Forms of $\varpi$-continuous functions between bitopological spaces

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Abstract

The paper introduces the concepts of $\varpi$-strongly (resp., $\varpi$-closure, $\varpi$-weakly) form of continuous functions on bitopological spaces, furthermore, we introduce theorems, characterizing on the class of functions, show how it can be studied from a different point of view.

Keywords: $\varpi$-strongly continuous, $\varpi$-closure continuous, $\varpi$-weakly continuous, bitopological spaces.

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1. Introduction and notations

Let $\mathcal{X}$ be anon empty set and $\mathcal{T}_1, \mathcal{T}_2$ are two topologies on $\mathcal{X}$, then the triple $(\mathcal{X}, \mathcal{T}_1, \mathcal{T}_2)$ is called bitopological spaces [1]. For other notions or notations not defined here we follow closely S. Willard [2].

Definition 1.1. A point $x$ of a space $\mathcal{X}$ is called a condensation point of the sub set $\mathcal{W} \subseteq \mathcal{X}$ if every neighbourhood of $x$ contains an uncountable subset of this set.

Definition 1.2. A subset $\mathcal{W}$ of a space $\mathcal{X}$ is called $\varpi$-closed if all its condensation points contains it. Also the $\varpi$-closure of a set $\mathcal{W}$ is the intersection of all $\varpi$-closed sets that contains $\mathcal{W}$, and denoted by $\text{Cl}^\varpi \mathcal{W}$, then $\mathcal{W}$ is $\varpi$-closed if and only if $\mathcal{W} = \text{Cl}^\varpi \mathcal{W}$. The complete of a $\varpi$-closed is called $\varpi$-open. Similarly, the $\varpi$-interior of a set $\mathcal{W}$ in a space $\mathcal{X}$, denoted by $\text{Int}^\varpi$, consists points $x$ of $\mathcal{W}$ such that for some open set $\mathcal{U}$ containing $x$ such that $\text{Cl}^\varpi \mathcal{U} \subseteq \mathcal{W}$, then $\mathcal{W}$ is $\varpi$-open if and only if $\mathcal{W} = \text{Int}^\varpi \mathcal{W}$, or we can write it as $\mathcal{X} - \mathcal{W}$ is $\varpi$-closed. Form above, we have every closed set is $\varpi$-closed and every open set is $\varpi$-open.

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Definition 1.3. A bitopological space \((X, \mathcal{F}_1, \mathcal{F}_2)\) is called pairwise Hausdorff space if for each pair of difference points \(x_1 \) and \(x_2 \) in \((X, \mathcal{F}_1, \mathcal{F}_2)\), then there is \(\mathcal{F}_1\)-open set \(A\) and \(\mathcal{F}_2\)-open set \(N\) such that \(x_1 \in A\) and \(x_2 \in N\), where \(A\) and \(N\) are disjoint.

Definition 1.4. A function \(f : (X, \mathcal{F}_1, \mathcal{F}_2) \rightarrow (Y, \mathcal{F}_1, \mathcal{F}_2)\) is called pairwise continuous if \(f : (X, \mathcal{F}_1) \rightarrow (Y, \mathcal{F}_1)\) and \(f : (X, \mathcal{F}_2) \rightarrow (Y, \mathcal{F}_2)\) are continuous.

Let \(f : (X, \mathcal{F}_1, \mathcal{F}_2) \rightarrow (Y, \mathcal{F}_1, \mathcal{F}_2)\) be a function, we will use the following symbol in this work as follow:

\(\mathcal{F}_1 \text{Cl}^\omega(\mathcal{A})\) denoted the \(\mathcal{F}_1\)-\(\omega\)-closed of a set \(\mathcal{A} \subseteq (X, \mathcal{F}_1)\)

\(\mathcal{F}_1 \text{Cl}^\omega(\mathcal{N})\) denoted the \(\mathcal{F}_1\)-\(\omega\)-closed of a set \(\mathcal{N} \subseteq (Y, \mathcal{F}_1)\)

\(\mathcal{F}_1 \text{Int}^\omega(\mathcal{A})\) denoted the \(\mathcal{F}_1\)-\(\omega\)-interior of a set \(\mathcal{A} \subseteq (X, \mathcal{F}_1)\)

\(\mathcal{F}_1 \text{Int}^\omega(\mathcal{N})\) denoted the \(\mathcal{F}_1\)-\(\omega\)-interior of a set \(\mathcal{N} \subseteq (Y, \mathcal{F}_1)\)

same as for \(\mathcal{F}_2\) and \(\mathcal{F}_2\) with respect to \((X, \mathcal{F}_2)\) and \((Y, \mathcal{F}_2)\) respectively.

A set \(\mathcal{A}\) is called \(\mathcal{F}_1\)-\(\omega\)-closed if and only if \(\mathcal{F}_1 \text{Cl}^\omega(\mathcal{A}) = \mathcal{A}\),

A set \(\mathcal{N}\) is called \(\mathcal{F}_1\)-\(\omega\)-open if and only if \(\mathcal{F}_1 \text{Int}^\omega(\mathcal{N}) = \mathcal{N}\),

A set \(\mathcal{A}\) is called \(\mathcal{F}_1\)-\(\omega\)-continuous if and only if \(\mathcal{F}_1 \text{Cl}^\omega(\mathcal{A}) = \mathcal{A}\),

A set \(\mathcal{N}\) is called \(\mathcal{F}_1\)-\(\omega\)-continuous if and only if \(\mathcal{F}_1 \text{Int}^\omega(\mathcal{N}) = \mathcal{N}\),

same as for \(\mathcal{F}_2\) and \(\mathcal{F}_2\) with respect to \((X, \mathcal{F}_2)\) and \((Y, \mathcal{F}_2)\) respectively.

2. Main Result

The author in [6, 7, 8, 9] define \(\omega\)-strongly (resp., \(\omega\)-closure, \(\omega\)-weakly) continuous functions as follows: A function \(f : X \rightarrow Y\) is called \(\omega\)-strongly (resp., \(\omega\)-closure, \(\omega\)-weakly) continuous, if for each point \(x \in X\) and every open set \(\mathcal{N}\) of \(f(x)\) in \(Y\), there exists an open set \(\mathcal{A}\) containing \(x\) in \(X\) such that \(f(\text{Cl}^\omega(\mathcal{A})) \subseteq \mathcal{N}\) (resp., \(f(\text{Cl}^\omega(\mathcal{A})) \subseteq \text{Cl}^\omega(\mathcal{N})\), \(f(\mathcal{A}) \subseteq \text{Cl}^\omega(\mathcal{N})\)).

Now, we present the main definition in this work.

Definition 2.1. A function \(f : (X, \mathcal{F}_1, \mathcal{F}_2) \rightarrow (Y, \mathcal{F}_1, \mathcal{F}_2)\) is call pairwise \(\omega\)-strongly continuous (resp., \(\omega\)-closure, \(\omega\)-weakly continuous) if, either \(f : (X, \mathcal{F}_1) \rightarrow (Y, \mathcal{F}_1)\) is \(\omega\)-strongly (resp., \(\omega\)-closure, \(\omega\)-weakly) continuous or \(f : (X, \mathcal{F}_2) \rightarrow (Y, \mathcal{F}_2)\) is \(\omega\)-strongly (resp., \(\omega\)-closure, \(\omega\)-weakly) continuous (i.e., for each point \(x \in (X, \mathcal{F}_1)\) and every \(\mathcal{F}_1\)-opening set \(\mathcal{N}_1\) of \(f(x)\) in \(Y\), there exists an \(\mathcal{F}_1\)-opening set \(\mathcal{A}_1\) contain \(x\) in \(X\) such that \(f(\text{Cl}^\omega(\mathcal{A}_1)) \subseteq \mathcal{N}_1\) (resp., \(f(\text{Cl}^\omega(\mathcal{A}_1)) \subseteq \text{Cl}^\omega(\mathcal{N}_1)\), \(f(\mathcal{A}_1) \subseteq \text{Cl}^\omega(\mathcal{N}_1)\)) or for each point \(x \in (X, \mathcal{F}_2)\) and every \(\mathcal{F}_2\)-opening set \(\mathcal{N}_2\) of \(f(x)\) in \(Y\), there exist an \(\mathcal{F}_2\)-opening set \(\mathcal{A}_2\) contain \(x\) in \(X\) such that \(f(\text{Cl}^\omega(\mathcal{A}_2)) \subseteq \mathcal{N}_2\) (resp., \(f(\text{Cl}^\omega(\mathcal{A}_2)) \subseteq \text{Cl}^\omega(\mathcal{N}_2)\), \(f(\mathcal{A}_2) \subseteq \text{Cl}^\omega(\mathcal{N}_2)\)).

Definition 2.2. If \((x_\alpha)\) is a net in a space \(X\), then \((x_\alpha)\) is called \(\omega\)-convergence to \(x \in X\) denoted by \((x_\alpha \xrightarrow{\omega} x)\), if for each neighbourhood \(\mathcal{A}\) of \(x\), there is some \(x_\alpha \in \Lambda\) such that \(\alpha \leq x_0\) implies \(x_\alpha \in \text{Cl}^\omega(\mathcal{A})\). Thus \(x_\alpha \xrightarrow{\omega} x\) if and only if each \(\omega\)-closure nbd of \(x\) contains a tail of \((x_\alpha)\), this is sometime said; \((x_\alpha)\) \(\omega\)-converges to \(x\) if it is eventually in every \(\omega\)-closure nbd of \(x\).

Theorem 2.3. For any \(f : (X, \mathcal{F}_1, \mathcal{F}_2) \rightarrow (Y, \mathcal{F}_1, \mathcal{F}_2)\) the follow are equivalent:

(a) \(f\) is pairase \(\omega\)-strongly continuous,

(b) The inverses images of every \(\mathcal{F}_1\)-closed sets is \(\mathcal{F}_1\)-\(\omega\)-closed and the inverses images of every \(\mathcal{F}_2\)-closed sets is \(\mathcal{F}_2\)-\(\omega\)-closed,

(c) The inverses images of every \(\mathcal{F}_1\)-opening set is \(\mathcal{F}_1\)-\(\omega\)-opening and the inverses images of every
Theorem 2.4. For any \( f : (X, T_1, T_2) \to (Y, F_1, F_2) \) the follow are equivalent:

(a) \( f \) is pairwise \( \varpi \)-continuous.

(b) The inverses images of every \( F_1 \)-\( \varpi \)-closed sets is \( F_1 \)-\( \varpi \)-closed and the inverses images of every \( F_2 \)-\( \varpi \)-closed sets is \( F_2 \)-\( \varpi \)-closed.

(c) The inverses images of every \( F_1 \)-\( \varpi \)-opening set is \( F_1 \)-\( \varpi \)-opening and the inverses images of every \( F_2 \)-\( \varpi \)-opening sets is \( F_2 \)-\( \varpi \)-open.

(d) For each \( x \in (X, T_1, T_2) \) and each net \( x \to x \), we have \( f(x) \to f(x) \).

Proof. (a) \( \Rightarrow \) (b) Let \( N_1 \) be \( F_1 \)-closed sets in \((Y, F_1)\) and \( N_2 \) be \( F_2 \)-closed set in \((Y, F_2)\). Suppose that \( f^{-1}(N_1) \) is \( F_1 \)-closed in \((X, T_1)\) and \( f^{-1}(N_2) \) is \( F_2 \)-closed in \((X, T_2)\). Then there is a point \( x \in f^{-1}(N_1) \cup f^{-1}(N_2) \) such that for every \( F_1 \)-open set \( A_1 \) and every \( F_2 \)-open set \( A_2 \) both containing \( x \) we have \( f(T_{\text{Cl}}^\varpi(A_1) \cap f^{-1}(N_1)) \neq \emptyset \) and \( f(T_{\text{Cl}}^\varpi(A_2) \cap f^{-1}(N_2)) \neq \emptyset \). Since \( f(x) \notin N_1 \cup N_2 \), \( \forall x \in N_1 \) is \( F_1 \)-open and \( \forall x \in N_2 \) is \( F_2 \)-open, both containing \( f(x) \), having the property that no \( \varpi \)-closed neighbourhood of \( x \) will map into \( \forall x \in N_1 \) and \( \forall x \in N_2 \under f \). Consequently, \( f \) is not pairwise \( \varpi \)-strongly continuous \( \square \) at \( x \).

This contradiction implies that \( f^{-1}(N_1) \) is \( \varpi \)-closed in \((X, T_1)\) and \( f^{-1}(N_2) \) is \( \varpi \)-closed in \((X, T_2)\).

(b) \( \Rightarrow \) (c) Let \( N_1 \) be \( F_1 \)-opening sets in \((Y, F_1)\) and \( N_2 \) be \( F_2 \)-open set in \((Y, F_2)\).

Then \( \forall x \in N_1 \) is \( F_1 \)-closed and \( \forall x \in N_2 \) is \( F_2 \)-closed. By (b) \( f^{-1}(\forall x \in N_1) \) is \( \varpi \)-closed in \((X, T_1)\) and \( f^{-1}(\forall x \in N_2) \) is \( \varpi \)-closed in \((X, T_2)\).

(c) \( \Rightarrow \) (d) Let \( x \in (X, T_1, T_2) \) and let a net \( x \to x \).

Let \( N_1 \) be \( F_1 \)-open set in \((Y, F_1)\) and \( N_2 \) be \( F_2 \)-open set in \((Y, F_2)\).

Thus there exists \( \varpi \)-open \( N_1 \) and \( \varpi \)-open \( N_2 \) such that \( x \in N_1 \subseteq T_{\text{Cl}}^\varpi(A_1) \subseteq f^{-1}(N_1) \) and \( x \in N_2 \subseteq T_{\text{Cl}}^\varpi(A_2) \subseteq f^{-1}(N_2) \).

The \( \varpi \)-convergence and \( \varpi \)-convergence of \( x \) is eventually in \( \forall x \in T_{\text{Cl}}^\varpi(A_1) \) and \( \forall x \in T_{\text{Cl}}^\varpi(A_2) \) respectively.

Now consider the directed sets \( D_1 = \{ x \alpha : \forall x \alpha \} \) and \( D_2 = \{ x \beta : \forall x \beta \} \) using by reverse inclusion where \( \forall x \alpha \) and \( \forall x \beta \) both contains \( x \) and \( x \in \forall x a \subseteq \forall x \alpha \cup \forall x \beta \) such that \( f(x) \subseteq N_1 \cup N_2 \).

Then the net \( g_1 : D_1 \to (X, T_1) \) and \( g_2 : D_2 \to (X, T_2) \) defined by \( g_1(x, \alpha) = x \alpha \), \( \forall x \alpha \)-converges to \( x \) and \( g_2(x, \alpha) = x \alpha \), \( \forall x \beta \)-converges to \( x \), but the net does not converge to \( f(x) \).

The contradiction we obtained implies that \( f \) is pairwise \( \varpi \)-strongly continuous function. \( \square \)

Similarly, we proving the following theorems:

Theorem 2.5. For any \( f : (X, T_1, T_2) \to (Y, F_1, F_2) \) the follow are equivalent:

(a) \( f \) is pairwise \( \varpi \)-weakly continuous.

(b) The inverses images of every \( F_1 \)-\( \varpi \)-closed sets is \( F_1 \)-\( \varpi \)-closed and the inverses images of every \( F_2 \)-\( \varpi \)-closed sets is \( F_2 \)-\( \varpi \)-closed.

(c) The inverses images of every \( F_1 \)-\( \varpi \)-opening set is \( F_1 \)-\( \varpi \)-opening and the inverses images of every \( F_2 \)-\( \varpi \)-opening sets is \( F_2 \)-\( \varpi \)-open.

(d) For each \( x \in (X, T_1, T_2) \) and each net \( x \to x \), we have \( f(x) \to f(x) \).
Definition 2.6. A bitopological space \((X, T_1, T_2)\) is called pairwise \(\varkappa\)-Urysohn if for each pairs of different point \(x_1\) and \(x_2\) in \((X, T_1, T_2)\) then there is a \(T_1\)-opening sets \(A\) and \(T_2\)-opening sets \(B\) such that \(x_1 \in A\) and \(x_2 \in B\). \(T_1Cl^\varkappa(A) \cap T_2Cl^\varkappa(B) \cap f^{-1}(\varkappa) = \phi\).

Theorem 2.7. If \(f : (X, T_1, T_2) \rightarrow (Y, F_1, F_2)\) be a pairwise \(\varkappa\)-strongly continuous injective and \((X, F_1, F_2)\) be pairwise Hausdorff. Then \((X, T_1, T_2)\) is pairwise \(\varkappa\)-Urysohn.

Proof. Let \(x_1, x_2 \in X\) such that \(x_1 \neq x_2\). Then \(f(x_1) \neq f(x_2)\). By hypothesis \((X, F_1, F_2)\) is pairwise Hausdorff, then there exist disjointing sets \(F_1\)-opening \(N_1\) and \(F_2\)-opening \(N_2\) contain \(f(x_1)\) and \(f(x_2)\) respective. Since \(f\) is pairwise \(\varkappa\)-strongly continuous, there exist \(T_1\)-opening sets \(A\) and \(T_2\)-opening sets \(B\) containing \(x_1\) and \(x_2\) respectively, such that \(f(T_1Cl^\varkappa(A)) \subseteq N_1\) and \(f(T_2Cl^\varkappa(B)) \subseteq N_2\). It follows that \(f^{-1}(f(T_1Cl^\varkappa(A))) \subseteq f^{-1}(N_1)\) and \(f^{-1}(f(T_2Cl^\varkappa(B))) \subseteq f^{-1}(N_2)\), therefore \(T_1Cl^\varkappa(A) \subseteq f^{-1}(N_1)\) and \(T_2Cl^\varkappa(B) \subseteq f^{-1}(N_2)\). Therefore \(T_1Cl^\varkappa(A) \cap T_2Cl^\varkappa(B) = \phi\), So \((X, T_1, T_2)\) is pairwise \(\varkappa\)-Urysohn.

Similarly, we can proving the follow theorems:

Theorem 2.8. If \(f : (X, T_1, T_2) \rightarrow (Y, F_1, F_2)\) be a pairwise \(\varkappa\)-closure continuous injectively and let \((X, F_1, F_2)\) be pairwise \(\varkappa\)-Urysohn. Then \((X, T_1, T_2)\) is pairwise \(\varkappa\)-Urysohn.

Theorem 2.9. If \(f : (X, T_1, T_2) \rightarrow (Y, F_1, F_2)\) be a pairwise \(\varkappa\)-closure continuous injectively and let \((X, F_1, F_2)\) be pairwise \(\varkappa\)-Urysohn. Then \((X, T_1, T_2)\) is pairwise Hausdorff.

Now, we are study the composition of difference form of pairwise \(\omega\)-continuous functions.

Theorem 2.10. If \(f : (X, T_1, T_2) \rightarrow (Y, F_1, F_2)\) be a pairwise \(\varkappa\)-strongly continuous and \(g : (Y, F_1, F_2) \rightarrow (P, K_1, K_2)\) be pairwise \(\varkappa\)-strongly continuous. Then \(gof : (X, T_1, T_2) \rightarrow (P, K_1, K_2)\) is pairwise \(\varkappa\)-strongly continuous.

Proof. take \(x \in (X, T_1, T_2)\). Let \(W_1\) be \(K_1\)-open set in \((P, K_1)\) and \(W_2\) be \(K_2\)-open set in \((P, K_2)\) both containing \((gof)(x)\) in \(P\), since \(g\) is pairwise \(\varkappa\)-strongly continuous, there is \(T_1\)-open set \(N_1\) in \((Y, F_1)\) and \(T_2\)-open set \(N_2\) in \((Y, F_2)\) both contain \(f(x)\) in \(Y\) such that \(g(T_1Cl^\varkappa(N_1)) \subseteq W_1\) and \(g(T_2Cl^\varkappa(N_2)) \subseteq W_2\). Since \(f\) is pairwise \(\varkappa\)-strongly continuous, there is \(T_1\)-opening sets \(A_1\) in \((X, T_1)\) and \(T_2\)-opening sets \(A_2\) in \((X, T_2)\) both contain \(x\) in \(X\) such that \(f(T_1Cl^\varkappa(A_1)) \subseteq N_1\) and \(f(T_2Cl^\varkappa(A_2)) \subseteq N_2\). Since \(N_1 \subseteq T_1Cl^\varkappa(A_1)\) and \(N_2 \subseteq T_2Cl^\varkappa(A_2)\), then \(f(T_1Cl^\varkappa(A_1)) \subseteq T_1Cl^\varkappa(N_1)\) and \(f(T_2Cl^\varkappa(A_2)) \subseteq T_2Cl^\varkappa(N_2)\). Therefore, found is \(T_1\)-opening sets \(A_1\) in \((X, T_1)\) and \(T_2\)-opening sets \(A_2\) in \((X, T_2)\) both contain \(x\) in \(X\) such that \((gof)(T_1Cl^\varkappa(A_1)) \subseteq W_1\) and \((gof)(T_2Cl^\varkappa(A_2)) \subseteq W_2\) and \(gof\) is pairwise \(\varkappa\)-strongly continuous.

Theorem 2.11. If \(f : (X, T_1, T_2) \rightarrow (Y, F_1, F_2)\) be a pairwise \(\varkappa\)-strongly continuous and \(g : (Y, F_1, F_2) \rightarrow (P, K_1, K_2)\) be pairwise continuous. Then \(gof : (X, T_1, T_2) \rightarrow (P, K_1, K_2)\) is pairwise \(\varkappa\)-strongly continuous.

Proof. Let \(W_1\) be \(K_1\)-open set in \((P, K_1)\) and \(W_2\) be \(K_2\)-open set in \((P, K_2)\) Since \(g\) is pairwise continuous, we have \(g^{-1}(W_1)\) is \(F_1\)-opening sets in \((Y, F_1)\) and \(g^{-1}(W_2)\) is \(F_2\)-opening set in \((Y, F_2)\). By Theorem 2.3 (c) we have \(f^{-1}(g^{-1}(W_1)) = (gof)^{-1}(W_1)\) is \(T_1\)-opening sets in \((X, T_1)\) and \(f^{-1}(g^{-1}(W_2)) = (gof)^{-1}(W_2)\) is \(T_2\)-opening set in \((X, T_2)\). Therefore, \(gof\) is pairwise \(\omega\)-strongly continuous. Both contain \((gof)(x)\) in \(P\), since \(g\) is pairwise \(\varkappa\)-strongly continuous, there is \(T_1\)-opening sets \(N_1\) in \((Y, F_1)\) and \(T_2\)-opening sets \(N_2\) in \((Y, F_2)\) both contain \(f(x)\) in \(Y\) such that
g(\mathcal{F}_1 \text{Cl}^\omega(\mathcal{A}_1)) \subseteq \mathcal{W}_1 \text{ and } g(\mathcal{F}_2 \text{Cl}^\omega(\mathcal{A}_2)) \subseteq \mathcal{W}_1. \text{ Since } f \text{ is pairwise } \omega \text{-strongly continuous, there is } \mathcal{T}_1 \text{-opening sets } \mathcal{A}_1 \text{ in } (\mathcal{X}, \mathcal{T}_1) \text{ and } \mathcal{T}_2 \text{-opening sets } \mathcal{A}_2 \text{ in } (\mathcal{X}, \mathcal{T}_2) \text{ both contain } x \text{ in } \mathcal{X} \text{ such that } f(\mathcal{F}_1 \text{Cl}^\omega(\mathcal{A}_1)) \subseteq \mathcal{N}_1 \text{ and } f(\mathcal{F}_2 \text{Cl}^\omega(\mathcal{A}_2)) \subseteq \mathcal{N}_2 \text{ since } \mathcal{N}_1 \subseteq \mathcal{F}_1 \text{Cl}^\omega(\mathcal{A}_1) \text{ and } \mathcal{N}_2 \subseteq \mathcal{F}_2 \text{Cl}^\omega(\mathcal{A}_2), \text{ then } f(\mathcal{F}_1 \text{Cl}^\omega(\mathcal{A}_1)) \subseteq \mathcal{F}_1 \text{Cl}^\omega(\mathcal{A}_1) \text{ and } f(\mathcal{F}_2 \text{Cl}^\omega(\mathcal{A}_2)) \subseteq \mathcal{F}_2 \text{Cl}^\omega(\mathcal{A}_2), \text{ so } g(f(\mathcal{F}_1 \text{Cl}^\omega(\mathcal{A}_1))) \subseteq g(\mathcal{F}_1 \text{Cl}^\omega(\mathcal{A}_1)) \text{ and } g(f(\mathcal{F}_2 \text{Cl}^\omega(\mathcal{A}_2))) \subseteq g(\mathcal{F}_2 \text{Cl}^\omega(\mathcal{A}_2)), \text{ also } g(f(\mathcal{F}_1 \text{Cl}^\omega(\mathcal{A}_1))) \subseteq g(\mathcal{F}_1 \text{Cl}^\omega(\mathcal{A}_1)) \text{ and } g(f(\mathcal{F}_2 \text{Cl}^\omega(\mathcal{A}_2))) \subseteq g(\mathcal{F}_2 \text{Cl}^\omega(\mathcal{A}_2)). \text{ Therefore, there is } \mathcal{T}_1 \text{-opening sets } \mathcal{A}_1 \text{ in } (\mathcal{X}, \mathcal{T}_1) \text{ and } \mathcal{T}_2 \text{-opening sets } \mathcal{A}_2 \text{ in } (\mathcal{X}, \mathcal{T}_2) \text{ both contain } x \text{ in } \mathcal{X} \text{ such that } (gof)(\mathcal{F}_1 \text{Cl}^\omega(\mathcal{A}_1)) \subseteq \mathcal{W}_1 \text{ and } (gof)(\mathcal{F}_2 \text{Cl}^\omega(\mathcal{A}_2)) \subseteq \mathcal{W}_2 \text{ and } gof \text{ is pairwise } \omega \text{-strongly continuous}. \square

Similarly, we can prove the following theorems:

**Theorem 2.12.** If \( f : (\mathcal{X}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{Y}, \mathcal{F}_1, \mathcal{F}_2) \) be a pairwise \( \omega \)-weakly continuous and \( g : (\mathcal{Y}, \mathcal{F}_1, \mathcal{F}_2) \to (\mathcal{P}, \mathcal{K}_1, \mathcal{K}_2) \) be pairwise \( \omega \)-strongly continuous. Then \( gof : (\mathcal{X}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{P}, \mathcal{K}_1, \mathcal{K}_2) \) is pairwise continuous.

**Theorem 2.13.** If \( f : (\mathcal{X}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{Y}, \mathcal{F}_1, \mathcal{F}_2) \) be a pairwise \( \omega \)-closure continuous and \( g : (\mathcal{Y}, \mathcal{F}_1, \mathcal{F}_2) \to (\mathcal{P}, \mathcal{K}_1, \mathcal{K}_2) \) be pairwise \( \omega \)-weakly continuous. Then \( gof : (\mathcal{X}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{P}, \mathcal{K}_1, \mathcal{K}_2) \) is pairwise \( \omega \)-weakly continuous.

**Theorem 2.14.** If \( f : (\mathcal{X}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{Y}, \mathcal{F}_1, \mathcal{F}_2) \) be a pairwise \( \omega \)-closure continuous and \( g : (\mathcal{Y}, \mathcal{F}_1, \mathcal{F}_2) \to (\mathcal{P}, \mathcal{K}_1, \mathcal{K}_2) \) be pairwise \( \omega \)-closure continuous. Then \( gof : (\mathcal{X}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{P}, \mathcal{K}_1, \mathcal{K}_2) \) is pairwise \( \omega \)-closure continuous.

**Theorem 2.15.** If \( f : (\mathcal{X}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{Y}, \mathcal{F}_1, \mathcal{F}_2) \) be a pairwise \( \omega \)-weakly continuous and \( g : (\mathcal{Y}, \mathcal{F}_1, \mathcal{F}_2) \to (\mathcal{P}, \mathcal{K}_1, \mathcal{K}_2) \) be pairwise \( \omega \)-weakly continuous. Then \( gof : (\mathcal{X}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{P}, \mathcal{K}_1, \mathcal{K}_2) \) is pairwise \( \omega \)-weakly continuous.

**Lemma 2.16.** If \( f : (\mathcal{X}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{Y}, \mathcal{F}_1, \mathcal{F}_2) \) be a pairwise \( \omega \)-strongly continuous if and only if for each pairwise sub basis \( \mathcal{F}_1 \)-open subset \( \mathcal{I} \) and \( \mathcal{F}_2 \)-open subset \( \mathcal{I} \) of \( (\mathcal{Y}, \mathcal{F}_1, \mathcal{F}_2) \), then \( f^{-1}(\mathcal{I}) \) and \( f^{-1}(\mathcal{J}) \) are \( \mathcal{F}_1 \)-open in \((\mathcal{X}, \mathcal{T}_1) \) and \( \mathcal{F}_2 \)-open in \((\mathcal{X}, \mathcal{T}_2) \).

**Proof.** \((\Rightarrow)\) Follows from Theorem 2.4.

\((\Leftarrow)\) Let \( (\mathcal{I}_a, \mathcal{J}_a) : \alpha \in \Lambda \) be a pairwise sub basis for \((\mathcal{Y}, \mathcal{F}_1, \mathcal{F}_2) \) and suppose that \( f^{-1}(\mathcal{I}_a) \) and \( f^{-1}(\mathcal{J}_a) \) are \( \mathcal{F}_1 \)-open sets and \( \mathcal{F}_2 \)-open sets for each \( \alpha \in \Lambda \). Every \( \mathcal{F}_1 \)-open subset \( \mathcal{I} \) and \( \mathcal{F}_2 \)-open subset \( \mathcal{J} \) of \((\mathcal{Y}, \mathcal{F}_1, \mathcal{F}_2) \) can be written as

\[
\mathcal{I} = \bigcup \{ \mathcal{I}_a \cap \mathcal{J}_a : a \in \Lambda \}; \quad \mathcal{J} = \bigcup \{ \mathcal{J}_a \cap \mathcal{I}_a : a \in \Lambda \}
\]

Then \( f^{-1}(\mathcal{I}) = \bigcup \{ f^{-1}(\mathcal{I}_a) \cap f^{-1}(\mathcal{J}_a) : a \in \Lambda \} \) and \( f^{-1}(\mathcal{J}) = \bigcup \{ f^{-1}(\mathcal{J}_a) \cap f^{-1}(\mathcal{I}_a) : a \in \Lambda \} \). The finite intersect of \( \mathcal{F}_1 \)-open sets is \( \mathcal{F}_1 \)-open and the finite intersect of \( \mathcal{F}_2 \)-open sets is \( \mathcal{F}_2 \)-open. Therefore \( f^{-1}(\mathcal{I}) \) is \( \mathcal{F}_1 \)-open and \( f^{-1}(\mathcal{J}) \) is \( \mathcal{F}_2 \)-open and hence by Theorem 2.3, \( f \) is pairwise \( \omega \)-strongly continuous. \( \square \)

Similarly, we can prove the following lemmas:

**Lemma 2.17.** If \( f : (\mathcal{X}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{Y}, \mathcal{F}_1, \mathcal{F}_2) \) be a pairwise \( \omega \)-closed continuous if and only if for each pairwise sub basis \( \mathcal{F}_1 \)-open subset \( \mathcal{I} \) and \( \mathcal{F}_2 \)-open subset \( \mathcal{J} \) of \((\mathcal{Y}, \mathcal{F}_1, \mathcal{F}_2) \), then \( f^{-1}(\mathcal{I}) \) and \( f^{-1}(\mathcal{J}) \) are \( \mathcal{F}_1 \)-open in \((\mathcal{X}, \mathcal{T}_1) \) and \( \mathcal{F}_2 \)-open in \((\mathcal{X}, \mathcal{T}_2) \).
Lemma 2.18. If \( f : (\mathcal{X}, \mathcal{I}_1, \mathcal{I}_2) \to (\mathcal{Y}, \mathcal{F}_1, \mathcal{F}_2) \) be a pairwise \( \omega \)-weakly continuous if and only if for each pairwise sub basis \( \mathcal{I}_1-\omega \)-open subset \( \mathcal{I} \) and \( \mathcal{I}_2-\omega \)-opening subset \( \mathcal{I} \) of \( (\mathcal{Y}, \mathcal{F}_1, \mathcal{F}_2) \), then \( f^{-1}(\mathcal{I}) \) and \( f^{-1}(\mathcal{I}) \) are \( \mathcal{I}_1 \)-open in \( (\mathcal{X}, \mathcal{I}_1) \) and \( \mathcal{I}_2 \)-open in \( (\mathcal{X}, \mathcal{I}_2) \).

Theorem 2.19. the function \( f : (\mathcal{X}, \mathcal{I}_1, \mathcal{I}_2) \to (\prod \mathcal{X}_\alpha, \mathcal{F}_1, \mathcal{F}_2) \) is a pairwise \( \omega \)-strongly continuous if and only if the compost with each pairwise continuous project function \( \prod \alpha \) is pairwise \( \omega \)-strongly continuous.

Proof. \((\Rightarrow)\) Follows from Theorem 2.11
\((\Leftarrow)\) Let \( \mathcal{I}_1 \) and \( \mathcal{I}_2 \) be a pairwise sub basis \( \mathcal{I}_1 \)-open set in \( (\prod \mathcal{X}_\alpha, \mathcal{F}_1) \) and \( \mathcal{I}_2 \)-open set in \( (\prod \mathcal{X}_\alpha, \mathcal{F}_2) \) for each \( \alpha \in \Lambda \). Then \( \mathcal{I}_1 = \prod \alpha^{-1}(\mathcal{I}_1) \) for some \( \mathcal{I}_1 \)-open set \( \mathcal{I}_1 \) in \( (\mathcal{X}_\alpha, \mathcal{F}_1) \) and \( \mathcal{I}_2 = \prod \alpha^{-1}(\mathcal{I}_2) \) for some \( \mathcal{I}_2 \)-open set \( \mathcal{I}_2 \) in \( (\mathcal{X}_\alpha, \mathcal{F}_2) \). Thus \( f^{-1}(\mathcal{I}_1) = f^{-1}(\prod \alpha^{-1}(\mathcal{I}_1)) = (\prod \alpha \text{of})^{-1}(\mathcal{I}_1) \) is \( \mathcal{I}_1 \)-\( \omega \)-open and \( f^{-1}(\mathcal{I}_2) = f^{-1}(\prod \alpha^{-1}(\mathcal{I}_2)) = (\prod \alpha \text{of})^{-1}(\mathcal{I}_2) \) is \( \mathcal{I}_2 \)-\( \omega \)-open. By Lemma 2.16, \( f \) is pairwise \( \omega \)-strongly continuous. \( \square \)

Similarly, we can prove the following theorems:

Theorem 2.20. the function \( f : (\mathcal{X}, \mathcal{I}_1, \mathcal{I}_2) \to (\prod \mathcal{X}_\alpha, \mathcal{F}_1, \mathcal{F}_2) \) is a pairwise \( \omega \)-closure continuous if and only if the compost with each pairwise continuous project function \( \prod \alpha \) is pairwise \( \omega \)-closure continuous.

Theorem 2.21. the function \( f : (\mathcal{X}, \mathcal{I}_1, \mathcal{I}_2) \to (\prod \mathcal{X}_\alpha, \mathcal{F}_1, \mathcal{F}_2) \) is a pairwise \( \omega \)-weakly continuous if and only if the compost with each pairwise continuous project function \( \prod \alpha \) is pairwise \( \omega \)-weakly continuous.

The following propositions are follow from Theorem 2.19, Theorem 2.20 and Theorem 2.21.

Proposition 2.22. If \( f : (\mathcal{X}, \mathcal{I}_1, \mathcal{I}_2) \to (\mathcal{Y}, \mathcal{F}_1, \mathcal{F}_2) \) be a function and let \( g : (\mathcal{X}, \mathcal{I}_1, \mathcal{I}_2) \to (\mathcal{X} \times \mathcal{Y}, \mathcal{I}_1 \times \mathcal{F}_1, \mathcal{I}_2 \times \mathcal{F}_2) \) be the pairwise graphic function of \( f \) given by \( g(x) = (x, f(x)) \) for every point \( x \in \mathcal{X} \). Then \( f \) is pairwise \( \omega \)-strongly continuous if and only if \( g \) is pairwise \( \omega \)-strongly continuous.

Proposition 2.23. If \( f : (\mathcal{X}, \mathcal{I}_1, \mathcal{I}_2) \to (\mathcal{Y}, \mathcal{F}_1, \mathcal{F}_2) \) be a function and let \( g : (\mathcal{X}, \mathcal{I}_1, \mathcal{I}_2) \to (\mathcal{X} \times \mathcal{Y}, \mathcal{I}_1 \times \mathcal{F}_1, \mathcal{I}_2 \times \mathcal{F}_2) \) be the pairwise graphic function of \( f \) given by \( g(x) = (x, f(x)) \) for every point \( x \in \mathcal{X} \). Then \( f \) is pairwise \( \omega \)-closure continuous if and only if \( g \) is pairwise \( \omega \)-closure continuous.

Proposition 2.24. If \( f : (\mathcal{X}, \mathcal{I}_1, \mathcal{I}_2) \to (\mathcal{Y}, \mathcal{F}_1, \mathcal{F}_2) \) be a function and let \( g : (\mathcal{X}, \mathcal{I}_1, \mathcal{I}_2) \to (\mathcal{X} \times \mathcal{Y}, \mathcal{I}_1 \times \mathcal{F}_1, \mathcal{I}_2 \times \mathcal{F}_2) \) be the pairwise graphic function of \( f \) given by \( g(x) = (x, f(x)) \) for every point \( x \in \mathcal{X} \). Then \( f \) is pairwise \( \omega \)-weakly continuous if and only if \( g \) is pairwise \( \omega \)-weakly continuous.

Lemma 2.25. Let \( (\mathcal{X}_a, \mathcal{I}_1, \mathcal{I}_2) \) be a bitopological spaces and let \( \mathcal{W}_a \) and \( \mathcal{A}_a \) be subsets of \( (\mathcal{X}_a, \mathcal{I}_1) \) and \( (\mathcal{X}_a, \mathcal{I}_2) \) respectively, for each \( i = 1, 2, ..., n \). Then \( \mathcal{W}_a \times \mathcal{W}_a \times ... \times \mathcal{W}_a \times \prod_{a \notin \alpha} \mathcal{I}_1 \subseteq \prod_{a \in A} \mathcal{I}_1, \) and \( \mathcal{A}_a \times \mathcal{A}_a \times ... \times \mathcal{A}_a \times \prod_{a \notin \alpha} \mathcal{I}_2 \subseteq \prod_{a \in A} \mathcal{I}_2 \) are \( \mathcal{I}_1 \)-\( \omega \)-open and \( \mathcal{I}_2 \)-\( \omega \)-open respectively if and only if \( \mathcal{W}_i \) is \( \mathcal{I}_1 \)-\( \omega \)-open in \( (\mathcal{X}_a, \mathcal{I}_1) \) and \( \mathcal{A}_i \) is \( \mathcal{I}_2 \)-\( \omega \)-open in \( (\mathcal{X}_a, \mathcal{I}_2) \) for each \( i = 1, 2, ..., n \).

Proof. \((\Rightarrow)\) Suppose that \( \mathcal{W}_i \) is \( \mathcal{I}_1 \)-\( \omega \)-open in \( (\mathcal{X}_a, \mathcal{I}_1) \) and \( \mathcal{A}_i \) is \( \mathcal{I}_2 \)-\( \omega \)-open in \( (\mathcal{X}_a, \mathcal{I}_2) \) for each \( i = 1, 2, ..., n \).
Then for each \( i \) and each \( x_i \in \mathcal{A}_i \subseteq \mathcal{I}_1 \mathcal{C}_\mathcal{W}(\mathcal{A}_i) \subseteq \mathcal{W}_i, \) \( x_i \in \mathcal{A}_i \subseteq \mathcal{I}_1 \mathcal{C}_\mathcal{W}(\mathcal{A}_i) \subseteq \mathcal{A}_i \).
Thus, for each \( \{ x_a \} \in \mathcal{W} \times \mathcal{W}_a \times \ldots \times \mathcal{W}_\alpha \times \prod_{\alpha \neq a}(\mathcal{Y}_\alpha, \mathcal{T}_\alpha) \subseteq \prod_{\alpha \in A}(\mathcal{Y}_\alpha, \mathcal{T}_\alpha) \),

\( \{ x_a \} \in \mathcal{A}_1 \times \mathcal{A}_2 \times \ldots \times \mathcal{A}_\alpha \times \prod_{\alpha \neq a}(\mathcal{Y}_\alpha, \mathcal{T}_\alpha) \subseteq \prod_{\alpha \in A}(\mathcal{Y}_\alpha, \mathcal{T}_\alpha) \subseteq \mathcal{T}_\mathcal{F}_\mathcal{C}_\mathcal{L}(\mathcal{F}_\mathcal{A}_1) \times \mathcal{T}_\mathcal{F}_\mathcal{C}_\mathcal{L}(\mathcal{F}_\mathcal{A}_2) \times \ldots \times \mathcal{T}_\mathcal{F}_\mathcal{C}_\mathcal{L}(\mathcal{F}_\mathcal{A}_\alpha) \times \prod_{\alpha \neq a}(\mathcal{Y}_\alpha, \mathcal{T}_\alpha) \subseteq \prod_{\alpha \in A}(\mathcal{Y}_\alpha, \mathcal{T}_\alpha) \).

This show that \( \mathcal{W}_1 \times \mathcal{W}_2 \times \ldots \times \mathcal{W}_\alpha \times \prod_{\alpha \neq a}(\mathcal{Y}_\alpha, \mathcal{T}_\alpha) \subseteq \prod_{\alpha \in A}(\mathcal{Y}_\alpha, \mathcal{T}_\alpha) \) is \( \mathcal{T}_1 \)-open.

By a similar way, we get \( \mathcal{A}_1 \times \mathcal{A}_2 \times \ldots \times \mathcal{A}_\alpha \times \prod_{\alpha \neq a}(\mathcal{Y}_\alpha, \mathcal{T}_\alpha) \subseteq \prod_{\alpha \in A}(\mathcal{Y}_\alpha, \mathcal{T}_\alpha) \) is \( \mathcal{T}_2 \)-open

\[ \Rightarrow \text{(⇒) Straightforward.} \]

Theorem 2.26. The function \( \prod_\alpha f_\alpha: \prod_\alpha (\mathcal{X}_\alpha, \mathcal{T}_\alpha, \mathcal{F}_\alpha, \mathcal{F}_\alpha) \rightarrow \prod_\alpha (\mathcal{Y}_\alpha, \mathcal{F}_\alpha, \mathcal{F}_\alpha) \) define by \( \{ \mathcal{X}_\alpha \} \rightarrow \{ f_\alpha(\mathcal{X}_\alpha) \} \) is a pairwise \( \mathcal{W} \)-strongly continuous if and only if each \( f_\alpha: (\mathcal{X}_\alpha, \mathcal{T}_\alpha, \mathcal{F}_\alpha, \mathcal{F}_\alpha) \rightarrow (\mathcal{Y}_\alpha, \mathcal{F}_\alpha, \mathcal{F}_\alpha) \) is pairwise \( \mathcal{W} \)-strongly continuous.

Proof. \( \Rightarrow \) Suppose that \( \prod_\alpha f_\alpha \) is pairwise \( \mathcal{W} \)-strongly continuous. Let \( \mathcal{W}_\alpha \) be \( \mathcal{T}_1 \)-open in \( (\mathcal{Y}_\alpha, \mathcal{T}_\alpha) \) and \( \mathcal{A}_\alpha \) be \( \mathcal{T}_2 \)-open in \( (\mathcal{Y}_\alpha, \mathcal{T}_\alpha) \). Then \( \mathcal{W} = \mathcal{W}_1 \times \prod_{\alpha \neq a}(\mathcal{Y}_\alpha, \mathcal{T}_\alpha) \) and \( \mathcal{A} = \mathcal{A}_1 \times \prod_{\alpha \neq a}(\mathcal{Y}_\alpha, \mathcal{T}_\alpha) \) are pairwise sub basic \( \mathcal{T}_1 \)-open in \( \prod_\alpha (\mathcal{Y}_\alpha, \mathcal{T}_\alpha) \) and \( \mathcal{T}_2 \)-open in \( \prod_\alpha (\mathcal{Y}_\alpha, \mathcal{T}_\alpha) \), respectively. And

\[ (\prod_\alpha f_\alpha)^{-1}(\mathcal{W}) = f_\alpha^{-1}(\mathcal{W}_\alpha) \times \prod_{\alpha \neq a}(\mathcal{Y}_\alpha, \mathcal{T}_\alpha) \]

is \( \mathcal{T}_1 \)-open and \( (\prod_\alpha f_\alpha)^{-1}(\mathcal{A}) = f_\alpha^{-1}(\mathcal{A}_\alpha) \times \prod_{\alpha \neq a}(\mathcal{Y}_\alpha, \mathcal{T}_\alpha) \) is \( \mathcal{T}_2 \)-open. Thus \( f^{-1}(\mathcal{W}_\alpha) \) is \( \mathcal{T}_1 \)-open in \( (\mathcal{X}_\alpha, \mathcal{T}_\alpha) \) and \( f^{-1}(\mathcal{A}_\alpha) \) is \( \mathcal{T}_2 \)-open in \( (\mathcal{X}_\alpha, \mathcal{T}_\alpha) \).

\[ \Rightarrow \text{(⇐) Suppose that } \mathcal{W}_\alpha \text{ is pairwise } \mathcal{W} \text{-strongly continuous.} \]

Then by Lemma 2.25 we have \( (\prod_\alpha f_\alpha)^{-1}(\mathcal{W}) = f^{-1}(\mathcal{W}) \times \prod_{\alpha \neq a}(\mathcal{X}_\alpha, \mathcal{T}_\alpha) \) is \( \mathcal{T}_1 \)-open in \( \prod_\alpha (\mathcal{X}_\alpha, \mathcal{T}_\alpha) \) and \( (\prod_\alpha f_\alpha)^{-1}(\mathcal{A}) = f^{-1}(\mathcal{A}) \times \prod_{\alpha \neq a}(\mathcal{X}_\alpha, \mathcal{T}_\alpha) \) is \( \mathcal{T}_2 \)-open in \( \prod_\alpha (\mathcal{X}_\alpha, \mathcal{T}_\alpha) \). This shows that \( \prod_\alpha f_\alpha \) is pairwise \( \mathcal{W} \)-strongly continuous.

Similarly, we can prove the following theorems:

Theorem 2.27. The function \( \prod_\alpha f_\alpha: \prod_\alpha (\mathcal{X}_\alpha, \mathcal{T}_\alpha, \mathcal{F}_\alpha, \mathcal{F}_\alpha) \rightarrow \prod_\alpha (\mathcal{Y}_\alpha, \mathcal{F}_\alpha, \mathcal{F}_\alpha) \) define by \( \{ \mathcal{X}_\alpha \} \rightarrow \{ f_\alpha(\mathcal{X}_\alpha) \} \) is a pairwise \( \mathcal{W} \)-closure continuous if and only if each \( f_\alpha: (\mathcal{X}_\alpha, \mathcal{T}_\alpha, \mathcal{F}_\alpha, \mathcal{F}_\alpha) \rightarrow (\mathcal{Y}_\alpha, \mathcal{F}_\alpha, \mathcal{F}_\alpha) \) is pairwise \( \mathcal{W} \)-closure continuous.

Theorem 2.28. The function \( \prod_\alpha f_\alpha: \prod_\alpha (\mathcal{X}_\alpha, \mathcal{T}_\alpha, \mathcal{F}_\alpha, \mathcal{F}_\alpha) \rightarrow \prod_\alpha (\mathcal{Y}_\alpha, \mathcal{F}_\alpha, \mathcal{F}_\alpha) \) define by \( \{ \mathcal{X}_\alpha \} \rightarrow \{ f_\alpha(\mathcal{X}_\alpha) \} \) is a pairwise \( \mathcal{W} \)-weakly continuous if and only if each \( f_\alpha: (\mathcal{X}_\alpha, \mathcal{T}_\alpha, \mathcal{F}_\alpha, \mathcal{F}_\alpha) \rightarrow (\mathcal{Y}_\alpha, \mathcal{F}_\alpha, \mathcal{F}_\alpha) \) is pairwise \( \mathcal{W} \)-weakly continuous.

References


