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Design of MRAC and modified MRAC for DC motor speed control

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Abstract

Due to nonlinear actuators or changing environmental conditions, the parameter of the process changes resulting in the failure of a conventional controller. To overcome the failure of the conventional controller and to achieve the desired output, an adaptive mechanism called Model Reference Adaptive Controller (MRAC) can be designed. Since the conventional MRAC is aimed at the first-order system for two-variable parameter adjustment, it cannot be applied for most practical systems, including DC motor. To overcome this a modified MRAC which is the combination of conventional MRAC and a PID controller is designed in this paper.

Keywords: Conventional controllers, PID, desired output, Model Reference Adaptive Controller (MRAC), modified MRAC.

1. Introduction

DC motor has increased need in most industrial processes. As this DC motor is nonlinear in nature, controlling is necessary. For varying process dynamics adaptive control must be implemented. Generally to adapt means to change so that one's behavior will confirm to new or changed behavior. When there is uncertainty in process parameters or environmental variables, adaptive control entails adjusting the controller parameters to produce the intended output. It is described as the use of a parameter estimator, which generates parameter estimates in real time, in conjunction with a control rule to regulate plants whose parameters are unknown or vary in an unpredictable way over time.

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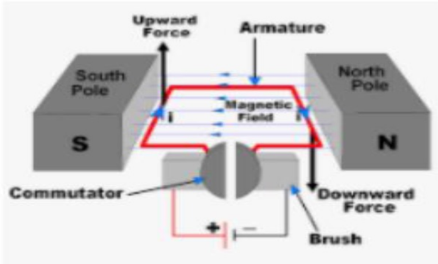

Figure 1: DC motor

There are two loops in adaptive systems. Second loop is a parameter adjustment loop with slower dynamics, while the first loop is a regular feedback loop with process and controller. Direct adaptive control and indirect adaptive control are the two types of adaptive control [1]. The plant model is parameterized in terms of desired controller parameters, which are then estimated directly without the need for intermediate calculations involving plant parameter estimates in direct adaptive control. The implicit plant model is another name for this. The plant parameters are evaluated online and used to calculate the controller parameters in indirect adaptive control. To put it another way, the estimated plant is built and treated as if it were the true plant when calculating the controller settings for each time t. Because the controller design is based on an explicit plant model, this approach is also known as explicit adaptive control.

2. Mathematical Model Of a DC Motor

As demonstrated in Figure 1, a DC motor turns direct current energy into mechanical energy. Variable supply voltage or adjusting the current in the field windings can control the DC motor's speed over a large range.

By Kirchhoff's voltage law

$$V_a = i_a R_a + L_a \frac{d_{i_a}}{dt} + e_b \tag{2.1}$$

Torque equation of DC motor is given by

$$T = K_t i_a \tag{2.2}$$

Differential equation governing the mechanical system is given by

$$T = J\frac{dw}{dt} + Bw \tag{2.3}$$

Equating equations (2.2) and (2.3)

$$K_t i_a = J\frac{dw}{dt} + Bw \tag{2.4}$$

Apply Laplace transform for (2.4)

$$K_t I_a(s) = JsW(s) + Bw(s) \tag{2.5}$$

$$I_a(s) = \frac{W(s)}{K_t}[Js + B] \tag{2.6}$$

Table 1: DC motor specifications

SYMBOL	DESCRIPTION	VALUE	UNIT
J	Moment of Inertia	0.00052	Kg/m^2
B	Fractional Coefficient	0.01	-
L_a	Armature Inductance	0.23	Henry
R_a	Armature Resistance	2	Ohm
K_t	Torque Constant	0.235	-
K_b	Back emf Constant	0.235	-

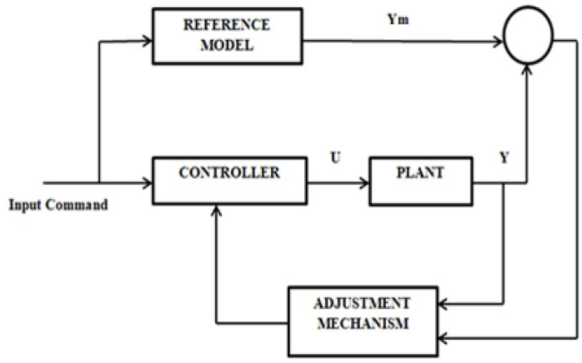

Figure 2: Block diagram of MRAC

Apply Laplace transform for (2.1)

$$V(s) = I_a(s)[R_a + L_a S] + E_b(s) \tag{2.7}$$

$$E_b(s) = K_b W(s) \tag{2.8}$$

Substitute (2.8) in (2.7)

$$V(s) = I_a(s)[R_a + L_a S] + K_b W(s) \tag{2.9}$$

Substituting (2.6) in (2.9) we get the transfer function for the DC motor [2].

$$\frac{W(s)}{V(s)} = \frac{K_t}{JL_a s^2 + (BL_a + JR_a)s + BR_a + K_b K_t} \tag{2.10}$$

The various specifications of DC motor is listed in table 1.

3. Design Of Controller

A conventional feedback loop with a process and a controller, as well as another control loop that alters the control parameters, make up the Model Reference Adaptive Controller. The inner loop is the standard loop, while the outer loop, as shown in Figure 2, is the loop that can alter the parameter.

Because the performance of MRAC is expressed in terms of the reference model, it is classified as an adaptive servo system. Gradient method and stability theory are two ways to implement the adjustment mechanism [2, 4, 6, 3]. The model specifies the expected behavior. The controller

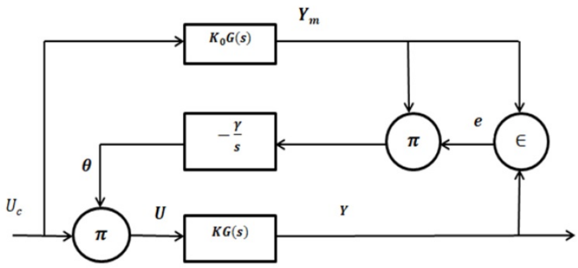

Figure 3: Block diagram for adaptation of a feed forward gain The error is given as

parameters are changed according to the error calculation. This mistake is the difference between the process's real output and the model's output. This MRAC was originally created for continuous-time systems, however it was later expanded to discrete-time systems.

3.1. Design of MRAC using MIT rule

MIT rule was first developed at the Instrumentation laboratory in MIT. Consider a closed loop system having one adjustable controller parameter θ. The output of the reference model is Y_m. The output of the process is Y. The difference between Y and Y_m is called as error e as shown in figure 3. The parameters can be adjusted by minimizing the loss function. Consider the following loss function

$$J(\theta) = \frac{1}{2}e^2 \tag{3.1}$$

In order to make the loss function J small, the parameters are changed in the direction of negative gradient.

$$\frac{d\theta}{dt} = -\gamma\frac{\partial J}{\partial t} = \gamma e\frac{\partial e}{\partial t} \tag{3.2}$$

The sensitivity derivative $\frac{de}{dt}$ says how the error is influenced by adjustable parameters. If it is assumed that the other variables in the system is faster than the parameter changes then this sensitivity derivative can be solved by the assumption that θ is a constant. The loss function can be chosen in many ways.

$$J(\theta) = \mid e \mid \tag{3.3}$$

$$\frac{d\theta}{dt} = \gamma\frac{\partial e}{\partial t}sign \tag{3.4}$$

This algorithm is called as sign algorithm which is used in areas like telecommunications where simple implementation and fast computations are required.

$$e = Y - Y_m = KG(p)\theta U_c - K_0G(p)U_c \tag{3.5}$$

Where, U_c is the command signal, Y_m is the model output, Y is the process output, θ is adjustable parameter, $\frac{d}{dt}$ is the differential operator.
The sensitivity derivative is given by

$$\frac{\partial e}{\partial \theta} = KG(p)U_C = \frac{K}{K_0}Y_m \tag{3.6}$$

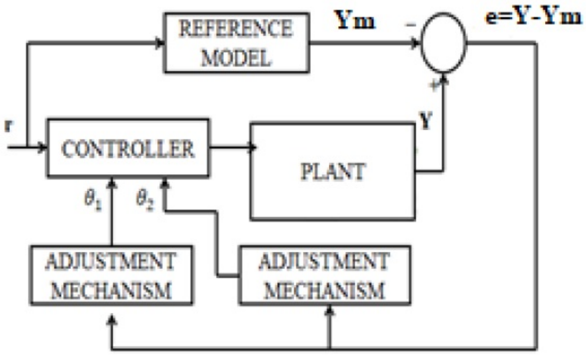

Figure 4: Block diagram of conventional MRAC

The adaptation law is given by the MIT rule

$$\frac{d\theta}{dt} = \gamma' \frac{K}{K_0} Y_m e = -\gamma Y_m e \tag{3.7}$$

Where $\gamma = \gamma' \frac{K}{K_0}$ is the adaptation gain.

The adaptation gain is determined as follows

$$Y = KG(p)U \tag{3.8}$$

$$Y_m = K_0 G(p) U_c \tag{3.9}$$

$$U = \theta U_C \tag{3.10}$$

$$e = Y - Y_m \tag{3.11}$$

$$\frac{d\theta}{dt} = -\gamma Y_m e \tag{3.12}$$

Substituting equations (3.8), (3.9), (3.10), (3.11) in equation (3.12) we get

$$\frac{d\theta}{dt} + \gamma Y_m (KG(p)\theta U_C) = \gamma Y_m^2 \tag{3.13}$$

$$S + \gamma Y_m U_C K G(s) = 0 \tag{3.14}$$

$$S + \mu G(s) = 0 \tag{3.15}$$

Where $\mu = \gamma Y_m U_C K$

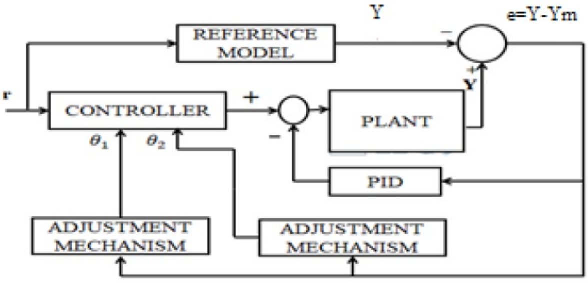

Figure 5: Block diagram of modified MRAC

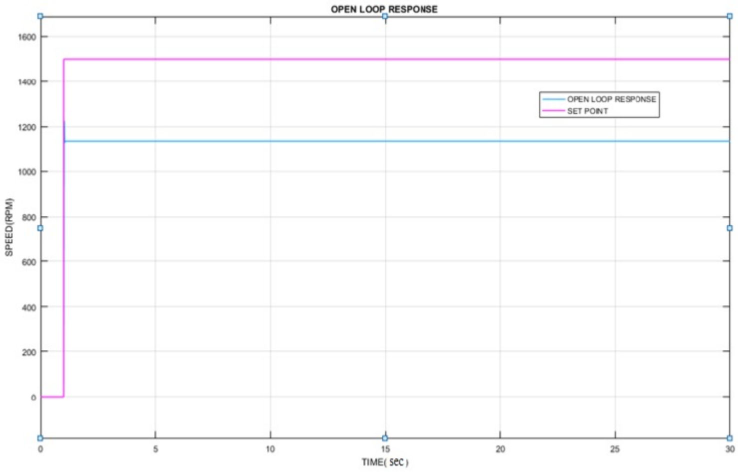

Figure 6: Open loop response

3.2. Design Of Conventional And Modified MRAC

Figure 4 depicts the Conventional MRAC with MIT rule. The updated MRAC [7] is implemented as illustrated in figure 5 to enhance the settling and convergence times even more.

The plant transfer function is given as

$$\frac{W(s)}{V(s)} = \frac{0.235}{0.00001196S^2 + 0.002404S + 0.075225} \tag{3.16}$$

For any system it is desired to have the damping ratio ξ to lie between 0.4 to 0.7 and the acceptable peak overshoot of the DC motor is found to be 1% to 3% [2].Considering these specifications the reference model is chosen as $\frac{1}{S^2+S+1}$ where it satisfies the considerations.

PID controller parameters for modified MRAC [7] shown in figure 5 is given as $P = 0.27827$, $I = 0.052017$, $D = -0.096434$.

4. Results And Discussion

From Figure 6 it is observed that the DC motor does not work as required for constant speed applications in the absence of a controller. For this purpose a conventional PID controller is designed.

From Figure 7, it is observed that the DC motor works for a desired set point. But when we use any actuators or if there is any change in the environmental conditions the PID controller fails. To overcome this an adaptive controller called MRAC is designed.

The Simulink response of conventional MRAC using MIT rule for various adaptation gain is shown in Figure 8.

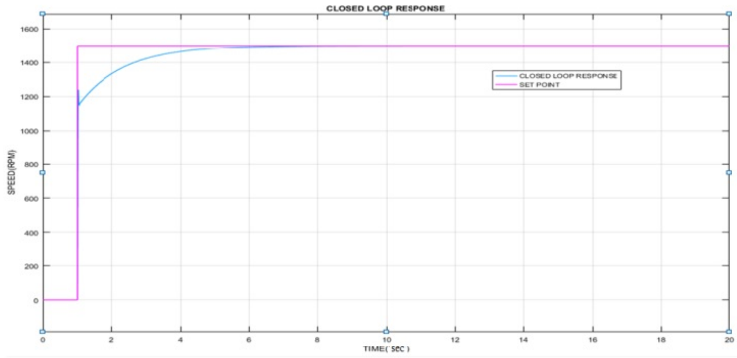

Figure 7: Closed loop response

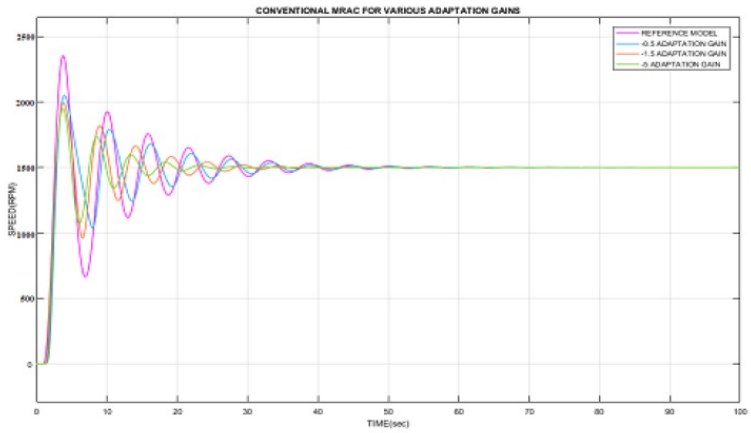

Figure 8: Simulink response of conventional MRAC

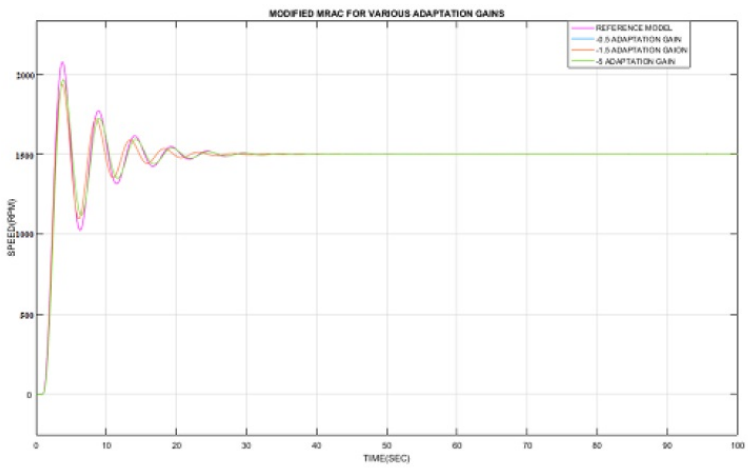

Figure 9: Simulink response of modified MRAC

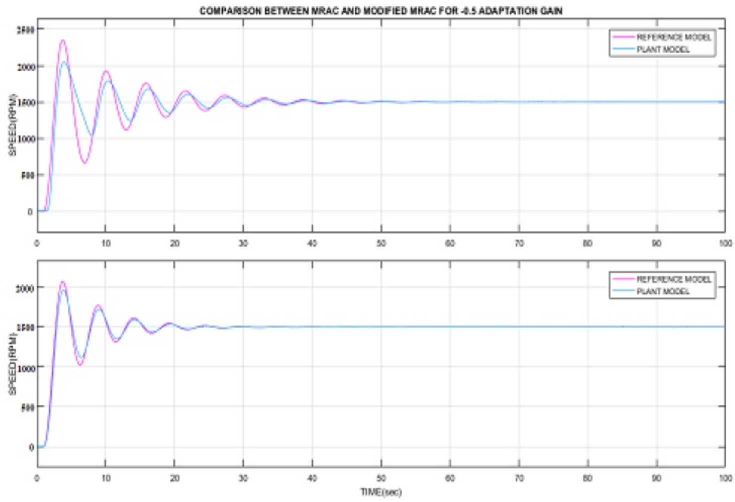

Figure 10: Comparison of conventional and modified MRAC

Table 2: Tabulation between MRAC and modified MRAC

TECHNIQUE USED	ADAPTATION GAIN	SETTLING TIME(sec)	CONVERGENCE TIME(sec)
MRAC WITH MIT	-0.5	40	50
	-1	50	40
	-1.5	51	38
	-2	53	30
	-5	55	18
MODIFIED MRAC WITH MIT	-0.5	30	20
	-1	29	19
	-1.5	28	18
	-2	25	15
	-5	23	11

The Simulink response of modified MRAC using MIT rule for various adaptation gain is shown in Figure 9.

From the comparison shown in Figure 10 it is observed that the settling time and convergence time is improved. The model following in modified MRAC is better compared to the model following in conventional MRAC.

The Settling time and convergence time for different adaptation gain values are obtained for conventional MRAC and modified MRAC as shown in Table 2.

5. Conclusion

Conventional MRAC and modified MRAC using MIT rule is applied for DC motor speed control. The response of conventional MRAC and modified MRAC shows improved performance in terms of settling time and convergence time. The parameters of PID and adaptation gain does not give optimal solution hence optimization techniques can be utilised.

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