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On the estimation the reliability stress-strength model for the odd Frèchet inverse exponential distribution

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Abstract

The maximum likelihood estimator employs in this paper to shrinkage estimation procedure for an estimate the system reliability R in the stress-strength model, when the stress and strength are independent and non-identically random variables and they follows the odd Frèchet inverse exponential distribution (OFIED). Comparisons among the proposed estimators were presented depend on simulation established on mean squared error (MSE) criteria.

Keywords: odd Frèchet inverse exponential distribution, Reliability Stress–Strength model, Maximum likelihood estimator, Shrinkage estimator, Single Stage Shrunken estimator, mean squared error.

1. Introduction

In the last year, interest in study increase the family odd Frećhet distribution has been proposed by Haq and Elgarhy [5]. The cdf and pdf of the odd Fréchet-G (OFr-G) distribution family are

$$Q\left(x;\theta,\zeta\right)=e^{-\left[\frac{1-G(x;\zeta)}{G(x;\zeta)}\right]^{\theta}}\ ,\ x\in R \tag{1.1}$$

$$q\left(x;\theta,\zeta\right)=\frac{\theta g\left(x;\zeta\right)\left[1-G(x;\zeta)\right]^{\theta-1}}{\left[G(x;\zeta)\right]^{\theta+1}}e^{-\left[\frac{1-G(x;\zeta)}{G(x;\zeta)}\right]^{\theta}}\ ,\ x\in R \tag{1.2}$$

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Where $G(x;\zeta)$ considers a pdf of base distribution [5].

In (2018), M.Elgarhy and A. Sharifah discussed the odd Frèchet inverse Rayleigh distribution and estimation the parameters used maximum likelihood method, [4]. In (2019), Sharifah Alrajhi introduced the odd Frèchet inverse exponential distribution with two parameters and estimated the parameters using maximum likelihood method [10]. In (2020), Ramadan A. Z. and et al introduced the Odd Frèchet Inverse Lomax Distribution and displayed some applied of this distribution [6].

The aim of this paper is to estimate the reliability (R) of system strengths X subjected to common stress Y (stress-strength model) based on the odd Frèchet inverse exponential distribution with unknown shape parameter θ and known shape $\alpha(\alpha = 1)$ via different estimation methods like MLE, as well as some shrinkage estimation methods and make a comparisons among the proposed estimator methods using simulation depends on mean squared error.

The odd Frèchet inverse exponential distribution when $G(x;\alpha) = e^{-\frac{\alpha}{x}}$, has pdf and cdf as below.[10]

$$f(x;\theta,\alpha) = \frac{\theta\alpha}{x^2} e^{\frac{\alpha}{x}} \left[e^{\frac{\alpha}{x}} - 1\right]^{\theta-1} e^{-\left[e^{\frac{\alpha}{x}}-1\right]^{\theta-1}} ;x,\alpha,\theta > 0 \tag{1.3}$$

$$F(x;\theta,\alpha) = e^{-\left[e^{-\frac{\alpha}{x}}-1\right]^{\theta}};\ x,\alpha,\theta > 0 \tag{1.4}$$

Then, when $\alpha = 1$, the pdf and cdf of the odd Frećhet inverse exponential (OFIE) distribution, will be

$$f(x;\theta) = \frac{\theta}{x^2} e^{\frac{1}{x}} \left[e^{\frac{1}{x}} - 1\right]^{\theta-1} e^{-\left[e^{\frac{1}{x}}-1\right]^{\theta-1}} ;\ x,\theta > 0 \tag{1.5}$$

$$F(x;\theta) = e^{-\left[e^{\frac{1}{x}}-1\right]^{\theta}} ;\ x,\theta > 0 \tag{1.6}$$

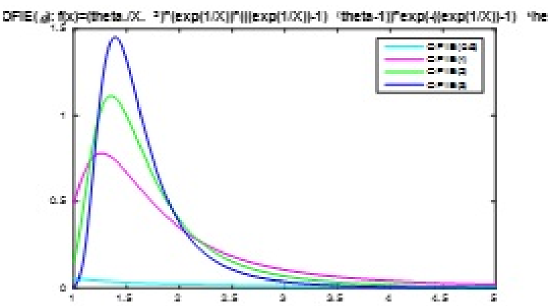

Figure 1

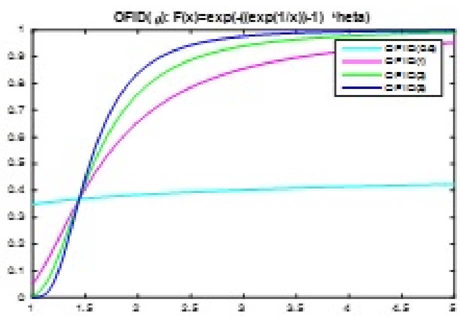

Figure 2

Figure 1 and Figure 2 shown the plot of pdf and cdf respectively of random variable X follows OFIE (θ) for some special choices of the parameter θ (0.5, 1, 2, 3).

In this paper, let (Y) refer to stress and (X) refer to the strength in stress-strength (S-S) model which be considered as random variables follow the odd Frèchet inverse exponential distributed respectively as $OFIE(\theta_1)$ and $OFIE(\theta_2)$. The strength in the distribution which will exceed the stress leads to the reliability system $R = P(Y < X)$ in the stress-strength model[3].

Now, the system reliability of this stress-strength (S-S) model can be derived as below

$$R = P(Y < X) = \int_0^\infty F_y(x)\, f(x)\, \mathrm{dx}$$
$$= \int_0^\infty e^{-\left[e^{\frac{1}{x}}-1\right]^{\theta_2}} \frac{\theta_1}{x^2} e^{\frac{1}{x}} \left[e^{\frac{1}{x}} - 1\right]^{\theta_1 - 1} e^{-\left[e^{\frac{1}{x}}-1\right]^{\theta_1}} \mathrm{dx}$$
$$= \int_0^\infty e^{-\left[e^{\frac{1}{x}}-1\right]^{\theta_1+\theta_2}} \frac{\theta_1}{x^2} e^{\frac{1}{x}} \left[e^{\frac{1}{x}} - 1\right]^{\theta_1 - 1} \mathrm{dx}$$

Assume that, w= $e^{\frac{1}{x}} - 1$, we get x= $\frac{1}{ln(w+1)} = [ln(w+1)]^{-1}$

So,$dx = -\frac{1}{[\ln(w+1)]^2} * \frac{1}{(w+1)}$ put on the assumption above, obtain

$$R = \theta_1 \int_0^\infty e^{-w^{\theta_1+\theta_2}} [w]^{\theta_1 - 1} \mathrm{dw} = \frac{\theta_1}{\theta_1+\theta_2} \Gamma(\frac{\theta_1}{\theta_1+\theta_2}) = \Gamma(\frac{\theta_1}{\theta_1+\theta_2} + 1) \tag{1.7}$$

2. Estimation Methods of R

2.1. Maximum Likelihood Estimator (MLE)

Let $x_1,\ x_2,\ \ldots,x_n$ from OFIE$(\theta_1, 1)$ and $y_1,\ y_2,\ \ldots,y_m$ from OFIE$(\theta_2, 1)$. Then, the likelihood function will be [10]

$$L = L(\theta_1, \theta_2; x, y) = \prod_{i=1}^n f(x_i) \prod_{j=1}^m f(y_j)$$
$$= \prod_{i=1}^n \frac{\theta_1}{(x_i)^2} e^{\frac{1}{x_i}} \left[e^{\frac{1}{x_i}} - 1\right]^{\theta_1 - 1} e^{-\left[e^{\frac{1}{x_i}}-1\right]^{\theta_1}} \prod_{j=1}^m \frac{\theta_2}{{y_j}^2} e^{\frac{1}{y_j}} \left[e^{\frac{1}{y_j}} - 1\right]^{\theta_2 - 1} e^{-\left[e^{\frac{1}{y_j}}-1\right]^{\theta_2}}$$
$$= {\theta_1}^n {x_i}^{-2n} e^{\sum_{i=1}^n \frac{1}{x_i}} \prod_{i=1}^n \left[e^{\frac{1}{x_i}} - 1\right]^{\theta_1 - 1} e^{-\sum_{i=1}^n \left[e^{\frac{1}{x_i}}-1\right]^{\theta_1}} {\theta_2}^m {y_j}^{-2m} e^{\sum_{j=1}^m \frac{1}{y_j}} \prod_{j=1}^m \left[e^{\frac{1}{y_j}} - 1\right]^{\theta_2 - 1} *$$
$$e^{-\sum_{j=1}^m \left[e^{\frac{1}{y_j}}-1\right]^{\theta_2}}$$

$$\ln(l) = n \ln \theta_1 - 2n \sum_{i=1}^n \ln(x_i) + \sum_{i=1}^n \frac{1}{x_i} + (\theta_1 - 1) \sum_{i=1}^n \ln\left(e^{\frac{1}{x_i}} - 1\right) - \sum_{i=1}^n \left[e^{\frac{1}{x_i}} - 1\right]^{\theta_1} +$$
$$m \ln \theta_2 - 2m \sum_{j=1}^m \ln y_j + \sum_{i=1}^n \frac{1}{y_j} + (\theta_2 - 1) \sum_{j=1}^m ln(e^{\frac{1}{y_j}} - 1) - \sum_{j=1}^m \left[e^{\frac{1}{y_j}} - 1\right]^{\theta_2}$$
$$\frac{\partial \ln(l)}{\partial \theta_1} = \frac{n}{\theta_1} - \sum_{i=1}^n \left[e^{\frac{1}{x_i}} - 1\right]^{\theta_1} * \ln\left(e^{\frac{1}{x_i}} - 1\right) + \sum_{i=1}^n \ln\left(e^{\frac{1}{x_i}} - 1\right) = 0$$
$$\frac{\partial \ln(l)}{\partial \theta_2} = \frac{m}{\theta_2} - \sum_{j=1}^m \left[e^{\frac{1}{y_j}} - 1\right]^{\theta_2} * \ln\left(e^{\frac{1}{y_j}} - 1\right) + \sum_{j=1}^m \ln(e^{\frac{1}{y_j}} - 1) = 0$$

Thus, the maximum likelihood estimator of the parameters θ_1 , $\theta_2$will be as follows:

$$\widehat{\theta}_{1\text{mle}} = \frac{n}{\sum_{i=1}^{n}\left[e^{\frac{1}{x_i}}-1\right]^{\theta_{01}} * ln(e^{\frac{1}{x_i}}-1) - \sum_{i=1}^{n}\ln\left(e^{\frac{1}{x_i}}-1\right)}$$
$$\widehat{\theta}_{2\text{mle}} = \frac{m}{\sum_{j=1}^{m}\left[e^{\frac{1}{y_j}}-1\right]^{\theta_{02}} * ln(e^{\frac{1}{y_j}}-1) - \sum_{j=1}^{m} ln(e^{\frac{1}{y_j}}-1)}$$

By substituting $\widehat{\theta}_{1\text{mle}}$ and $\widehat{\theta}_{2\text{mle}}$ in equation (1.7), we get the reliability estimation model $\widehat{\text{R}}_{\text{mle}}$

$$\widehat{R}_{\text{mle}} = \Gamma(\frac{\widehat{\theta}_{1\text{mle}}}{\widehat{\theta}_{1\text{mle}}+\widehat{\theta}_{2\text{mle}}}+1) \tag{2.1}$$

3. Shrinkage Estimation Method

Thompson in 1968, proposed the problem of shrink a usual estimator $\widehat{\theta}$ of the parameter θ to prior information θ_0 using shrinkage weight factor $k(\widehat{\alpha})$, such that $0 \le k(\widehat{\alpha}\ ) \le 1$. He believes that θ_0 is closed to the true value of θ or θ_0 may be near the true value of θ. Thus, the form of Thompson - Type shrinkage estimator of θ say $\widehat{\theta}_{\text{sh}}$ will be [11].

$$\widehat{\theta}_{\text{sh}} = k\widehat{\theta}_{\text{mle}} + (1-k)\theta_0 \tag{3.1}$$

Therefore the shrinkage estimator using shrinkage weight function of $\theta_1$and θ_2 which is defined in equation (3.1), will take the following formula:

$$\widehat{\theta}_{1\text{sh}} = k_1(\theta_1)\,\widehat{\theta}_{1\text{mle}} + (1-K_1(\theta_1))\theta_{10} \tag{3.2}$$
$$\widehat{\theta}_{2\text{sh}} = k_2(\theta_2)\,\widehat{\theta}_{2\text{mle}} + (1-K_2(\theta_2))\theta_{20} \tag{3.3}$$

Where θ_{10} and θ_{20} are prior information (prior estimate) of $\theta_1$and θ_2 respectively.

3.1. The constant Shrinkage Weight Factor (Sh1)

Constant shrinkage weight factor $k_1(\theta_1) = 0.05$, and $k_2(\theta_2) = 0.05$ will be considered in this subsection, [8]. Therefore, the shrinkage estimator of θ_1 and θ_2 through specific constant shrinkage weight factor will be as follows:

$$\widehat{\theta}_{1sh1} = (0.05)\,\widehat{\theta}_{1\text{mle}} + (0.95)\theta_{10} \tag{3.4}$$
$$\widehat{\theta}_{2sh1} = (0.05)\widehat{\theta}_{2\text{mle}} + (0.95)\theta_{20} \tag{3.5}$$

Also, as Thompson mentioned, θ_{1_0} and θ_{1_0} are closed to the real value of θ_1 and θ_2 respectively. Then Substitute equation (3.4) and (3.5) in equation (1.7) obtain the shrinkage estimation of R using the above constant shrinkage weight factor as below:

$$\widehat{R}_{sh1} = \Gamma(\frac{\widehat{\theta}_{1sh1}}{\widehat{\theta}_{1sh1}+\widehat{\theta}_{2sh1}}+1) \tag{3.6}$$

3.2. The Shrinkage Weight Function (Sh2)

We suggest in this subsection the shrinkage weight factor as a function of sample sizes n and m respectively will be considered as the form below.[8]

$$k_1(\theta_1) = \frac{|sin(n)|}{n} \quad , and \quad k_2(\theta_2) = \frac{|sin(m)|}{m}$$

the shrinkage estimator of $\theta_1$and $\theta_2$using above the considered shrinkage weight function

$$\widehat{\theta}_{i\text{sh}} = k_i(\theta_i)\,\widehat{\theta}_{i\text{mle}} + (1 - K_i(\theta_i))\theta_{i0}, \quad i = 1, 2$$

The shrinkage estimation $\widehat{R}_{sh2}$ in equation (1.7) using shrinkage weight function will be:-

$$\widehat{R}_{sh2} = \Gamma\left(\frac{\widehat{\theta}_{1sh2}}{\widehat{\theta}_{1sh2} + \widehat{\theta}_{2sh2}} + 1\right) \tag{3.7}$$

3.3. Preliminary Test Single Stage Shrinkage Estimator

As Thompson recommended shrinking the natural estimator $\widehat{\theta}$ of θ towards the prior guess point θ_0, the pre-test shrinkage estimator defined as.[9, 1, 2]:

$$\widehat{\theta}_{i_{ss}} = \begin{cases} \Psi\left(\widehat{\theta}_i\right)\ \widehat{\theta}_{i_{mle}} + \left(1 - \Psi\left(\widehat{\theta}_i\right)\right)\theta_{i0}, & if \quad \theta_{i_0} \in R \\ \widehat{\theta}_{i_{mle}}, & if \quad \theta_{i0} \notin R \end{cases} \tag{3.8}$$

And,

$$\Psi\left(\widehat{\theta}_1\right) = \frac{n}{n+1}, \qquad \Psi\left(\widehat{\theta}_2\right) = \frac{m}{m+1}. \tag{3.9}$$

Where n and m are sample size of x and y respectively.

Where i=1,2 and R refers to the pre- test region for acceptance the null hypothesis H_0:θ_i= θ_{i0} beside $H_A : \theta_i \neq \theta_{i0}, \widehat{\theta}_{mle}$ is the maximum likelihood estimator of θ, $\Psi\left(\widehat{\theta}_i\right)$ is a shrinkage weight factor such that $0 \leq \Psi(\widehat{\theta}_i) \leq 1$ which may be a function of or may be a constant.

In this paper we assume the region as follow,$R = [\theta_{i0} - \varepsilon, \theta_{i0} + \varepsilon]$, $\varepsilon = 0.001$.

Hence, via equation (1.7), the reliability estimation in (S-S) models using single stage shrinkage estimator become:

$$\widehat{R}_{ss} = \Gamma\left(\frac{\widehat{\theta}_{1ss}}{\widehat{\theta}_{1ss} + \widehat{\theta}_{2ss}} + 1\right) \tag{3.10}$$

4. Simulation Experiments

In this section, numerical results were studied to compare the performance of the different estimators of system reliability using several sample sizes (30, 70 and 100) based on 1000 replication via MSE criteria. For this purpose, Monte Carlo simulation was employed by generating the random sample from the continuous uniform distribution defined on the interval $(0, 1)$ as $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_m$. Transform uniform random samples to follows some distributions special case of n using (c. d. f.),[7].

$$F(x_i) = e^{-[e^{\frac{1}{x_i}} - 1]^{\theta_1}}$$

$$U_i = e^{-[e^{\frac{1}{x_i}} - 1]^{\theta_1}}$$

$$x_i = \frac{1}{ln[1 + (-lnU_i)^{\frac{1}{\theta_1}}]}$$

And, by the same method we get y_j, $y_j = \frac{1}{ln[1+(-lnV_j)^{\frac{1}{\theta_2}}]}$.

The following steps, Compute the real value of R in equation (1.7) and the value of estimation methods of all proposal methods $\widehat{R}_{\mathrm{mle}}$, $\widehat{R}_{\mathrm{sh1}}$, $\widehat{R}_{sh2}$ and $\widehat{R}_{ss}$ in equations (2.1), (3.6), (3.7) and (3.10) respectively.

Based on (L=1000) replication, we calculate the MSE for all proposed estimation methods of $\widehat{R}$ as ollows:

$$MSE = \frac{1}{L}\sum_{i=1}^{L}\left(\widehat{R}_i - R\right)^2$$

At this instant, the estimation of reliability system of stress- strength model for some different assumption parametersofθ_1 *and* θ_2 were put on the following tables:

Table 1: Values of the $\widehat{R}$ when $R = 0.906402$, $\theta_1 = 3$ *and* $\theta_2 = 4$

n	m	$\widehat{R}_{mle}$	$\widehat{R}_{sh1}$	$\widehat{R}_{sh2}$	$\widehat{R}_{SS}$
n=30	m=30	0.951272	0.963536	0.963516	0.919142
	m=70	0.999472	0.999472	0.980423	0.938686
	m=100	0.999999	0.995343	0.967492	0.917937
n=70	m=30	0.999264	0.996531	0.992513	0.928833
	m=70	0.999848	0.999588	0.999535	0.974482
	m=100	0.977798	0.984733	0.96264	0.919273
n=100	m=30	0.998605	0.994641	0.953074	0.892030
	m=70	0.999999	0.997214	0.997593	0.907344
	m=100	0.999647	0.994825	0.994778	0.991673

Table 2: Values MSE of the $\widehat{R}$ when R = 0.906402 , $\theta_1 = 3$ *and* $\theta_2 = 4$.

n	m	$\widehat{R}_{mle}$	$\widehat{R}_{sh1}$	$\widehat{R}_{sh2}$	$\widehat{R}_{SS}$
n=30	m=30	0.002013	0.003264	0.003262	0.000016
	m=70	0.008662	0.005479	0.001042	0.000018
	m=100	0.008760	0.007911	0.003732	0.000013
n=70	m=30	0.008623	0.008123	0.007415	0.000015
	m=70	0.008732	0.008683	0.008674	0.004634
	m=100	0.005097	0.006136	0.003163	0.000017
n=100	m=30	0.008501	0.007786	0.002178	0.000021
	m=70	0.008760	0.008247	0.008316	0.886E-7
	m=100	0.008694	0.007819	0.007810	0.007271

Table 3: Values of the $\widehat{R}$ when R = 0.892979 , $\theta_1 = 4$ *and* $\theta_2 = 3$.

n	m	$\widehat{R}_{mle}$	$\widehat{R}_{sh1}$	$\widehat{R}_{sh2}$	$\widehat{R}_{SS}$
n=30	m=30	0.998412	0.995802	0.995797	0.929887
	m=70	0.999999	0.970353	0.979399	0.911653
	m=100	0.990498	0.951571	0.939923	0.892998
n=70	m=30	0.993856	0.981772	0.977867	0.894436
	m=70	0.998875	0.970794	0.970713	0.890616
	m=100	0.977218	0.952922	0.943603	0.913665
n=100	m=30	0.999511	0.994421	0.939702	0.887668
	m=70	0.999156	0.986442	0.985752	0.910973
	m=100	0.999998	0.963872	0.963823	0.936929

Table 4: Values MSE of the $\widehat{R}$ when R = 0.892979 , $\theta_1 = 4$ *and* $\theta_2 = 3$.

n	m	$\widehat{R}_{mle}$	$\widehat{R}_{sh1}$	$\widehat{R}_{sh2}$	$\widehat{R}_{SS}$
n=30	m=30	0.011116	0.010572	0.010571	0.001362
	m=70	0.011453	0.005987	0.007468	0.000034
	m=100	0.011362	0.007937	0.009675	0.001721
n=70	m=30	0.010176	0.007884	0.007205	0.212E-6
	m=70	0.011214	0.006055	0.006043	0.558E-8
	m=100	0.007096	0.003593	0.002568	0.428E-4
n=100	m=30	0.011349	0.010290	0.002182	0.283E-5
	m=70	0.011273	0.008735	0.008607	0.324E-4
	m=100	0.011453	0.005027	0.005019	0.001932

Table 5: Values of the $\widehat{R}$ when R = 0.886226 , $\theta_1 = 1.5$ *and* $\theta_2 = 2$.

n	m	$\widehat{R}_{mle}$	$\widehat{R}_{sh1}$	$\widehat{R}_{sh2}$	$\widehat{R}_{SS}$
n=30	m=30	0.999947	0.993657	0.993624	0.970792
	m=70	0.925078	0.956121	0.990913	0.885739
	m=100	0.999849	0.999161	0.994818	0.942200
n=70	m=30	0.998826	0.936092	0.983023	0.917693
	m=70	0.998790	0.989937	0.989656	0.927501
	m=100	0.995334	0.987577	0.979245	0.972512
n=100	m=30	0.999861	0.999533	0.999852	0.928696
	m=70	0.999999	0.997177	0.998775	0.886443
	m=100	0.999909	0.998860	0.998564	0.982891

Table 6: Values MSE of the $\widehat{R}$ when R = 0.886226, $\theta_1 = 1.5$ *and* $\theta_2 = 2$.

n	m	$\widehat{R}_{mle}$	$\widehat{R}_{sh1}$	$\widehat{R}_{sh2}$	$\widehat{R}_{SS}$
n=30	m=30	0.012932	0.011541	0.011534	0.007151
	m=70	0.001509	0.004885	0.010959	0.238E-7
	m=100	0.012909	0.012754	0.011792	0.003133
n=70	m=30	0.012679	0.002486	0.009369	0.990E-4
	m=70	0.012670	0.010756	0.010698	0.001703
	m=100	0.011904	0.010272	0.008652	0.007445
n=100	m=30	0.012913	0.012838	0.012910	0.001804
	m=70	0.012944	0.012309	0.012667	0.466E-8
	m=100	0.012924	0.012686	0.012619	0.009344

Table 7: Values of the $\widehat{R}$ when R = 0.902745 , $\theta_1 = 2$ *and* $\theta_2 = 1.5$.

n	m	$\widehat{R}_{mle}$	$\widehat{R}_{sh1}$	$\widehat{R}_{sh2}$	$\widehat{R}_{SS}$
n=30	m=30	0.995813	0.991060	0.991023	0.926309
	m=70	0.999476	0.944481	0.979004	0.886856
	m=100	0.999836	0.976609	0.968417	0.892464
n=70	m=30	0.999984	0.994944	0.992641	0.975126
	m=70	0.999993	0.996914	0.996860	0.913482
	m=100	0.994587	0.991545	0.992746	0.909157
n=100	m=30	0.996112	0.992668	0.995993	0.885712
	m=70	0.999949	0.984829	0.980964	0.893556
	m=100	0.999999	0.998384	0.998234	0.913439

Table 8: Values MSE of the $\widehat{R}$ when R = 0.902745, $\theta_1 = 2$ *and* $\theta_2 = 1.5$.

n	m	$\widehat{R}_{mle}$	$\widehat{R}_{sh1}$	$\widehat{R}_{sh2}$	$\widehat{R}_{SS}$
n=30	m=30	0.008662	0.007799	0.007793	0.000056
	m=70	0.009357	0.001742	0.005815	0.000025
	m=100	0.009427	0.005456	0.004313	0.000011
n=70	m=30	0.009455	0.008501	0.008081	0.005239
	m=70	0.009457	0.008868	0.008858	0.000012
	m=100	0.008435	0.007885	0.008100	0.000004
n=100	m=30	0.008717	0.008086	0.008695	0.000029
	m=70	0.009449	0.006738	0.006118	0.000008
	m=100	0.009458	0.009146	0.009118	0.000011

5. Results Analysis

From the tables above, for all n and m =30,70,100, we conclude that ,the minimum (MSE) for $\widehat{R}_{ss}$ held using preliminary test single stage shrinkage estimator (SS) so it is the best for all n and m , follows ordered best estimators as following:

1. when $\theta_1 = 3, \theta_2 = 4$, (n,m)=(30,30) and $\theta_1 = 1.5, \theta_2 = 2$, (n,m)=(30,70) . The second best methods the Maximum Likelihood Estimator (MLE) and follow the shrinkage weight function and the last was method constant shrinkage weight factor

2. when $\theta_1 = 3, \theta_2 = 4$, (n,m)=(100,70) , $\theta_1 = 4, \theta_2 = 3$, (n,m)=(30,70) ,(30,100), $\theta_1 = 1.5, \theta_2 = 2$, (n,m)=(70,30), (100,30),(100,70) and $\theta_1 = 2, \theta_2 = 1.5$, (n,m)=(30,70), (70,100),(100,30), the second best methods the constant shrinkage weight factor and follow the shrinkage weight function and the last method was Maximum Likelihood Estimator (MLE).

3. As for the other, (n,m) and θ_1, θ_2, the second best methods the shrinkage weight function and follow the constant shrinkage weight factor and the last one was method Maximum Likelihood Estimator (MLE)

6. Conclusion

The simulation results exhibited that, the preliminary test single stage shrinkage estimator (SS) is the best way. Then the resulting estimator $\widehat{R}_{ss}$ perform well and will be the best estimator than the other in the sense of MSE.

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