621224	$\widehat{s}$	0.388878157841871	0.396930908207690	0.331700916643145
	MSE	0.009451268505276	0.023837799347524	0.026389254051596
0.322327344300367	$\widehat{s}$	0.376113873994750	0.385023904545571	0.300701416577442
	MSE	0.008632022582661	0.028332027565969	0.031410910342425
0.318988130211524	$\widehat{s}$	0.374620778000015	0.383629655861392	0.297159012004256
	MSE	0.008530156448210	0.028839063062578	0.031980010456263
0.313278491123817	$\widehat{s}$	0.372053931366948	0.381232004252963	0.291112743444020
	MSE	0.008352707112633	0.029699145511508	0.032946777906039
0.296755248056510	$\widehat{s}$	0.364519301170059	0.374188449207589	0.273695577948755
	MSE	0.007817954889310	0.032128048590595	0.035687794977243
0.286541427190329	$\widehat{s}$	0.359774648822300	0.369748581957226	0.262992171404407
	MSE	0.007473591510550	0.033574140682376	0.037328716975029
0.255444437065445	$\widehat{s}$	0.344842337883347	0.355751295338016	0.230730014357814
	MSE	0.006377501612783	0.037624998579900	0.041976060750856
0.179619248239917	$\widehat{s}$	0.303988928883367	0.317236548689449	0.154535279594821
	MSE	0.003639367586803	0.044008011722502	0.049741705462659
0.168497468571656	$\widehat{s}$	0.297230399828402	0.310828982007816	0.143712042231168
	MSE	0.003259023213814	0.044354995052614	0.050267809448448
0.164978609288017	$\widehat{s}$	0.295037261845094	0.308747282660034	0.140309228785947
	MSE	0.003141009695320	0.044425167754147	0.050392531019816
0.136694871539504	$\widehat{s}$	0.276276415606463	0.290887610837565	0.113363778235844
	MSE	0.002242684490201	0.044213331331215	0.050571400562476
0.109667806589924	$\widehat{s}$	0.255911300557886	0.271385208255352	0.088366687307079
	MSE	0.001489088268292	0.042490957733312	0.049109776396596
0.0974157602149612	$\widehat{s}$	0.245599485799619	0.261459317114676	0.077311626866668
	MSE	0.001188770307654	0.041114226774337	0.047797443904221
0.0968037869691725	$\widehat{s}$	0.245062658145785	0.260941580216785	0.0767643877981703
	MSE	0.001174514929579	0.041034399729291	0.0477197103004680
0.0932039537724501	$\widehat{s}$	0.241858698588227	0.257849440486260	0.0735552607965320
	MSE	0.001092147941593	0.040542275531166	0.0472374870985198
0.0831928829850746	$\widehat{s}$	0.232494147280371	0.248790313831421	0.0647232790629106
	MSE	0.000876858185394	0.038959923342861	0.0456586505230837
0.0722001858340905	$\widehat{s}$	0.221300143540447	0.237917392847783	0.0551933974600285
	MSE	0.0006649260051545	0.036822894780599	0.0434764929152250

5. Conclusion

A new method was proposed by mixed Simplex Downhill Algorithm with Moment Method (SMOM) to estimate the parameters of Log-Logistic distribution depend on Survival functions. A simulation analysis was employed to investigate and compare the performance of the proposed method with two classical methods (Maximum Likelihood Estimator and with Moment Method). The result showed that the SMOM delivers good results than MLE and MOM based on Mean Square Error (MSE).

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