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Performance evaluation of firefly algorithm with unconstrained optimization issues

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Abstract

In this paper, we have investigated a new spectral Quasi-Newton (QN) algorithm. New search directions of the proposed algorithm increase its stability and increase the arrival to the optimum solution with a lowest cost value and our numerical applications on the standard Firefly Algorithm (FA)and the new proposed algorithm are powerful as in meta-heuristic field. Our new proposed algorithm has quite common uses in several sciences and engineering problems. Finally, our numerical results show that the proposed technique is the best and its accuracy higher than the accuracy of the standard FA. These numerical results are compared using statistical analysis to evaluate the efficiency and the robustness of new proposed algorithm.

Keywords: QN-method, self-scaling QN, conjugate gradient, unconstrained Optimization, Firefly algorithm.

1. Introduction

Optimization techniques deal with finding the minimum or the maximum of any nonlinear complicated test function. FA is considered as a new swarm intelligence planned algorithm. Optimization techniques, in general, haven't capable to solve this type of test problems.
However, the population-based techniques are considered as resolution optimization issues, with their own hardiness rather than the normal standard optimization issues. Indeed, the FA is an optimization technique impressed from flashing of sunshine from firefly. The flashing properties may be divided to a few rules:

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Rule (1): Each one firefly is interested in alternative fireflies; as a result of their unisex firefly.
Rule (2): There was proportional amidst the attraction and their burnish. In order for the less brighter of two flashing fireflies to migrate towards the brighter one. The attraction is proportional to the burnish, and it decreases as the distance between them grows. It will move at random if no one is brighter than a particular firefly.
Rule (3): The landscape influences or determines the firefly burnish.
Optimized objective function may be outlined as $f(x)$ for $x \in S \subset R^n$ and

$$"f(x^*) = \min_{x \in S} f(x)" \tag{1.1}$$

Iteratively, the firefly algorithmic rule can be utilized to solve an optimization problem. Let's say there's a swarm of (n) fireflies, and (x_i) returns a solution for each firefly at each iteration (i), where $f(x_i)$ represents its value. In the beginning, all fireflies disarticulate in $S_m (m = 1, \ldots, k)$ (randomly or by recruiting some deterministic approach). The current levels of interest Iteratively, the firefly algorithmic rule can be utilized to solve an optimization problem. Let's say there's a swarm of (n) fireflies, and (x_i) returns a solution for each firefly at each iteration, where represents its value. In the beginning, all fireflies disarticulate in $S_m (m = 1, \cdots, k)$ (randomly or by recruiting some deterministic approach). The current levels of interest

2. The Formulation of Standard Firefly Algorithm (FA)

The FA will be divided into substantial situation as well as variance intensity of sunshine and therefore the formula of the attraction. Displays the barriers of the best FA optimization during a condition that, the distance amidst fireflies and their level of attractionω are proportional to each other. Therefore, the burnish L of a firefly at a special position x can be chosen as $L(x) \alpha f(x)$. This relies on the distance of fireflies r_{ij} from the earth and the fireflies' variations jth.
Thus, ω mutations with capacity rate, which inversely relies on the square law as, explain by:

$$L(r) = \frac{L_s}{r^2}. \tag{2.1}$$

Where r, represents the space amidst every two fireflies;$L_s$is that the intensity at the supply and $L(r)$represents the sunshine intensity varies with distance (r) monotonically and exponentially. i.e.:

$$L(r) = L_0 e^{-\rho r}. \tag{2.2}$$

In above equation L_0 represents to the original light intensity. Thus, on avoid the singularity at $r = 0$ within the expressionL_s/r^2, the composition result of the inverse square law and talent will be approximated mistreatment Gaussian formula form

$$L(r) = L_0 e^{-\rho r^2}. \tag{2.3}$$

Sometimes, we tend to may need a function that decreases monotonically at slower rate. During this situation, we are able to use the subsequent equation:

$$L(r) = \frac{L_0}{1 + \rho r^2}.$$

The relation amidst the level of firefly's attraction ω and the space size r can be calculated by:

$$\beta(r) = \omega_0 e^{-\rho r^2}. \tag{2.4}$$

Because, the firefly's attraction is proportional on to the sunshine intensity, it is accepted and recognizable amongst neighboring fireflies (adjacent firefly's). Where, ρ represents to the constancy of environmental, that regulates the average of sunshine intensity decrease, if the two fireflies are located at identical point of search space S, then r=0, during this case the firefly's attraction level was described by

$$\beta(r) = \omega_0$$

In general,$\omega_0 \epsilon [0, 1]$ should be employed, and there are two limiting scenarios to consider:
Situation 1: ω_0 =0, that is only non-commune distributed random search is try to obtain.
Situation 2: ω_0 =1, which is Valente t to the sketch the commune local search with the brightest firefly powerfully confirm the firefly's situations, primarily in its neighborhood.

The motion of the firefly i is attracted to another additional enticing (brighter) firefly (j) is illustrated by:

$$x_i = x_i + \omega_0 e^{-\rho r_{ij}^2} (x_j - x_i) + \tau \varepsilon_i, \tag{2.5}$$

The presentation of the middle part within the last equation due to the firefly's attraction towards the brighter firefly. At an equivalent time, randomization τ otherwise be appeared on the last half in equation (2.5). This symbol denoted to the parameter of randomization with the transmitter of stochastic variable$\varepsilon_i$being drawn from aGaussian distribution and ($\tau \epsilon [0, 1]$) (Gope, et al., 2016 ; Yang, 2008) .

3. Distance and Movement Of Two Flies In FA

The distance amidst any two fireflies may be calculated by exploitation Cartesian distance:

$$r_{ij} = \|x_i - x_j\| = \sqrt{\sum_{k=1}^{K} (x_{ik} - x_{jk})^2} \tag{3.1}$$

Where r_{ij} is that the space amidst the fireflies i and j at x_i and x_j respectively and K is that the dimensional size of given downside and x_{ik} is that the k-th element of the special coordinate x_i of i-th firefly (Babaeizadeh, et al., 2016).In 2D situation, we have:

$$r_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \tag{3.2}$$

The move of a firefly(i) is attracted to another a lot of drawn to another brighter firefly (j) is outlined as:

$$x_i = x_i + \omega e^{-\rho r_{ij}^2} (x_j - x_i) + \tau \left(rand - \frac{1}{2} \right) \tag{3.3}$$

where the second term within the higher than equation is pointed to the attraction; whereas the last term in equation (3.3) is randomization with ρ being the organization variable;$rand$ could be a random number generator uniformly distributed in interval [0, 1].For the most situations of implementation, $\omega_0 = 1$and $\rho \epsilon [0, 1]$. The scalar ρ distinguish the variation of the attraction and its value is substantial to work out the convergence speed and the way the FA behaves. Within the most applications, it always varies from (0.01) to (100).
Finally, when utmost of the generation is close to finished, the fireflies square measure labeled relying on their burnish and therefore the superior firefly cluster is discovered from every category. At the ultimate, the fireflies begin moving towers to their items. The intensity of burnish of all fireflies is

lately improved with the data as long as, in fitness function for every cluster. Moreover, once the feasible square measure good on the teams, the firefly with the supreme level of burnish marks the best quantity of fitness .this can be determined because the optimal solution to non-linear programming (Gope et al., 2016 ; Babaeizadeh, et al., 2016).

4. Standard Quasi Newton Method

The Quasi-Newton (QN) strategies is that the one in all effectiveness strategies for finding the solution of any non-linear programming problems, epically if we try to use the Broyden-Fletcher-Goldfarb-Shanno (BFGS) method. Several modifications are done for finding similar updates used to solve these types of problems see (Al-Bayati et al., 1994). Let us consider the following unconstrained optimization problem:

$$" \min_{x\in R^n} f(x)" \tag{4.1}$$

Where $f : R^n \to R$ is twice continuously differentiable, i.e. $f \in C^2$. In this paper we foucs on Quasi-Newton formula. This is an iterative formula, which is formulated by:

$$"x_{k+1} = x_k + \alpha_k d_k". \tag{4.2}$$

Where$d_k$represents to the search directions and $\alpha_k$is a positive parameter represents to the step-length of the formula (4.2) see (Hamed et al.,2020), that is chosen to satisfy the inexact line search conditions; using the well-known Weak Wolfe-Powell conditions (WWP):

$$"f\,(x_k + \alpha_k d_k) \leq f\,(x_k) + \ \tau_1 + \alpha_k g_k^T d_k, \tag{4.3}$$

$$g(x_k + \alpha_k d_k)^T d_k \leq \tau g_k^T d_k". \tag{4.4}$$

Where τ_1 *and* $\tau_2$constants are satisfying $0 < \tau_1 < \tau_2 < 1$ *and* $\tau_1 < \frac{1}{2}$.
BFGS technique is mostly contemplate as the most noted one as a result of it's the foremost economical technique among alternative variable metric to resolve unconstrained optimization nonlinear problems. Recently it's derived from the Newton's technique that is employed first and second derivatives for finding the stationary point. The additional best-known update formula outlined by:

$$B_{k+1} = B_k + \left[1 + \frac{y_k^T B_k y_k}{v_k^T y_k}\right] \frac{v_k v_k^T}{v_k^T y_k} - \left[\frac{v_k y_k^T B_k + B_k y_k v_k^T}{v_k^T y_k}\right]. \tag{4.5}$$

The misreckoning and rounding error need to add some sort of spectral scaling techniques to the above sequence. This problem will sometime be taken away by sensible scaling.

5. Standard Conjugate Gradient Method

There are several varieties of numerical methods to deal with equation (4.1); for example, the Steepest Descent (SD); Newton (N) and Quasi-Newton (QN) methods. The CG-method is one in every of the alternatives for finding massive scaling algorithms, to ensure that it doesn't need any storage of matrices (Ahmed et al.,2019).
The search direction in the standard Conjugate Gradient (CG) are usually defined by:

$$"d_{k+1} = \begin{cases} -g_k, & \text{if } k = 0 \\ -g_{k+1} + \beta_k d_k, & \text{if } k \geq 1 \end{cases} " . \tag{5.1}$$

Where $g_{k+1} = \nabla f\left(x_{k+1}\right)$ and $\beta_k \in R$ is scalar parameter which characterize CG-methods.

Usually, several kinds of formulas for β_k has been proposed. For example, the Fletcher-Reeves (FR), Polak-Ribière-Polyak (PRP), Hestenes-Stiefel (HS) and Dai-Yuan (DY) formulas are well known and they are defined by:

$$\left.\begin{array}{c} ''\beta_k^{FR} = \frac{\|g_{k+1}\|^2}{\|g_k\|^2},\ \beta_k^{PR} = \frac{g_{k+1}^T y_k}{\|g_k\|^2} \\ \beta_k^{HS} = \frac{g_{k+1}^T y_k}{d_k^T y_k} \text{ and } \beta_k^{DY} = \frac{\|g_{k+1}\|^2}{d_k^T y_k}'' \end{array}\right\} \tag{5.2}$$

where $y_k = g_{k+1} - g_k$ and $\|.\|$ denote the Euclidean norm see (Al-Namat et al.,2020; Dai et al.,1999; Huda et al., 2018).

In this work, we have a tendency to summarize another work of this subject, Ibrahim et al (Ibrahim et al.,2017) planned another direction that outlined by:

$$d_{k+1} = \left\{\begin{array}{ll} -B_{k+1}^{-1} g_{k+1}, & k=0 \\ -B_{k+1}^{-1} g_{k+1} + \gamma(g_{k+1} + \beta_k d_k), & k \geq 1 \end{array}\right\}$$

Where γ is positive scaler and $\beta_k$is conjugacy coefficient defined in (5.2).

6. Derivation of a New Scaling Parameter

In this work, we try to investigate a generalization of a new scaling parameter for Ibrahim et al (Ibrahim et al.,2017) CG-method; this parameter will deal with our new proposed QN-procedure.

QN-methods are one amongst several methods that projected for resolution problem (Huda et al., 2018), they avoid expensive computations of Hessian matrices and perform well done in the apply, several modifications are projected for resolution the BFGS problems, that is that the preferred methods (Hamed et al., 2019).

In this section, we try to derive a new spectral scaling parameter, which deals with the standard BFGS method. We consider the following spectral scaling QN search direction:

$$d_{k+1} = -\vartheta_k B_{k+1}^{-1} g_{k+1} \tag{6.1}$$

where B_{k+1} is nonsingular, symmetric and positive definite in (4.5).

Ibrahim et al (4.2) search directions in their CG-algorithm defined by:

$$''d_{k+1} = \left\{\begin{array}{ll} -B_{k+1}^{-1} g_{k+1}, & k=0 \\ -B_{k+1}^{-1} g_{k+1} + \gamma(g_{k+1} + \beta_k d_k), & k \geq 1 \end{array}\right\} \tag{6.2}$$

Now, multiplying both sides of (6.1) and (6.2) by(y_k), to get:

$$-\vartheta_k y_k^T B_{k+1}^{-1} g_{k+1} = -t s_k^T g_{k+1} + \gamma y_k^T g_{k+1} + \gamma \beta_k y_k^T d_k.$$

Therefore

$$\vartheta_k = \frac{t s_k^T g_{k+1} - \gamma y_k^T g_{k+1} - \gamma \beta_k y_k^T d_k}{y_k^T B_{k+1}^{-1} g_{k+1}}'' \tag{6.3}$$

7. Converge Analysis Property for Scaled QN-Method

To prove that the new propose technique satisfy the QN-condition, we tend to rewrite BFGS formula as:

$$B^{+}_{\text{new}}{=}B^{(a)}{+}\vartheta_k B^{(b)} \tag{7.1}$$

Where

$$B^{(a)} = B_k - \frac{B_k y_k v_k^T}{v_k^T y_k}$$

$$B^{(b)} = \left[1 + \frac{y_k^T B_k y_k}{v_k^T y_k}\right] \frac{v_k v_k^T}{v_k^T y_k} - \frac{v_k y_k^T B_k}{v_k^T y_k}$$

They have fulfilled the requirement of QN i.e.

$$B^{+}_{new} y_k = \vartheta_k v_k$$

If we tend to multiplies each side of the equation (7.1) by y_k, we obtain

$$\left(B^{(a)}{+}\vartheta_k B^{(b)}\right) y_k {=} \left[[B_k - \frac{B_k y_k v_k^T}{v_k^T y_k}]{+}\frac{ts_k^T g_{k+1} - \gamma y_k^T g_{k+1} - \gamma\beta_k y_k^T d_k}{y_k^T B_{k+1}^{-1} g_{k+1}} \left[\left(1{+}\frac{y_k^T B_k y_k}{v_k^T y_k}\right)\frac{v_k v_k^T}{v_k^T y_k}\right] - \frac{v_k y_k^T B_k}{v_k^T y_k}\right] y_k.$$

We can rewrite the equation above to get:

$$\text{''}\ \left(B^{(a)}{+}\vartheta_k B^{(b)}\right) y_k = \left[[B_k y_k - \frac{B_k y_k v_k^T y_k}{v_k^T y_k}] + \frac{ts_k^T g_{k+1} - \gamma y_k^T g_{k+1} - \gamma\beta_k y_k^T d_k}{y_k^T B_{k+1}^{-1} g_{k+1}} \left[\left(1 + \frac{y_k^T B_k y_k}{v_k^T y_k}\right) \frac{v_k v_k^T y_k}{v_k^T y_k}\right] - \frac{ts_k^T g_{k+1} - \gamma y_k^T g_{k+1} - \gamma\beta_k y_k^T d_k}{y_k^T B_{k+1}^{-1} g_{k+1}} \left(\frac{v_k y_k^T B_k y_k}{v_k^T y_k}\right)\right]$$

After some mathematical calculations, yields

$$\left(rmB^{(a)}{+}\vartheta_k B^{(b)}\right) y_k = \frac{ts_k^T g_{k+1} - \gamma y_k^T g_{k+1} - \gamma\beta_k y_k^T d_k}{y_k^T B_{k+1}^{-1} g_{k+1}} v_k\text{''}.$$

this means $B^{+}_{new} y_k = \vartheta_k v_k$ is holds.

Theorem . The new formula $B^{+}_{new}{=}B^{(a)}{+}\vartheta_k B^{(b)}$, generates conjugate search directions.

Proof: Let $f\left(x\right) = \frac{1}{2}x^T G x + b^T x + c$ is a quadratic function, we choose approximation matrix which is symmetric and positive definite $B_1 = B$. we must prove that if the exact line search used the direction d_{k+1} satisfies

$$B_{i+1} g_{k+1}{=}B_i g_{k+1} \quad , \quad 0{\leq}k < i \leq n. \tag{7.2}$$

By using the mathematical induction, let $(i = 0)$ this yield

$$B_1 g_{k+1}{=}B^{+}_{new} g_{k+1},$$

We suppose that this property is true at (i), i.e.

$$B_i g_{k+1}{=}B^{+}_{new} g_{k+1}.$$

Now, we must prove that this property it's true for $(i + 1)$

$$B_{i+1} g_{k+1}{=}\left(B^{(a)}{+}\vartheta_k B^{(b)}\right) g_{k+1}$$

$$= B_i g_{k+1} - \frac{B_i y_i v_i^T g_{k+1}}{v_i^T y_i} + \vartheta_k \frac{v_i v_i^T g_{k+1}}{v_i^T y_i} + \vartheta_k \frac{y_i^T B_i y_i v_i v_i^T g_{k+1}}{v_i^T y_i} - \vartheta_k \frac{v_i y_i^T B_i g_{k+1}}{v_i^T y_i}.$$

If an exact line search is used, then $v_i^T g_{k+1} = 0$, and $y_i^T B_i g_{k+1} = 0\ for\ i < k$. Hence, we get that (7.2) is true.

8. Outline Of The New Proposed Algorithm

In this paragraph, we tend to establish our planned incorporate **FA** and the **NEW** algorithm. to attain the optimal resolution by dominant to the conduct firefly and the new method with scaled BFGS update so as to eliminate the truncation and misreckoning to get additional activity than before.

Algorithm:
Step1: Start with initial point x_0, put $B_1 = I$, begin a population of fireflies x_i $(i = 1, 2, \ldots, n)$, and put $i = 1$.
Step2: Put $d_i = -B_i g_i$.
Step3: Compute $x_{i+1} = x_i + \alpha_i d_i$, with α_i by using Cubic interpolation line search.
Step4: **While** (t <Max Generation)
For $i = 1 : n$ (all n fireflies)
For $j = 1 : i$
Light intensity L_i at x_i is computed by $f(x_i)$
If $(L_i > L_j)$
Movement of FA i towards j in all d
dimensions (used Eq. (2.5))
Else
Movement of FA i randomly
End if
Attraction varies with distance r via $\left(e^{-\rho r^2}\right)$.
Compute the new solutions and update light
Intensity
End for j
Step5: Compute scaling parameter ϑ by using eq. (6.3)
Step6: Update B_{i+1}^{-1} by using BFGS and **End for** i
Degree the fireflies and find the current best
Step7: If $n = I$, then go to Step2, otherwise we get the solution.

9. Numerical Results

In order to demonstrate analytical results and to get the effectiveness of such proposed NEW algorithm by specific value of the performance parameters (Fmin, max, min, mean, stdDev).
In **Table (I)**we 've compared the new proposed method (**NEW**) with the normal firefly algorithmic (**FA**) algorithm.
Whereas **Table (II)** ensures that, the new algorithm (**NEW**) with BFGS update is superior to each customary firefly algorithm.

Table (I): Comparison (FA) Algorithm Against Modified Firefly Algorithm (NEW)

	FA					NEW				
Fun.	**F**$_{min}$	**max**	**min**	**mean**	**stdDev**	**F**$_{min}$	**max**	**Min**	**mean**	**StdDev**
1	0.015	10.2783	0.0542	3.5826	5.7976	0.0131	8.9365	0.0540	3.1341	5.0282
2	5.000E+13	5.0249	0.2500	2.1237	2.0459	5.000E+13	9.9300	0.2500	2.9743	4.0078
3	3.8377E+17	3.2161	1.3000	2.1720	0.9696	3.8377E+17	7.6262	1.3000	3.6421	3.4681
4	1.0973E+16	9.2631	0.0500	3.5377	4.9976	1.0973E+16	9.7011	0.0500	3.6837	5.2486
5	5.9646E+15	6.7633	0.0500	2.7044	3.5702	5.9646E+15	5.5883	0.0500	2.3128	2.9047
6	1.4618E+05	9.8140	1.2228	4.2065	4.8596	6.5335E+03	7.4436	1.2767	3.4199	3.4871
7	2.0100E+15	2.000	0.0500	1.0166	0.9751	2.0100E+15	2.000	0.0500	1.0167	0.9751
8	8.5805E+14	12.9194	1.2864	5.1686	6.7124	8.5805E+14	8.8892	1.2865	3.8252	4.855
9	2.5459E+18	15.000	1.300	6.100	7.7156	2.5459E+18	15.000	1.300	6.100	7.7156
10	3.1597E+6	6.7255	0.7237	2.8881	3.3324	3.0536E+05	6.6114	0.9244	2.8687	3.2421
11	3.3498E+04	9.6969	0.7690	3.7622	5.1397	1.7161E+06	9.3523	0.5018	3.7005	4.7005
12	1.000E+15	4.6226	1.0508	2.5578	1.8501	1.000E+15	10.5489	1.300	4.6163	5.1497
13	1.4292E+15	9.8301	0.2500	3.9954	5.1204	1.4292E+15	8.1349	0.2500	3.4304	4.1575
14	2.5006E+04	9.9083	0.2401	3.7962	5.3168	4.1241E+04	7.9014	0.1518	3.0683	4.2153
15	1.000E+15	7.8073	0.2500	3.3524	3.9560	1.000E+15	8.0378	0.2500	3.4293	4.0859

Table (II): Comparison (FA) algorithm against Modified firefly algorithm (NEW)

	FA					NEW				
Fun.	**F**$_{min}$	**Max**	**min**	**mean**	**stdDev**	**F**$_{min}$	**Max**	**Min**	**mean**	**stdDev**
1	0.015	10.2783	0.0542	3.5826	5.7976	0.0205	8.	0.0596	2.8959	4.4738
2	5.000E+13	5.0249	0.2500	2.1237	2.0459	5.000E+13	7.7339	0.2500	3.2196	3.9739
3	3.8377E+17	3.2161	1.3000	2.1720	0.9696	3.8377E+17	9.8624	1.300	4.3875	4.7543
4	1.0973E+16	9.2631	0.0500	3.5377	4.9976	1.0973E+16	6.6833	0.0500	2.6778	3.5247
5	5.9646E+15	6.7633	0.0500	2.7044	3.5702	5.9646E+15	13.0339	0.0500	4.7946	7.1627
6	1.4618E+05	9.8140	1.2228	4.2065	4.8596	2.4750E+06	8.2647	1.2811	3.6939	3.9605
7	2.0100E+15	2.000	0.0500	1.0166	0.9751	5.9646E+15	2.000	0.0500	1.0166	0.9751
8	8.5805E+14	12.9194	1.2864	5.1686	6.7124	8.5805E+14	8.6677	1.2865	3.7514	4.2577
9	2.5459E+18	15.000	1.300	6.100	7.7156	2.5459E+18	15.000	1.300	6.100	7.7156
10	3.1597E+6	6.7255	0.7237	2.8881	3.3324	1.6788E+05	10.2830	0.6928	4.06695	5.3877
11	3.3498E+04	9.6969	0.7690	3.7622	5.1397	9.6904E+04	5.9981	0.7297	2.5444	2.9923
12	1.000E+15	4.6226	1.0508	2.5578	1.8501	1.000E+15	11.2811	0.9745	4.7519	5.6777
13	1.4292E+15	9.8301	0.2500	3.9954	5.1204	1.4292E+15	11.7988	0.2500	4.6517	6.2447
14	2.5006E+04	9.9083	0.2401	3.7962	5.3168	1.0442E+03	6.8231	0.1269	2.6923	3.6122
15	1.000E+15	7.8073	0.2500	3.3524	3.9560	1.000E+15	8.4117	0.2500	3.5539	4.29

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