



# Solving a multicriteria problem in a hierarchical method

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## Abstract

The problem of minimizing a function of three criteria maximum earliness, the sum of completion times and maximum lateness in a hierarchical method is presented in this paper. A set of  $n$  independent jobs has to be scheduled on a single machine that is continuously available from time zero onwards and that can handle no more than one job at a time. Job  $j$  ( $j = 1, 2, \dots, n$ ) requires processing during a given positive uninterrupted time  $p_j$ . An algorithm to find the best possible solution is proposed for the problem of three criteria maximum earliness, the sum of completion times and maximum lateness in a hierarchical case.

*Keywords:* Single machine, Hierarchical, Lexicographical, Multicriteria scheduling.

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## 1. Introduction

In the real life situations, decisions to be made are often constrained by specific requirements conflicting in nature. The decision making process gets increasingly more complicated with increment in the number of constraints. Modeling and development of solution methodologies for these scenarios have been the challenge for operations researchers from the outset. A variety of algorithms and formulations has been developed for various classes of problems [11].

The basic scheduling problem can be described as finding for each of the tasks, which are also called jobs, an execution interval on one of the machines that are able to execute it, such that all side constraints are met; obviously, this should be done in such a way that the resulting solution, which is called a schedule, is best possible, that is, it minimizes the given objective function [5]. Scheduling theory has been developed to solve problems occurring in for instance nurse scheduling [8]. In this study, the one machine case is considered because the one machine problem provides

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a useful laboratory for the development of ideas for heuristics and interactive procedure that may prove to be useful in more general models.

There are two approaches for the multicriteria problems; the hierarchical and the simultaneous approach. In the hierarchical approach, one of the two criteria is considered as the primary criterion and the other one as the secondary criterion. The problem is to minimize the primary criterion while breaking ties in favor of the schedule that has the minimum secondary criterion value. In the simultaneous approach, two criteria are considered simultaneously. This approach typically generates all efficient schedules and selects the one that yields the best composite objective function value of the two criteria. Most multicriteria scheduling problems are NP-hard in nature [1]. In recent years, as a powerful optimization tool [2, 3], evolutionary algorithms (EAs) have been introduced to solve the order scheduling problems.

Erne [4] gave a heuristic method for multicriteria scheduling problem with sequencing dependent setup time for minimizing the weighted sum of total completion time, maximum tardiness and maximum earliness by integer programming model. Nelson et al. [10] presented some algorithms for the three, two criteria problems utilizing mean flow time  $F$ , maximum tardiness  $T_{\max}$  and number of tardy job  $n_T$ . Hooegeven [6] presented an algorithm to minimize a non-decreasing function of K performance regular criteria. A schedule  $\sigma$  defines for each job  $j$  its completion time  $C_j(\sigma)$  such that the jobs do not overlap in their execution. The cost of completing  $j$  at time  $C_j(j = 1, 2, \dots, n)$  is measured by penalty function  $f_j$ . In this paper the multicriteria problem concerns the hierarchical minimization of the performance measure sum of completion time  $\sum C_j$  and maximum cost  $f_{\max}$ , maximum cost defines  $f_j(C_j)$ , where each  $f_j$  denotes regular or irregular cost function; regular means that  $f_j(C_j)$  does not decrease when  $C_j$  increases such as Tmax, Lmax and  $\sum C_j$ . Otherwise the function is called irregular such as Emax.

## 2. Notation and basic concepts

In this paper the notation is used for single machine, jobs  $j(j=1,2,\dots,n)$  have:

$N$  : set of jobs.

$n$  : the number of jobs in a known sequence.

$p_j$  : a processing time for job  $j$ .

$d_j$  : the date when the job  $j$  should ideally be completed.

$d_{\bar{j}}$  : the deadline for job  $j$ .

$C_j$  : the completion time of job  $j$ .

$C_1 = p_1$

$C_j = C_{j-1} + p_j, \quad j = 2, \dots, n.$

$s_j = d_j - p_j$  : the slack time of job  $j$ .

$L_j = C_j - d_j$  : the lateness of job  $j$ .

$E_j = \max\{0, d_j - C_j\}$  : the earliness of job  $j$ .

$C_{\max} = \max_j\{C_j\}$  : maximum completion time.

$E_{\max} = \max_j\{E_j\}$  : maximum earliness.

$L_{\max} = \max_j\{L_j\}$  : maximum lateness.

SPT= shortest processing time rule, that is, sequencing the jobs in non-decreasing order of their processing times.

EDD= earliest due date rule, that is, sequencing the jobs in non-decreasing order of their due dates.

**Theorem 2.1.** [9] *The  $1//f_{\max}$  problem is minimized as follows:*

*While there are unassigned jobs, assign the job that has minimum cost when scheduled in the last unassigned position in the schedule.*

**Theorem 2.2.** [7] *The  $1//E_{\max}$  problem is solved by sequencing the jobs according to the minimum slack time (MST) rule, that is, in non-decreasing order of  $d_j - p_j$ .*

**Definition 2.3.** [6] *Hierarchical minimization : the performance criteria  $f_1, f_2, \dots, f_k$  are indexed in order of decreasing importance. First,  $f_1$  is minimized. Next,  $f_2$  is minimized subject to the constraint that the schedule has minimal  $f_1$  value. If necessary,  $f_3$  is minimized subject to the constraint that the values for  $f_1$  and  $f_2$  are equal to the values determined in the previous step.*

**3. The  $1//Lex(E_{\max}, \sum_{j=1}^n C_j, L_{\max})$  problem**

This problem can be defined as follows:

$$\left. \begin{array}{l} \text{Min}\{L_{\max}\} \\ \text{S.t} \\ E_{\max} = E^*, \quad E^* = E_{\max}(MST) \\ \sum_{j=1}^n C_i \leq c^*, \quad C^* \in \left[ \sum_{j=1}^n C_j(SPT), \sum_{j=1}^n C_j(MST) \right] \end{array} \right\} \quad (P)$$

Since in this problem (P), the  $E_{\max}$  is the more important function and should be optimal, then the following algorithm (ECL) gives the best possible solution.

**Algorithm (ECL)**

- Step 1: Solve  $1//E_{\max}$  problem to find  $E^*$ .
- Step 2: Determine  $\bar{d}_j = d_j + E^* \quad \forall j \in N, N = 1, 2, \dots, n$ .
- Step 3: Let  $t = \sum_{j=1}^n p_j, \quad k = 1$ .
- Step 4: Find a job  $j^* \in N$  satisfies  $d_{j^*} \leq t$  ( if there is a tie choose the job with smallest processing time and if a tie is still, choose the job with smallest due date ).
- Step 5: Set  $t = t - p_{j^*}, k = k + 1, N = N - \{j^*\}, \sigma = (\sigma, \sigma(k)),$  if  $N = \phi$  go to step 6 else go to step 4.
- Step 6: For a schedule  $\sigma$  compute  $E_{\max}, \sum_{j=1}^n C_j$  and  $L_{\max}$  .

**Example 3.1.** Consider the problem (P) with the following data.

<i>j</i>	1	2	3	4	5
<i>p<sub>j</sub></i>	3	2	4	7	7
<i>d<sub>j</sub></i>	4	12	14	8	7

$E^* = 0, \quad t = 32, \quad \bar{d}_1 = 4, \bar{d}_2 = 12, \bar{d}_3 = 14, \bar{d}_4 = 8, \bar{d}_5 = 7$

<i>k</i>	<i>t</i>	<i>j*</i>
1	23	2
2	21	1
3	18	3
4	14	5
5	7	4

Hence the schedule (2, 1, 3, 5, 4) gives  $(E_{\max}, \sum_{j=1}^n C_j, L_{\max}) = (10, 55, 15)$  according to algorithm (ECL).

#### 4. Conclusion

For the multicriteria scheduling problem  $1//(\sum_{j=1}^n C_j, L_{max})$ , an algorithm to find the best possible solution for the hierarchical case is proposed.

It is hoped that the contribution of this paper would provide an incentive research effort in the multicriteria field especially three hierarchically criteria. A future research topic would involve experimentation with the following machine scheduling problem:  $1//Lex(\sum_{j=1}^n C_j, L_{max})$ .

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