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ω-α-open sets ω-α-continuity in bitopological spaces

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Abstract

The purposes of this article are to introduce and characterize the notions of (i, j)-ω-α-open sets in bitopological spaces. Besides, It introduces and studies the concepts of (i, j)-ω-α-continuous functions. Furthermore, (i, j)-ω-α-connected and (i, j)-ω-α-set-connected functions are defined in bitopological spaces and some of their properties are studied.

Keywords: (i, j)-α-ω-open sets, (i, j)-ω-α-continuous function, (i, j)-ω-α-connected, (i, j)-ω-α-set-connected function.
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1. Introduction and preliminaries

Kelly [7] in 1963 introduced the notion of bitopological space, after that, many researchers have studied this concept in the general topology. This concept has been studied in different field of the general topology, for example: Granados [3] in 2020 took the notion of α^m-open sets and defined it on bitopological spaces. Besides, Jelić [6] in 1990 defined the notions of (i, j)-α-open sets and (i, j)-α-continuous functions in bitopological spaces, these sets were taking for introducing the sets which are studied in this paper. Otherwise, Carpintero et al. [1] in 2015, took the notions defined by [6] for introducing the concepts of (i, j)-ω-semi open sets. On the other hand, The notion of ω-closed set was originally introduced by Hdeib [4]. Taking into account the concepts of bitopological spaces and ω-closed sets, Hussein et al. [5] in 2013 introduced a new notion related with the concepts mentioned above and defined pairwise $\omega\beta$-continuous functions. Later, Granados [2] in 2020 took those notions and showed new properties over ω-$\mathbb{N}$-α-open sets, as well as, some variants of ω-$\mathbb{N}$-α-continuity.

In this paper, motivated by the authors mentioned above, it took the ideas showed by [2] and it defines the concept of (i, j)-ω-α-open set, besides it studies those notion on (i, j)-ω-α-continuous

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functions where it gets some important results. Besides, (i,j)-ω-α-connected and (i,j)-ω-α-set-connected functions are defined, it also proves and investigates some of their properties.

Throughout this paper, the space (X, τ_i, τ_j) or simply X always means a bitopological spaces on which no separation axioms area assumed unless otherwise mentioned.

Definition 1.1. *A subset A of (X,τ) is said to be ω-closed [4] if it contains all of its condensation points.*

Remark 1.2. *The complement of a ω-closed is called ω-open.*

Definition 1.3. *A subset A of (X,τ) is α-open [9] if $A \subseteq Int(Cl(Int(A)))$.*

Definition 1.4. *A subset A of (X,τ_i,τ_j) is (i,j)-α-open [6] if $A \subset Int_{\tau_i}(Cl_{\tau_j}(Int_{\tau_i}(A)))$.*

Definition 1.5. *Let $f : (X,\tau_i,\tau_j) \to (Y,\sigma_i,\sigma_j)$ be a function. Then, f is said to be (i,j)-α-continuous [6] if $f^{-1}(V)$ is (i,j)-α-open set of X for every σ_i-open set V of Y.*

Theorem 1.6. *Let $f : (X,\tau_i,\tau_j) \to (Y,\sigma_i,\sigma_j)$ be a function. Then, f is said to be pairwise continuous [8] if the induced functions $f_i : (X,\tau_i) \to (Y,\sigma_i)$ and $f_j : (X,\tau_j) \to (Y,\sigma_j)$ are both continuous.*

2. (i,j)-ω-α-open sets

In this section, it defines (i,j)-ω-α-open sets and it shows some properties.

Definition 2.1. *Let (X,τ_i,τ_j) be a bitopological space and $A \subseteq X$. Then A is said to be (i,j)-ω-α-open if for each $x \in A$ there exits an (i,j)-α-open U_x containing x such that $U_x - A$ is a countable set. The complement of an (i,j)-ω-α-open set is an (i,j)-ω-α-closed set.*

Remark 2.2. *The collection of all (i,j)-ω-α-open sets and (i,j)-ω-α-closed sets are denoted by $\omega\alpha BO(X)$ and $\omega\alpha BC(X)$.*

Lemma 2.3. *Every (i,j)-α-open set is (i,j)-ω-α-open set.*

Proof . It follows from the Definition 2.1. □ The converse of the Lemma 2.3 need not be true as can be seen in the following example:

Example 2.4. *Let $X = \{r,t,y\}$, $\tau_i = \{\emptyset, X, \{r,t\}\}$, $\tau_j = \{\emptyset, X, \{t,y\}\}$. Then, $\{r,y\}$ is an (i,j)-ω-α-open set, but it is not an (i,j)-α-open set.*

Lemma 2.5. *Let A and Y be subsets of (X,τ_i,τ_j) such that $A \subseteq Y$. If A is (i,j)-ω-α-open set of X, then A is (i,j)-α-ω-open set of $(Y, \tau_i|_Y, \tau_j|_Y)$.*

Proof . Let A be an (i,j)-α-ω-open set of X, for every $x \in A$, there exits an (i,j)-ω-α-open set U of X containing x such that $U - A$ is a countable. In consequence, it has that U is (i,j)-ω-α-open set of $(Y, \tau_i|_Y, \tau_j|_Y)$ containing x. This is shown that A is (i,j)-α-open set of $(Y, \tau_i|_Y, \tau_j|_Y)$. □

Theorem 2.6. *Let (X,τ_i,τ_j) be a bitopological space and $A \subseteq X$. Then A is said to be (i,j)-ω-α-open if and only if for every $x \in A$, there exits a (i,j)-α-open set U_x containing x and a countable subset B such that $U_x - B \subseteq A$.*

Proof . Let A be an (i,j)-ω-α-open set and $x \in A$, then there exits an (i,j)-α-open subset U_x containing x such that $U_x - A$ is countable. Now, let $B = U_x - A = U_x \cap (X - A)$. Then, $U_x - B \subseteq A$. Conversely, let $x \in A$. Then, there exits an (i,j)-ω-α-open subset U_x containing x and a countable subset B such that $U_x - B \subseteq A$. Therefore, $U_x - A \subseteq B$ and $U_x - A$ is countable. □

Definition 2.7. *Let $\{U_\delta : \delta \in \Delta\}$ a collection of (i,j)-α-open sets in a bitopological space X is called an (i,j)-α-open cover of a subset A of X if $A \subseteq \bigcup_{\delta\in\Delta} U_\delta$.*

Definition 2.8. *Let (X,τ_i,τ_j) be a bitpological space. Then, X is said to be (i,j)-α-Lindeloff, if every (i,j)-α-open cover of X has a countable sub-cover.*

Theorem 2.9. *Let (X,τ_i,τ_j) be a bitopological space. Then, the following statements are equivalent:*

1. *X is (i,j)-α-Lindeloff.*
2. *Every countable cover of X by (i,j)-α-open sets has a countable sub-cover.*

Proof . (2) $\Rightarrow$ (1) : Since every (i,j)-α-open set is (i,j)-ω-α-open set, the proof follows.

(1) $\Rightarrow$ (2) : Let $\{U_\delta : \delta \in \Delta\}$ be a cover of X by (i,j)-ω-α-open sets of X. Now, for each $x \in X$ there exits an $\delta_x \in \Delta$ such that $x \in U_{\delta_x}$. Since U_{δ_x} is an (i,j)-ω-α-open. Then, there exits an (i,j)-α-open set V_{δ_x} such that $x \in V_{\delta_x}$ and $V_{\delta_x} - U_{\delta_x}$ is countable. Then, the family $\{V_\delta : \delta \in \Delta\}$ is an (i,j)-α cover of X and X is (i,j)-α-Lindeloff. Therefore, there exits a countable sub-cover δ_{x_i} with $i \in I$ such that $X = \bigcup_{i\in I} V_{\delta_{x_i}}$. Since $X = \bigcup_{i\in I}[(V_{\delta_{x_i}} - U_{\delta_{x_i}}) \cup U_{\delta_{x_i}}] = \bigcup_{i\in I}[(V_{\delta_{x_i}} - U_{\delta_{x_i}}) \cup \bigcup_{i\in I} U_{\delta_{x_i}}]$. Since $V_{\delta_{x_i}} - U_{\delta_{x_i}}$ is a countable set, for each $\delta(x_i)$, there exits a countable subset $\Delta_{\delta(x_i)}$ of Δ such that $V_{\delta_{x_i}} - U_{\delta_{x_i}} \subseteq \bigcup_{\Delta_{\delta(x_i)}} U_\delta$ and therefore $X \subseteq \bigcup_{i\in I}(\bigcup_{\delta\in\Delta_{\delta(x_i)}} U_\delta) \cup (\bigcup_{i\in I} U_{\delta_{x_i}})$. □

Theorem 2.10. *Let (X,τ_i,τ_j) be a bitopological space and $C \subseteq X$. If B is (i,j)-ω-α-closed set. Then, $C \subseteq J \cup B$, for some (i,j)-ω-α-closed subset J and a countable subset B.*

Proof . If C is (i,j)-ω-α-closed set. Then, $X - C$ is (i,j)-ω-α-open set and hence by Theorem 2.6, for every $x \in X - C$, there exits an (i,j)-ω-α-open set U containing x and a countable set B such that $U - B \subseteq X - C$. Thus, $C \subseteq X - (U - B) = X - (U \cap (X - B)) = (X - U) \cup B$, let $J = X - U$. Then, J is an (i,j)-ω-α-closed set such that $C \subseteq J \cup B$. □

Theorem 2.11. *The union of any family of (i,j)-ω-α-open sets is (i,j)-ω-α-open set.*

Proof . Let $\{A_\delta : \delta \in \Delta\}$ is a collection of (i,j)-ω-α-open subsets of X. Then, for every $x \in \bigcup_{\delta\in\Delta} A_\delta$, $x \in A_\delta$, for some $\delta \in \Delta$. Hence, there exits an (i,j)-ω-α-open subset U containing x, such that $U - A_\delta$ is countable. Now, as $U - (\bigcup_{\delta\in\Delta} A_\delta) \subseteq U - A_\delta$, and thus $U - (\bigcup_{\delta\in\Delta} A_\delta)$ is countable. Therefore, $\bigcup_{\delta\in\Delta} A_\delta$ is an (i,j)-ω-α-open set. □

Remark 2.12. *Let (X,τ_i,τ_j) be a bitopological space, then the following statements hold:*

1. *If a subset A of X is (i,j)-ω-α-open and $U \in \omega O(X,\tau_i) \cap \omega O(X,\tau_j)$, then $A \cap U$ is (i,j)-ω-α-open set.*
2. *The union of the arbitrarily many (i,j)-ω-α-open sets is (i,j)-ω-α-open set.*

Definition 2.13. *The union of all (i,j)-ω-α-open sets contained in $A \subseteq X$ is called (i,j)-ω-α-interior of A and is denoted by $\omega\alpha BInt(A)$.*

Definition 2.14. *The intersection of all (i,j)-ω-α-closed sets of X containing A is called (i,j)-ω-α-closure of A and is denoted by $\omega\alpha BCl(A)$*

The $\omega\alpha BInt(A)$ is an (i,j)-ω-α-open set and the $\omega\alpha BCl(A)$ is an (i,j)-ω-α-closed set.

Theorem 2.15. *Let (X,τ_i,τ_j) be a bitopological space and $A, B \subseteq X$. Then, the following statements hold:*

1. $\omega\alpha BInt(\omega\alpha BInt(A)) = \omega\alpha BInt(A)$.
2. *if* $A \subset B$, *then* $\omega\alpha BInt(A) \subset \omega\alpha BInt(B)$.
3. $\omega\alpha BInt(A \cap B) \subset \omega\alpha BInt(A) \cap \omega\alpha BInt(B)$.
4. $\omega\alpha BInt(A) \cup \omega\alpha BInt(B) \subset \omega\alpha BInt(A \cup B)$.
5. $\omega\alpha BInt(A)$ *is the largest* (i,j)-ω-α-*open subset of* X. *contained in* A.
6. A *is* (i,j)-ω-α-*open if and only if* $A = \omega\alpha BInt(A)$.
7. $\omega\alpha BCl(\omega\alpha BCl(A)) = \omega\alpha BCl(A)$.
8. *If* $A \subset B$, *then* $\omega\alpha BCl(A) \subset \omega\alpha BCl(B)$.
9. $\omega\alpha BCl(A) \cup \omega\alpha BCl(B) \subset \omega\alpha BCl(A \cup B)$.
10. $\omega\alpha BCl(A \cap B) \subset \omega\alpha BCl(A) \cap \omega\alpha BCl(B)$.

Proof . (1), (2), (6), (7) and (8) are follow directly from the Definition 2.1. (3), (4) and (5) are follow from part (2) of this Theorem. (9) and (10) are follow by applying part (8) of this Theorem. □

Theorem 2.16. *Let (X,τ_i,τ_j) be a bitopological space and $A \subseteq X$. Then, $x \in \omega\alpha BCl(A)$ if and only if $U \cap A \neq \emptyset$ for every $U \in \omega\alpha BO(X,x)$.*

Proof . Suppose that $x \in \omega\alpha BCl(A)$ and it knows that $U \cap A \neq \emptyset$, for all $U \in \omega\alpha BO(X,x)$. Now, suppose the contrary that there exits $U \in \omega\alpha BO(X,x)$ such that $U \cap A = \emptyset$, then $A \subseteq X - U$ and $X - U$ is an (i,j)-ω-α-closed set. This follows that $\omega\alpha BCl(A) \subseteq \omega\alpha BCl(X-U) = X - U$. Since $x \in \omega\alpha BCl(A)$, it has $x \in X - U$ and hence $x \notin U$. Which is a contradiction. Therefore, $U \cap A \neq \emptyset$. Conversely, suppose that $U \cap A \neq \emptyset$ for every $U \in \omega\alpha BO(X,x)$. Now, it has to prove that $x \in \omega\alpha BCl(A)$. Suppose the contrary that $x \notin \omega\alpha BCl(A)$. Now, let $U = X - \omega\alpha BCl(A)$, then $U \in \omega\alpha BO(X,x)$ and $U \cap A = (X - (\omega\alpha BCl(A)) \cap A \subseteq (X - A) \cap A = \emptyset$ and this is a contradiction. Therefore, $x \in \omega\alpha BCl(A)$. □

Theorem 2.17. *Let (X,τ_i,τ_j) be a bitopological space and $A \subset X$. Then, the following statements hold:*

1. $\omega\alpha BCl(X - A) = X - \omega\alpha BCl(A)$.
2. $\omega\alpha BInt(X - A) = X - \omega\alpha BInt(A)$.

Proof .

1. Let $x \in X - \omega\alpha BCl(A)$. Then, there exits $U \in \omega\alpha BO(X,x)$ such that $U \cap A = \emptyset$ and hence it has $x \in \omega\alpha BInt(A)$. This shows that $X - \omega\alpha BCl(A) \subset \omega\alpha BInt(X - A)$. Now, take $x \in \omega\alpha BInt(X - A)$. Since $\omega\alpha BInt(X - A) \cap A = \emptyset$, it gets that $x \notin \omega\alpha BCl(A)$. In consequence, $\omega\alpha BCl(X - A) = X - \omega\alpha BInt(A)$.

2. Let $x \in X - \omega\alpha BInt(X - A)$. Since $\omega\alpha BInt(X - A) \cap A = \emptyset$, it has $x \notin \omega\alpha BCl(A)$ and this implies that $x \in X - \omega\alpha BCl(A)$. Now, take $x \in X - \omega\alpha BCl(A)$. Then, there exist $U \in \omega\alpha BO(X, x)$ such that $U \cap A = \emptyset$. Therefore, $\omega\alpha BInt(X - A) = X - \omega\alpha BCl(A)$.

□

Definition 2.18. *Let (X, τ_i, τ_j) be a bitopological space and $A \subseteq X$. Then A is said to be (i, j)-ω-α-neighbourhood of a point $x \in X$ if there exists an (i, j)-ω-α-open set J such that $x \in J \subset A$.*

Theorem 2.19. *Let (X, τ_i, τ_j) be a bitopological space and $A \subseteq X$. Then, A is (i, j)-ω-α-open set if and only if it is an (i, j)-ω-α-neighbourhood of each of its points.*

Proof . Let A be an (i, j)-ω-α-open set of X. Then by the Definition 2.18 A is (i, j)-ω-α-neighbourhood of each of its points. Conversely, If A is an (i, j)-ω-α-neighbourhood of each of its points. Then, for each $x \in A$, there exits $D_x \in \omega\alpha B(X, x)$ such that $D_x \subset A$. In consequence, $A = \bigcup\{D_x : x \in A\}$. Since, each D_x is an (i, j)-ω-α-open and arbitrary union of (i, j)-ω-α-open sets is an (i, j)-ω-α-open set. therefore, A is an (i, j)-ω-α-open set of X. □

Theorem 2.20. *Let (X, τ_i, τ_j) be a bitopological space and for each non-empty (i, j)-ω-α-open set of X is uncountable. Then, $\alpha BCl(A) = \omega\alpha BCl(A)$, for each subset $A \in \tau_i \cup \tau_j$.*

Proof . The implication $\omega\alpha BCl(A) \subseteq \alpha BCl(A)$ is clear. On the other hand, let $x \in \alpha BCl(A)$ and B be an (i, j)-ω-α-open set containing x. By the Theorem 2.6, there exits an (i, j)-α-open set U containing x and a countable set C such that $U - C \subseteq B$. Then, $(U - C) \subseteq B \cap A$ and $(U \cap A) - C \subseteq B \cap A$. Now, let $x \in U$ and $x \in \alpha BCl(A)$ such that $U \cap A \neq \emptyset$ where $U \cap A$ is an (i, j)-ω-α-open set, since U is (i, j)-α-open set and $A \in \tau_i \cup \tau_j$. By hypothesis, each non-empty (i, j)-ω-α-open set of X is uncountable, thus $(U \cap A) - C$. Therefore, $B \cap A$ is uncountable. In consequence, $B \cap A \neq \emptyset$, this implies that $x \in \omega\alpha BCl(A)$. □

Theorem 2.21. *Let (X, τ_i, τ_j) be a topological space. If every (i, j)-ω-α-open set of X is τ_i-open of X. Then, $(X, \omega\alpha BO(X))$ is a topological space.*

Proof .

1. $\emptyset, X \in \omega\alpha BO(X)$.
2. Let $U, V \in \omega\alpha BO(X)$ and $x \in U \cap V$. Then , there exits (i, j)-α-open sets J, K of X containing x such that $J - U$ and $K - V$ are countable. Since $(J \cap K) - (U \cap V) = (J \cap K) \cap ((X - U) \cup (X - V)) \subseteq (J \cap (X - U)) \cup (K \cap (X - V))$, this implies that $(J \cap K) - (U \cap V)$ is a countable set and by hypothesis, the intersection of two (i, j)-α-open sets is an (i, J)-α-open set. Therefore, $U \cap V \in \omega\alpha BO(X)$.
3. The union is followed directly.

□

3. (i, j)-ω-α-continuous functions

In this section, it defines the concept of (i, j)-ω-α-continuous functions. Moreover, it proves some properties.

Definition 3.1. *Let $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be a functions. Then, f is said to be (i, j)-ω-α-continuous if $f^{-1}(V)$ is (i, j)-ω-α-open set of X for every σ_i-open set of Y, where $i \neq j$.*

Theorem 3.2. *Every (i,j)-α-continuous function is (i,j)-ω-α-continuous functions.*

Proof . It follows form the fact that every (i,j)-α-open set is (i,j)-ω-α-open set. $\square$ The converse of the Theorem 3.2 need not be true as can be seen in the following example:

Let $X = \{r,t,y\}$, $\tau_i = \{\emptyset, X, \{r\}, \{t\}, \{r,t\}\}$, $\tau_j = \{\emptyset, X, \{r\}\}$, $\sigma_i = \{\emptyset, X, \{r,t\}\}$, $\sigma_j = \{\emptyset, X, \{r,y\}\}$. Then, the identify function $f:(X,\tau_i,\tau_j) \to (Y,\sigma_i,\sigma_j)$ is (i,j)-ω-α-continuous, but it is not (i,j)-α-continuous.

Proposition 3.3. *Let $f:(X,\tau_i,\tau_j) \to (Y,\sigma_i,\sigma_j)$ be a function. If f is (i,j)-ω-α-continuous and $A \in \omega O(X,\tau_i) \cap \omega O(X,\tau_j)$, then the restriction $f|_A : (A, \tau_i|_A, \tau_j|_A) \to (Y,\sigma_i,\sigma_j)$ is (i,j)-ω-α-continuous*

Proof . Since f is (i,j)-ω-α-continuous, for any $U \in \sigma_i$ of Y, $f^{-1}(U)$ is (i,j)-ω-α-open set of X. By the Remark 2.12 part (1), $f^{-1}(U) \cap A$ is (i,j)-ω-α-open set of X. Therefore, by the Lemma 2.5 $(f|A)^{-1}(U)$ is (i,j)-ω-α-open set of $(A,\tau_i|_A,\tau_j|_A)$. $\square$

Proposition 3.4. *Let $f:(X,\tau_i,\tau_j) \to (Y,\sigma_i,\sigma_j)$ be a function and $X = \bigcup\{V_\delta \in \tau_i : \delta \in \Delta\}$. If the restriction $f|_{V_\delta} : (V_\delta, \tau_i|_{V_\delta}, \tau_j|_{V_\delta}) \to (Y,\sigma_i,\sigma_j)$ is (i,j)-ω-α-continuous for each $\delta \in \Delta$, then f is (i,j)-ω-α-continuous.*

Proof . Let U be σ_i-open set of Y. Since $f|_{V_\delta}$ is (i,j)-ω-α-continuous for each $\delta \in \Delta$, $(f|_{V_\delta})^{-1}(U) = f^{-1}(U) \cap V_\delta$ is (i,j)-ω-α-open set of V_δ. Now, by the Lemma 2.5, $f^{-1}(U) \cap V_\delta$ is (i,j)-ω-α-open set of X for each $\delta \in \Delta$. Now, taking $f^{-1}(U) = {}_{\delta\in\Delta}(f^{-1}(U) \cap V_\delta)$. Now, by the Remark 2.12 part (2), $f^{-1}(U)$ is (i,j)-ω-α-open set of X. Therefore, f is (i,j)-ω-α-continuous. $\square$

Theorem 3.5. *Let $f:(X,\tau_i,\tau_j) \to (Y,\sigma_i,\sigma_j)$ be a function. Then, the following statements are equivalent:*

1. *f is (i,j)-ω-α-continuous.*
2. *For each point $x \in X$ and each σ_i-open set J of Y such that $f(x) \in J$, there is an (i,j)-ω-α-open set A of X such that $x \in A$ and $f(A) \subset J$.*
3. *The inverse image of each σ_i-closed set of Y is an (i,j)-ω-α-closed set of X.*
4. *For $A \subseteq X$, then $f(\omega\alpha BCl(A)) \subset \sigma_i\text{-}Cl(f(A))$.*
5. *For $A \subseteq Y$, then $\omega\alpha BCl(f^{-1}(A)) \subset f^{-1}(\sigma_i\text{-}Cl(A))$.*
6. *For $A \subseteq Y$, then $f^{-1}(\sigma_i\text{-}Int(A)) \subset \omega\alpha BInt(f^{-1}(A))$.*

Proof . (1) $\Rightarrow$ (2) : Let $x \in X$ and J be a σ_i-open set of Y containing $f(x)$. By part (1) of this Theorem, $f^{-1}(J)$ is (i,j)-ω-α-open of X. Let $B = f^{-1}(J)$. Then, $x \in B$ and $f(B) \subset J$.

(2) $\Rightarrow$ (1) : Let J be a σ_i-open of Y and $x \in f^{-1}(J)$. Then, $f(x) \in J$- By part (2) of this Theorem, there is (i,j)-ω-α-open set U_x of X such that $x \in U_x$ and $f(U_x) \subset J$. This implies that $x \in U_x \subset f^{-1}(J)$. Therefore, $f^{-1}(J)$ is an (i,j)-ω-α-open set of X.

(1) $\Leftrightarrow$ (3) : It follows from the fact that for any set A of Y, $f^{-1}(Y-B) = X - f^{-1}(B)$.

(3) $\Rightarrow$ (4) : Let $A \subseteq X$. Since $A \subset f^{-1}(f(A))$, it has that $A \subset f^{-1}(\sigma_i\text{-}Cl(f(A))$. By hypothesis, $f^{-1}(\sigma_i\text{-}Cl(f(A))$ is an (i,j)-ω-α-closed set of Y and hence $\omega\alpha BCl(A) \subset f^{-1}(\sigma_i\text{-}Cl(f(A))$. Then, $f((\omega\alpha Bl(A))) \subset f(f^{-1}(\sigma_i\text{-}Cl(f(A)))) \subseteq \sigma_i\text{-}Cl(f(A))$.

(4) $\Rightarrow$ (3) : Let J be a σ_i-closed set of Y. Then, $f(\omega\alpha BCl(f^{-1}(J)) \subset \sigma_i\text{-}Cl(f(f^{-1}(J))) \subset \sigma_i\text{-}Cl(J) = J$. Therefore, $\omega\alpha BCl(f^{-1}(J)) \subset f^{-1}(J)$. In consequence, $f^{-1}(J)$ is an (i,j)-ω-α-closed set of X.

(4) $\Rightarrow$ (5) : Let $A \subseteq Y$. Now, $f((\omega\alpha BCl(A))) \subset \sigma_i\text{-}Cl(f(f^{-1}(A))) \subset \sigma_i\text{-}Cl(A)$. In consequence, $\omega\alpha BCl(f^{-1}(A)) \subset f^{-1}(\sigma_i\text{-}Cl(A))$.

(5) $\Rightarrow$ (4) : Let $A = f(B)$ where $B \subseteq X$. Then, $\omega\alpha BCl(A) \subset \omega\alpha BCl(B) \subset \omega\alpha BCl(f^{-1}(A)) \subset f^{-1}(\sigma_i\text{-}Cl(A)) = f^{-1}(\sigma_i\text{-}Cl(f(A)))$. Therefore, $f(\omega\alpha BCl(B) \subset \sigma_i\text{-}Cl(f(A))$.

(1) $\Rightarrow$ (6) : Let $A \subseteq Y$. It is clear that $f^{-1}(\sigma_i\text{-}Int(A))$ is an (i,j)-ω-α-open and it has $f^{-1}(\sigma_i\text{-}Int(A)) \subset \omega\alpha BInt(f^{-1}(\sigma_i\text{-}Int(A))) \subset \omega\alpha BInt(f^{-1}(A)$.

(6) $\Rightarrow$ (1) : Let A be a σ_i-open set of Y. Then, $\sigma_i\text{-}Int(A) = A$ and $f^{-1}(A) \subset f^{-1}(\sigma_i\text{-}Int(A)) \subset \omega\alpha BInt(f^{-1}(A))$. Therefore, it has $f^{-1}(A) = \omega\alpha BInt(f^{-1}(A))$. This implies that $f^{-1}(A)$ is an (i,j)-ω-α-open set of X. □

Definition 3.6. *Let $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be a functions. Then, f is said to be (i,j)-ω-α-irresolute if $f^{-1}(V)$ is an (i,j)-ω-α-open set of X for every (i,j)-ω-α-open set V of Y, where $i \neq j$.*

Theorem 3.7. *Let $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ and $g : (Y, \sigma_i, \sigma_j) \to (Z, \theta_i, \theta_j)$ be two functions. Then, the following statements hold:*

1. *$g \circ f$ is (i,j)-ω-α-irresolute, if f is (i,j)-ω-α-irresolute and g is (i,j)-ω-α-irresolute.*
2. *$g \circ f$ is (i,j)-ω-α-continuous, if f is (i,j)-ω-α-continuous and g is pairwise continuous.*
3. *$g \circ f$ is (i,j)-ω-α-continuous, if f is (i,j)-ω-α-irresolute and g is (i,j)-ω-α-continuous.*
4. *$g \circ f$ is (i,j)-ω-α-continuous, if f is (i,j)-ω-α-irresolute and g is pairwise continuous.*

Proof . It begins proof the part (1): Let V be a (i,j)-ω-α-open set of Z, since g is (i,j)-ω-α-irresolute, then $g^{-1}(V)$ is an (i,j)-ω-α-open set of Y. Now, since f is(i,j)-ω-α-irresolute, $f^{-1}(g^{-1}(V))$ is (i,j)-ω-α-open set of X, therefore $g \circ f$ is (i,j)-ω-α-irresolute.

The proof of (2), (3) and (4) are made in the same way to the part (1). □

Definition 3.8. *Let $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be a functions. Then, f is said to be (i,j)-ω-α-open if $f(V)$ is an (i,j)-ω-α-open set of Y for every (i,j)-ω-α-open set or τ_i-open V of X, where $i \neq j$.*

Proposition 3.9. *Let $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be a pairwise continuous and pairwise open function. Then, f is (i,j)-ω-α-open.*

Proof . Let V be an (i,j)-ω-α-open set of X and $y \in f(V)$. Then, there exits $x \in V$ such that $f(x) = y$. Since V is (i,j)-ω-α-continuous, there exits (i,j)-α-open set V_1 of X containing x such that $U_1 - U \subseteq A$, where A is a countable set. Therefore, $f(U_1) - f(U) \subseteq f(A)$. Since f is pairwise continuous and pairwise open, by the Theorem 1.6, $f(U_1)$ is an (i,j)-α-open set of Y containing $y = f(x)$ and hence $f(U)$ is (i,j)-ω-α-open of Y. □

Definition 3.10. *Let $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be a function. Then, f is said to be (i,j)-ω-α-closed if $f(U)$ is an (i,j)-ω-α-closed set of Y for every τ_i-closed set of X.*

Theorem 3.11. *Let $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be a function. Then, the following properties are equivalent:*

1. *f is an (i,j)-ω-α-open.*
2. *$f(\tau_i\text{-}Int(U) \subseteq \omega\alpha BCl(f(U))$, for each subset U of X.*
3. *$\tau_i\text{-}Int(f^{-1}(U)) \subseteq \omega\alpha BInt(U)$, for each subset U of X.*

Proof . (1) $\Rightarrow$ (2) : Let U be a subset of X. Then, τ_i-$Int(U)$ is a τ_i-open set of X. Then, $f(\tau_i\text{-}Int(U))$ is (i,j)-ω-α-open set of Y. Since $f(\tau_i\text{-}Int(U)) \subseteq f(U)$, $f(\tau_i\text{-}Int(U)) = \omega\alpha BInt(f(\tau_i\text{-}Int(U))) \subseteq \omega\alpha BInt(f(U))$.

(2) $\Rightarrow$ (3) : Let U be a subset of Y. Then, $f(\tau i\text{-}Int(f^{-1}(V))) \subseteq \omega\alpha BInt(f(f^{-1}(U)))$. Therefore, $\tau_i\text{-}Int(f^{-1}(U)) \subseteq f^{-1}(\omega\alpha BInt(U))$.

(3) $\Rightarrow$ (1) : Let U be a τ_i-open set of X. Then, $\tau - i\text{-}Int(U) = U$. Now, $V = \tau_i\text{-}Int(V) \subseteq \tau_i\text{-}Int(f^{-1}(f(V)) \subseteq f^{-1}(\omega\alpha BInt(f(V))$. This implies that $f(V) \subseteq f(f^{-1}(\omega\alpha BInt(f(V)))) \subseteq \omega\alpha BInt(f(V))$. In consequence, $f(V)$ is an (i,j)-ω-α-open set of Y. Therefore, f is (i,j)-ω-α-open. $\square$

Theorem 3.12. *Let $f : (X,\tau_i,\tau_j) \to (Y,\sigma_i,\sigma_j)$ be a function. Then, f is an (i,j)-ω-α-closed function if and only if for each subset U of X, the $\omega\alpha BCl(f(U)) \subseteq f(\tau_i\text{-}Cl(U))$.*

Proof . Let f be an (i,j)-ω-α-closed function and U be a subset of X. Then, $f(U) \subseteq f(\tau_i\text{-}Cl(U))$ and $f(\tau_i\text{-}Cl(U))$ is an (i,j)-ω-α-closed set of Y. Therefore, $\omega\alpha BCl(f(U)) \subseteq \omega\alpha BCl(f(\tau_i\text{-}Cl(U))) = f(\tau_i\text{-}Cl(U))$. Conversely, let U be a τ_i-closed of X. Then, $f(U) \subseteq \omega\alpha BCl(f(U)) \subseteq f(\tau_i\text{-}Cl(U)) = f(U)$. Hence, $f(U)$ is an (i,j)-ω-α-closed set of Y. In conclusion, f is an (i,j)-ω-α-closed function. $\square$

Definition 3.13. *Let $f : (X,\tau_i,\tau_j) \to (Y,\sigma_i,\sigma_j)$ be a function. Then, the graph $G(f)$ of f is said to be (i,j)-ω-α-closed in $X \times Y$ if for each $(x,y) \in (X \times Y) - G(f)$, there exits $U \in \omega\alpha BO(X,x)$, and a σ_i-open set V of Y containing y such that $(U \times V) \cap G(f) = \emptyset$.*

Lemma 3.14. *Let $f : (X,\tau_i,\tau_j) \to (Y,\sigma_i,\sigma_j)$ be a function. Then, the graph $G(f)$ of f is (i,j)-ω-α-closed in $X \times Y$ if and only if for each $(x,y) \in (X \times Y) - G(f)$, there exits $U \in \omega\alpha BO(X,x))$, and a σ_i-open set V of Y containing y such that $f(U) \cap V = \emptyset$.*

Proof . It follows by the Definition 3.13. $\square$

Theorem 3.15. *Let $f : (X,\tau_i,\tau_j) \to (Y,\sigma_i,\sigma_j)$ be a function. If f is (i,j)-ω-α-continuous and (Y,σ_i) is T_1, then $G(f)$ is (i,j)-ω-α-closed.*

Proof . Let $(x,y) \in (X \times Y) - G(f)$. Then, $y \neq f(x)$. Since, (Y,σ_i) is T_1, there exits a σ_i-open sets V, U of Y such that $f(x) \in V$ and $y \notin U$ and $U \cap V = \emptyset$. Since f is (i,j)-ω-α-continuous, there exits $W \in (i,j)$-ω-α-open, such that $f(W) \subset V$. Therefore, $f(W) \cap U = \emptyset$. In consequence, by the Lemma 3.14, $G(f)$ is (i,j)-ω-α-closed. $\square$

Definition 3.16. *Let (X,τ_i,τ_j) be a bitopological space. Then, X is said to be pairwise Lindeloff [1] if each pairwise open cover of X has a countable sub-cover.*

Theorem 3.17. *Let $f : (X,\tau_i,\tau_j) \to (Y,\sigma_i,\sigma_j)$ be an (i,j)-ω-α-continuous function. If X is (i,j)-α-Lindeloff, then Y is pairwise Lindeloff.*

Proof . Let $\{U_\delta : \delta \in \Delta\}$ be a cover of Y by σ_i-open sets. Then, $\{f^{-1}(U_\delta) : \delta \in \Delta\}$ is an (i,j)-ω-α-open cover of X. Since X is (i,j)-α-Lindeloff and by the Definition 2.8, there exits a countable subset Δ_0 of Δ such that $X = \bigcup_{\delta\in\Delta_0} U_\delta$. Therefore, Y is pairwise Lindeloff. $\square$

Definition 3.18. *Let (X,τ_i,τ_j) be a bitopological space and $A \subseteq X$. The (i,j)-ω-α-frontier of A is defined as (i,j)-ω-α-$Fr(A)$= $\omega\alpha BCl(A) \cap \omega\alpha BCl(X - A)$=$\omega\alpha BCl(A) - \omega\alpha BInt(A)$.*

Theorem 3.19. *Let $f:(X,\tau_i,\tau_j)\to(Y,\sigma_i,\sigma_j)$ be a function. Then, $X-\omega\alpha Bc(f)=\bigcup\{(i,j)$-$\omega$-$\alpha$-$Fr(f^{-1}(V)):V\in\sigma_i, f(x)\in V, x\in X\}$, where $\omega\alpha Bc(f)$ denotes the set of points at which f is (i,j)-ω-α-continuous.*

Proof . Let $x\in X-\omega\alpha Bc(f)$. Then, there exits a σ_i-open set V of Y containing $f(x)$ such that $U\cap(X-f^{-1}(V))\neq\emptyset$ for every (i,j)-ω-α-open set U of X containing x. Thus, $x\in\omega\alpha BCl(X-f^{-1}(V))$. Then, $x\in f^{-1}(V)\cap\omega\alpha BCl(X-f^{-1}(V))\subseteq(i,j)$-$\omega$-$\alpha$-$Fr(f^{-1}(V))$. Hence, $X-\omega\alpha Bc(f)\subseteq\bigcup\{(i,j)$-$\omega$-$\alpha$-$Fr(f^{-1}(V)):V\in\sigma_i, f(x)\in V, x\in X\}$. Conversely, let $x\notin X-\omega\alpha Bc(f)$. Then, for each σ_i-open set V of Y containing $f(x)$, there exits and (i,j)-ω-α-open set U containing x such that $f(U)\subseteq V$ and hence $x\in U\subseteq f^{-1}(V)$. Therefore, $x\in\omega\alpha BInt(f^{-1}(V))$, in consequence $x\notin(i,j)$-ω-α-$Fr(f^{-1}(V))$ for each σ_i-open set V in Y containing $f(x)$. □

Definition 3.20. *Let (X,τ_i,τ_j) be a bitopological space. Then, X is said to be (i,j)-ω-α-T_2 space i for each pair of distinct points $x,y\in X$, there exits (i,j)-ω-α-open sets U and V containing x and y, respectively, such that $U\cap V=\emptyset$.*

Theorem 3.21. *Let $f:(X,\tau_i,\tau_j)\to(Y,\sigma_i,\sigma_j)$ be an (i,j)-ω-α-continuous and injective function, and Y is a T_2 space, then X is a ω-α-T_2 space.*

Proof . Let x and y be two distinct points of X. Then, $f(x)\neq f(y)$. Since Y is T_2, there exist a τ_i-open set U and τ_j-open set V such that $f(x)\in U$, $f(y)\in V$ and $U\cap V=\emptyset$. Therefore, $f^{-1}(U)\cap f^{-1}(V)=\emptyset$. Since f is (i,j)-ω-α-continuous, then $f^{-1}(U)$ is (i,j)-ω-α-open, $f^{-1}(V)$ is (i,j)-ω-α-open , $x\in f^{-1}(U)$ and $y\in f^{-1}(V)$. Which implies that X is ω-α-T_2 space. □

Theorem 3.22. *Let $f:(X,\tau_i,\tau_j)\to(Y,\sigma_i,\sigma_j)$ be an (i,j)-ω-α-continuous and injective function, and Y is a ω-α-T_2 space, then X is a ω-α-T_2 space.*

Proof . The proof is similar to the Theorem 3.21. □

4. (i,j)-ω-α-continuous functions and (i,j)-ω-α-connected spaces

In this section, it defines the concepts of (i,j)-ω-α-connected space, (i,j)-ω-α-set-connected, (i,j)-ω-α-extremally disconnected and (i,j)-ω-α-C-compact space. Besides it proves some properties on (i,j)-ω-α-continuous functions.

Definition 4.1. *Let (X,τ_i,τ_j) be a bitopological space. Then, X is said to be (i,j)-ω-α-connected if X cannot be expressed as the union of two non-empty disjoint (i,j)-ω-α-open sets.*

Remark 4.2. *If A is both (i,j)-ω-α-open set and (i,j)-ω-α-closed set of a bitopological space X, then A is called (i,j)-ω-α-coplen, where $i\neq j$.*

Definition 4.3. *Let (X,τ_i,τ_j) be a bitopological space. Then, X is said to be pairwise connected [10] if it cannot be expressed as the union of two non-empty disjoint sets U and V such that U is τ_i-open and V is τ_j-open, where $i\neq j$.*

Proposition 4.4. *If $f:(X,\tau_i,\tau_j)\to(Y,\sigma_i,\sigma_j)$ is a (i,j)-ω-α-continuous and surjection function, besides X is (i,j)-ω-α-connected, then Y is pairwise connected.*

Proof . Suppose that Y is not pairwise connected. Then, there exits $U \in \sigma_i$ and $V \in \sigma_j$ such that $U, V \neq \emptyset$, $U \cap V = \emptyset$ and $U \cup V = Y$. Since f is surjection, it has $f^{-1}(U) \neq \emptyset$ and $f^{-1}(V) \neq \emptyset$. Besides, since f is (i,j)-ω-α-continuous, $f^{-1}(U)$ is (i,j)-ω-α-continuous, it has $f^{-1}(U)$ is (i,j)-ω-α-open and $f^{-1}(V)$ is (i,j)-ω-α-open such that $f^{-1}(U) \cap f^{-1}(V) = \emptyset$ and $f^{-1}(U) \cup f^{-1}(V) = X$. This implies that X is not (i,j)-ω-α-connected, which is a contradiction. In consequence Y is pairwise connected. □

Proposition 4.5. *If $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ is a (i,j)-ω-α-irresolute and surjection function, besides X is (i,j)-ω-α-connected, then Y is (i,j)-ω-α-connected.*

Proof . The proof is similar to the Proposition 4.4. □

Definition 4.6. *A function $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ is said to be (i,j)-ω-α-set-connected if $f(x)$ is (i,j)-ω-α-connected between $f(A)$ and $f(B)$ in the bitopological space X which is (i,j)-ω-α-connected.*

Theorem 4.7. *Let $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be a function (i,j)-ω-α-set-connected if and only if $f^{-1}(F)$ is an (i,j)-ω-α-coplen set of X for any (i,j)-ω-α-coplen F set of Y.*

Proof . Necessity: Let f be (i,j)-ω-α-set-connected and F be (i,j)-ω-α-coplen set of Y. Now, suppose that $f^{-1}(F)$ is not (i,j)-ω-α-copplen set of X, then X is (i,j)-ω-α-connected between $f^{-1}(F)$ and $X - f^{-1}(F)$. Since f is (i,j)-ω-α-set-connected, Y is (i,j)-ω-α-connected between $f(f^{-1}(F))$ and $f(X - f^{-1}(F))$. But, $f(f^{-1}(F)) = F \cap Y = F$ and $f(X - f^{-1}(F)) = Y - F$, in consequence F is not (i,j)-ω-α-clopen set of Y and this is a contradiction. Therefore, $f^{-1}(F)$ is an (i,j)-ω-α-coplen set of X.

Sufficiency: Let $f^{-1}(F)$ be an (i,j)-ω-α-clopen set of X for any (i,j)-ω-α-coplen F set of Y and let X be (i,j)-ω-α-connected between A and B. Now, suppose that Y is not (i,j)-ω-α-connected between $f(A)$ and $f(B)$, then there exits an (i,j)-ω-α-coplen F set of Y such that $f(A) \subset F \subset Y - f(B)$. But, $A \subset f^{-1}(F) \subset X - B$ and $f^{-1}(F)$ is an (i,j)-ω-α-coplen set of X and this is a contradiction, because X is (i,j)-ω-α-connected. Therefore, f is (i,j)-ω-α-connected. □

Lemma 4.8. *Let $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be a function (i,j)-ω-α-set-connected and $A \subset X$ such that $f(A)$ is an (i,j)-coplen set of Y. Then, the restriction $f|_A : A \to Y$ is (i,j)-ω-α-set-connected.*

Proof . Let A be (i,j)-ω-α-connected space between B and C. Then, X is (i,j)-ω-α-connected between B and C of Y is (i,j)-ω-α-connected between $f(B)$ and $f(C)$. Since $f(A)$ is an (i,j)-coplen set of Y, then $f(A)$ is an (i,j)-ω-α-connected between $f(B)$ and $f(C)$. □

Definition 4.9. *Let (X, τ_i, τ_j) be a bitopological space. Then, X is said to be (i,j)-ω-α-extremally disconnected if the (i,j)-ω-α-closure of any (i,j)-ω-α-open set is (i,j)-ω-α-open set, where $i \neq j$.*

Theorem 4.10. *Let $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be a function (i,j)-ω-α-set-connected. If Y is (i,j)-ω-α-T_2 space and (i,j)-ω-α-extremally disconnected, then $f|_A : A \to Y$ is constant for every (i,j)-ω-α-connected subset A of X.*

Proof . Let $x, y \in A$ and $x \neq y$. Suppose that $f(x) \neq f(y)$ in Y. Since Y is (i,j)-ω-α-T_2 space and (i,j)-ω-α-extremally disconnected, there exits (i,j)-ω-α-coplen set U of Y such that $f(x) \in U$ and $f(y) \notin U$. Now, since f is (i,j)-ω-α-set-connected, it has $f^{-1}(U)$ is (i,j)-ω-α-coplen set of X. And so, by the Lemma 2.5, $f^{-1}(U) \cap A$ is a non-empty proper (i,j)-ω-α-coplen set of the subset A, this implies that A is not (i,j)-ω-α-connected space and this is a contradiction. Therefore, $f(x) = f(y)$ and hence $f|_A : A \to Y$ is constant. □

Definition 4.11. *Let (X, τ_i, τ_j) be a bitopological space. Then, X is said to be (i,j)-ω-α-C-compact if given and (i,j)-ω-α-closed set A of X and a cover $\{V_\delta : \delta \in \Delta\}$ of A by (i,j)-ω-α-open sets of X, then there exits a finite subset $\Delta 0$ of Δ such that $A \subset \bigcup\{\omega\alpha BCl(V_\delta : \delta \in \Delta_0)\}$, where $i \neq j$.*

Theorem 4.12. *Let Y be (i,j)-ω-α-extremally disconnected, (i,j)-ω-α-C-compact and (i,j)-ω-α-T_2. Then, $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ is (i,j)-ω-α-irresolute if and only if it is (i,j)-ω-α-set-connected.*

Proof . Necessity: The proof is easy following the Definition.

Sufficiency: Let f be not (i,j)-ω-α-irresolute. Then, there exits an (i,j)-ω-α-closed set J of Y such that $f^{1-}(J)$ is not an (i,j)-ω-α-closed set of X. Now, let $x \in \omega\alpha BCl(f^{-1}(J)) - f^{-1}(J)$. Then X is (i,j)-ω-α-connected between $f^{-1}(J)$ and x. Hence, Y is (i,j)-ω-α-connected between $f(f^{-1}(J))$ and $f(x)$. In consequence Y is (i,j)-ω-α-connected between J and $f(x)$. Since Y is (i,j)-ω-α-T_2, for each $y \in J$ there exits an (i,j)-ω-α-open set U_y containing y in Y such that $f(x) \notin \omega\alpha BCl(U - y)$. Then, the family $\{U_y : y \in J\}$ is a cover of F by (i,j)-ω-α-open sets of Y. Now, since Y is (i,j)-ω-α-C-compact, there exist a finite number of points $y_1, y_2, ..., y_n$ in J such that $J \subset \bigcup_{i=1}^{n} \omega\alpha BCl(U_{y_i}) = U$. Then, U is (i,j)-ω-α-coplen set of Y since Y is (i,j)-ω-α-extremally disconnected. Besides, $f(x) \notin U$ since $f(x)\omega\alpha BCl(U_y)$ for any $y \in J$ an this is a contradiction. Hence f is (i,j)-ω-α-irresolute. □

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