



Estimation of the general spatial regression model (SAC) by the maximum likelihood method

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Abstract

That there are indicators or statistical transactions that have appeared in a large way in recent times to describe, summarize and analyse spatial data, when a study is done of many phenomena or a disease is studied, whether it is on humans or animals, we need to analyze the spatial data resulting from those phenomena, as it includes observations of the spatial units. For example, countries or provinces ... etc., all of these are linked to certain points or locations. The study uses the maximum likelihood method to estimate the parameters of the General Spatial Model by employing the model to study cancer which shows the relationship between the dependent variable Y represented by the number of patients and the explanatory variables represented (average age, tumor size, treame, hormone, immunity) in light of the effect of spatial juxtaposition and using Rook neigh boring criteria. One of the most important conclusions reached is the emergence of significant effects of some explanatory variables on the dependent variable Y , and the estimated values of the dependent variable Y are close to the real values of the same variable.

Keywords: The general spatial regression model, Spatial contiguity matrix, Rook neighboring criteria, Maximum Likelihood Method, Cancer.

1. Introduction

Spatial regression is methods for capturing the spatial dependence, and the spatial dependence of the regression model can be entered as relationships between the independent variables and the dependent variables [9]. As spatial regression depends largely on the data obtained by the researcher, any data that the researcher collects for any phenomenon he wants to study is not independent in itself, but rather depends on the place from which the data was taken. The spatial data is

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distinguished from the time series data by the spatial arrangement of the observations, and spatial econometrics deals with spatial dependency and spatial heterogeneity. These characteristics may make traditional econometric techniques become unsuitable, and spatial economic measurement is concerned with following up on spatial effects such as the spatial dependence of observations in points different from the place.

The problem here is that the general linear regression model does not estimate spatial contiguity in its calculations, and this may lead to the loss of important data about the studied phenomenon, which ultimately affects the statistical results. Cancer in particular and its spread among neighboring areas varies from one area to another. This disease is considered one of the diseases that pose a danger to humans, as in many cases the health status of the neighboring areas is analyzed without following a correct scientific approach that takes into account the appropriate spatial or spatial effects to consider consideration.

2. Objective of the research

The aim of this research is to estimate the General Spatial Model, which suffers from the problem of spatial dependence, using the Maximum Likelihood Method, which describes the relationship between the dependent and represented variable (number of infected) and explanatory variables (average age, tumor size, treatment, hormone, immunity) for cancer under the Rook spatial contiguity criterion for the regular and modified spatial neighborhoods matrixes (W_{ij}, W_{ij}^{Adj}).

3. Matrices of criterion and spatial response

The Rook contiguity criterion, one of the criteria for spatial contiguity, was used in the formation of the spatial contiguity matrix, as a method for this contiguity where a single value is taken if the two adjacent regions are finite and have a relationship between two regions on any side, meaning that ($W_{R=1}$), otherwise, the value will be zero ($W_{R=0}$) since the main diameter elements of the matrix are zero because the point does not adjoin itself and that this contiguity has more than one point in one row of the matrix W_R , and the use of this matrix is more than others.[2].

Ten regions will be developed to determine the spatial contiguity matrix, as follows: A: represents the Kadhimiya area, B: represents the Adhamiya area, C: represents the city center, D: represents Palestine, E: represents the center of Rusafa, F: represents the new Baghdad, G: represents Eastern Karada, H: represents the safe, I: represent Al-Mansour, and J: represents the Karkh center.

Show the common boundaries between cell (A) and cells (B,C) and also between cell (B) and cells (A,D) through the matrix is as in the following formula:

$$W_R = \begin{matrix} & \begin{matrix} A & B & C & D & E & F & G & H & I & J \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \\ I \\ J \end{matrix} & \left[\begin{array}{cccccccccc} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{array} \right] \end{matrix} \tag{3.1}$$

It is noticeable that the Rook adjacency matrix shown in formula (3.1) can be formed as follows:

The point (A) is not adjacent to itself, so it takes the value ($W_{R11} = 0$), If the point (A) is adjacent to the point (B) it takes the value ($W_{R12} = 1$), If point (A) is adjacent to point (C) it takes the value ($W_{R13} = 1$), If point (A) is not adjacent to point (D) it takes the value ($W_{R14} = 0$), etc....

The modified spatial adjacency matrix can be found which is denoted by the symbol W_{ij}^{Adj} .

It is calculated by the following formula.

$$W_{ij}^{Adj} \left\{ \begin{array}{ll} \frac{W_{ij}}{\sum W_{ij}} & i \text{ neighbor } j \quad 0 < W_{ij}^{std} \leq 1 \\ 0 & \text{other wise} \end{array} \right\} \tag{3.2}$$

That is, each value of any row of the ordinary spatial adjacency matrix W_{ij} divide by the total row as shown in the matrix below:

$$W_{ij}^{Adj} = \begin{bmatrix} 0 & 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/3 & 0 & 0 & 1/3 & 1/3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 0 & 0 & 1/3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 & 1/3 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/3 & 1/3 & 0 & 0 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/3 & 0 & 0 & 1/3 & 1/3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/3 & 1/3 & 0 & 0 & 1/3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1/2 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/2 & 1/2 & 0 \end{bmatrix} \tag{3.3}$$

4. Study model

The general spatial regression model consists of two parts: the spatial lag and a spatially linked error structure. (SAC) represents an appropriate approach to modeling this type of dependence in error and it is explained as in the following:

$$Y = \rho W_1 Y + X\beta + u \quad , \quad u = \lambda W_2 u + \varepsilon \quad , \quad \varepsilon \sim N(0, \sigma^2 \varepsilon) \tag{4.1}$$

Since: Y : it represents a vector ($n \times 1$) of the dependent variable. X : represents an array ($n \times k$) of explanatory variables. W_1, W_2 : they represent spatial weight matrices with dimensions ($n \times n$), where (W_1) represents the spatial adjacency matrix between the views and the contiguous regions and (W_2) it represents the contiguity between the views and the city center, usually the proximity relationship or the distance function, which are fixed and predetermined. They can be equal $W_1 = W_2$ what do you represent W_1 [4]. ρ : represents a parameter of spatial dependence. β : it represents a vector of parameters ($k \times 1$) that are associated with the matrix of explanatory variables X . λ : the spatial autoregressive parameter of errors is the spatial lag coefficient of the error and u : spatially related errors.[4].

5. The maximum likelihood method

Estimation of spatial regression models is usually performed by estimating the greatest possible, where the probability of the joint distribution (possibility) of all observations is maximized with respect to the number of relevant parameters. It is strong in small deviations from the assumption of normality, and it is also considered one of the most important methods because it gives the best estimate of the parameter among several possible estimates.[7].

To extract the estimation equations, it is explained as follows:

$$\varepsilon = u - \lambda W u \tag{5.1}$$

$$u = (I - \lambda W)^{-1} \varepsilon \tag{5.2}$$

$$Y (I - \rho W) - X \beta = (I - \lambda W)^{-1} \varepsilon \tag{5.3}$$

$$\varepsilon' \varepsilon = [(I - \lambda W) Y (I - \rho W) - (I - \lambda W) X \beta]' [(I - \lambda W) Y (I - \rho W) - (I - \lambda W) X \beta] \tag{5.4}$$

$$L (\beta, \rho, \lambda, \sigma^2 / Y, X) = - \frac{n}{2} \text{Ln} 2\pi - \frac{n}{2} \text{Ln} \sigma^2 + \text{Ln} |I - \rho W| + \text{Ln} |I - \lambda W| - \frac{1}{2\sigma^2} \varepsilon' \varepsilon \tag{5.5}$$

$$\frac{\partial (\beta, \rho, \lambda, \sigma^2 / Y, X)}{\partial \beta} = - \frac{1}{2\sigma^2} * [-2X' (I - \lambda W)' (I - \lambda W) Y (I - \rho W) + X' (I - \lambda W)' (I - \lambda W) X \hat{\beta}_{mle}] \tag{5.6}$$

$$\hat{\beta}_{mle} = [X' A' A X]^{-1} [X' A' A Y B], \text{ and } A = (I - \lambda W) \tag{5.7}$$

To estimate the value of the correlation parameter (ρ) they are found using iterative methods for the probability function, as follows:

$$|I - \rho W| = \prod_{i=1}^n (1 - \rho w_i) \tag{5.8}$$

$$\text{Ln} |I - \rho W| = \sum_{i=1}^n \text{Ln} (1 - \rho w_i) \tag{5.9}$$

As well as to estimate the value of the parameter (λ) they are found using iterative methods as follows:

$$|I - \lambda W| = \prod_{i=1}^n (1 - \lambda w_i)$$

$$\text{Ln} |I - \lambda W| = \sum_{i=1}^n \text{Ln} (1 - \lambda w_i) \tag{5.10}$$

$$s^2 = \frac{\varepsilon' \varepsilon}{n} \tag{5.11}$$

6. Moran's Test

This test is a measure to show whether there is a spatial dependency in the data or not, and it is a general measure and depends on a model (GLM) $[Y = X \beta + \varepsilon]$, Or how one of the observations is similar to the other observations surrounding it in each region and neighboring regions through the matrix of spatial weight [8], the idea in Moran's test depends on that the close things have more relationship than the distant things of any phenomenon related to each other, if the value of Moran's coefficient close to (3.1) means that there is a spatial autocorrelation[9], Moran's formula is as follows:

$$I_m = n \left(\varepsilon' W \varepsilon \right) / S_o \left(\varepsilon' \varepsilon \right) \tag{6.1}$$

Since: S_0 : the sum of each element in the W matrix. W: square weight matrix ($n * n$). n : sample volume. ε : the error vector (residuals) has dimensions ($n * 1$).

When a standard row is used and the row sum is equal to (3.1) in this case ($n = S_0$) It simplifies the above formula as follows:

$$I_m = \varepsilon' w \varepsilon / \varepsilon' \varepsilon \tag{6.2}$$

To find out if the value of Moran's coefficient is (I_m) Statistically significant at a certain degree of confidence, the Moran (Z) test is used:

$$Z = (I - E(I)) / \sqrt{var(I)} \tag{6.3}$$

$$E(I) = \text{tr}(MW) / (n - k) \tag{6.4}$$

$$var(I) = \frac{\text{tr}(MWMW') + \text{tr}(MWMW) + (\text{tr}(MW))^2}{(n-k)(n-k+2)} - (E(I))^2 \tag{6.5}$$

Since: $M = I - X(X'X)^{-1}X'$: A deaf matrix is square and symmetric. $\text{tr}(MWMW')$: the sum of the diagonal elements of the matrix. k : The number of explanatory variables.

7. Lagrange Multiplier Test

The multiplexed Lagrange test is more used than Moran's test because Moran is used only to test the spatial dependency whether it exists or not, and it is not possible to test what is the alternative model of the GLM model by Moran's test, while the Lagrange test gives what is the alternative model (ρ or λ).) This approach indicates that if (ρ) is important or (λ) or both, [9].

7.1. Lagrange Multiplier for (ρ)

$H_0: \rho = 0$ Spatial dependence exists

$H_1: \rho \neq 0$ At least one of ρ is not equal to 0 i.e. there is no spatial dependence

The test format is as follows:

$$LM_\rho = \frac{\left(\frac{\varepsilon' W Y}{S^2}\right)^2}{D} \tag{7.1}$$

$$D = \frac{(WXb)' M (WXb)}{S^2} + \text{tr}(W'W + WW) \tag{7.2}$$

S^2 : is the error variance of the general linear regression model.

We compare the calculated value with the tabular value of $\chi^2(1, \alpha)$ and then hypotheses are determined.

7.2. Lagrange Multiplier for (λ)

$H_0 : \lambda = 0$ The spatial dependence is in the wrong

$H_1 : \lambda \neq 0$ At least one of λ is not equal to zero, the spatial dependence does not exist by mistake

Where rejecting the null hypothesis and accepting the alternative hypothesis means that the spatial dependence exists and the alternative model is (λ)

$$LM_\lambda = \frac{\left(\frac{\varepsilon' w \varepsilon}{S^2}\right)^2}{T} \tag{7.3}$$

$$T = \text{tr}[(w + w')w] \tag{7.4}$$

To compare (LM_ρ, LM_λ) with a tabular value of $\chi^2(1, \alpha)$, where the Lagrange test (ρ or λ) for spatial dependence in each of them needs a strong test and a strong role for the (ρ and λ) model.

8. The practical side

Despite the performance of the health side and the hospitals related to it and its support in increasing the medical requirements of doctors, nurses and other cadres, but there is an important matter, which is how to cut and dissipate diseases between areas that have a direct impact on human life, as well as the impact of neighboring areas on the spread of diseases, in this research was Focusing on the spatial aspect, i.e. spatial juxtapositions to know its impact on the spread of diseases between regions, knowing the distribution of disease incidence and building a spatial model for predicting cancer, as this aspect included estimating the general spatial regression model using the regular and modified spatial juxtapositions W_{ij} , W_{ij}^{Adj} In light of the spatial juxtaposition criterion Rook, and using the spatial data collected by taking a random sample and according to the geographical areas of the two sides of Karkh and Rusafa in Baghdad governorate from hospitals affiliated with the Baghdad Health Department / Ministry of Health, as they were collected from records and drums belonging to each patient, as well as using the Department of The Central Statistical Organization of the Ministry of Planning in designing the map that includes the Karkh and Rusafa areas according to their administrative division. The data collected was characterized by the following variables:

Cancer disease was used, which shows the relationship between the dependent variable Y , represented by the number of patients, and (5.1) of the explanatory variables were used with their levels after agreement on them with the specialized doctors, as follows:

Y : represents the number of injured; X_1 : represents the average age number;

X_2 : represents the Tumor size; X_3 : represents the hormone; X_4 : represents the immunity;

X_5 : represents the treatment.

The data in the above was obtained through a questionnaire for people with cancer, which included all the ten regions of the Karkh and Rusafa sides of Baghdad Governorate, according to the administrative division and knowledge, And Figure 1 represents a map of the Karkh and Rusafa sides of the city of Baghdad, divided by the ten regions according to the administrative division:

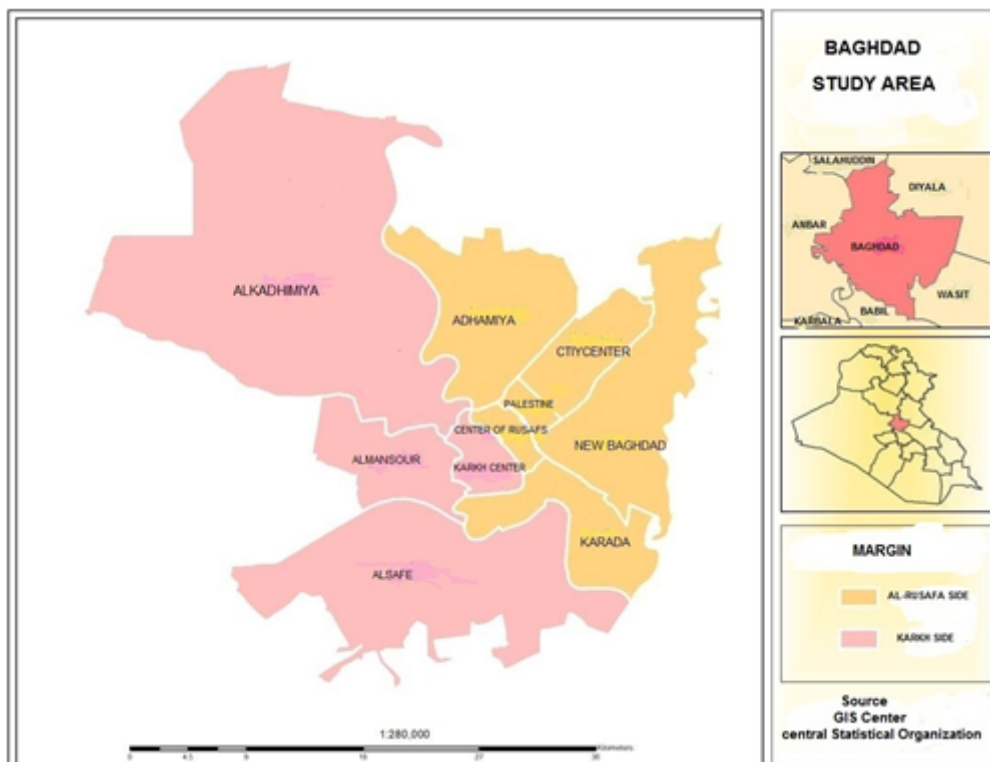


Figure 1: Map of the Karkh and Rusafa sides of the city of Baghdad.

After the application of the statistical program Matlab, the study model test was conducted in formula (4.1) using Moran’s Z test in formula (6.3) to detect the spatial dependence. From the table (3.1) below, which shows the some Statistical indicate, it was found that the value of Moran’s Z-test statistic is 9.606904 when using the ordinary spatial adjacency matrix W_{ij} and that its P-Value is (1.96) which is less than the significance level 0.05 and this indicates the significance of the test, meaning that there is a spatial dependence between the Karkh and Rusafa areas of Baghdad governorate.

Table 1: shows the some Statistical indicate

F	R ²	R ² adj	Z	LM _ρ SACML	LM _λ SACML	MAPE
14.93335	0.3663	0.3176	9.6069	3.857293	244.8722	0.53095699

The data was analyzed according to the usual spatial adjacency matrix W_{ij} with dimension (85×85) by using the greatest possibility method in estimating the parameters of the general spatial regression model (SAC) in formula (4.1) and also under the Rook criterion, as these parameters were tested using the F test and extracting the significant value of the test and comparing it With the significance level of 0.05, so we note that the F-test value equals 14.93335 and its P-Value is (0.217185) which is less than the significance level 0.05. This indicates that the differences are significant, that is, there is at least one of the explanatory variables (age rate, tumor size Treatment, hormone, immunity) have a significant effect on the Y-dependent variable (the number of infected), It is also noted that the value of the coefficient of the value of the coefficient of determination R² is 0. 366343, which indicates that 36% of the differences in the number of injured people under spatial

influences are caused by the explanatory variables.

$$R^2 = \frac{\sum_{i=1}^n (\hat{y}_i - \bar{y})^2}{\sum_{i=1}^n (y_i - \bar{y})^2} \tag{8.1}$$

$$R_{adj}^2 = 1 - \frac{(1 - R^2)(n - 1)}{(n - k - 1)} \tag{8.2}$$

Table 2: shows the real and estimated values of the dependent variable Y of using the usual spatial adjacency matrix W_{ij} under the Rook.

	Y	Y_hat		Y	Y_hat		Y	Y_hat		Y	Y_hat
1	17	34.30839	23	25	59.95965	45	50	92.41552	67	8	32.7784
2	51	75.64811	24	24	48.22248	46	49	99.64901	68	3	14.33783
3	40	71.95312	25	42	78.66708	47	202	406.5243	69	14	35.01177
4	25	37.3608	26	40	85.61975	48	94	174.8316	70	42	60.6197
5	21	50.00752	27	246	487.8845	49	38	71.01928	71	39	78.02265
6	45	76.93921	28	76	129.0298	50	8	25.40555	72	18	33.43757
7	184	321.1754	29	19	42.81988	51	1	16.89708	73	42	80.21287
8	59	113.6461	30	26	43.52387	52	4	16.9209	74	25	56.68784
9	26	53.69172	31	94	127.6302	53	4	24.34964	75	124	248.8755
10	27	46.64975	32	59	112.7837	54	3	18.60641	76	43	96.49516
11	45	74.52909	33	29	56.14945	55	29	72.4466	77	21	45.2633
12	24	50.35123	34	19	44.46662	56	49	92.70032	78	11	36.16298
13	49	80.2287	35	16	35.0339	57	241	480.4666	79	18	29.27032
14	43	88.28071	36	20	47.73456	58	86	159.3303	80	19	32.06947
15	35	64.38263	37	14	43.63961	59	33	65.22252	81	17	43.16309
16	40	80.45722	38	80	183.1086	60	12	31.40908	82	10	36.86993
17	34	80.29568	39	25	54.40315	61	21	49.08453	83	9	27.45094
18	217	458.8325	40	7	24.60467	62	24	52.03069	84	36	79.9632
19	77	147.5653	41	28	56.85412	63	25	55.16188	85	28	46.29183
20	22	45.30557	42	42	86.01443	64	28	71.51414			
21	10	25.06506	43	38	61.68985	65	118	247.3315			
22	20	48.01115	44	47	87.89959	66	30	61.27334			

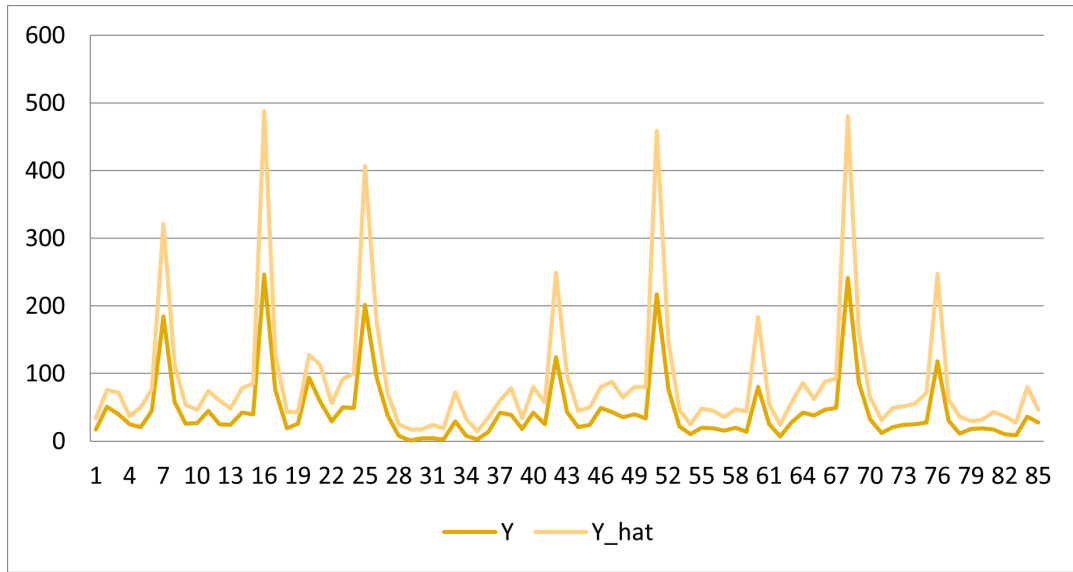


Figure 2: shows the real and estimated values of the dependent variable (Y).

From the table 3 below, which shows the some Statistical indicate, it was found that the value of Moran’s Z-test statistic is 9.4636 when using the ordinary spatial adjacency matrix W_{ij}^{Adj} and that its P-Value is (9.4636) which is less than the significance level 0.05 and this indicates the significance of the test, meaning that there is a spatial dependence between the Karkh and Rusafa areas of Baghdad governorate.

Table 3: shows the some Statistical indicate

F	R ²	R ² adj	Z	LM _ρ SAC	LM _λ SAC	MAPE
14.74235	0.288	0.541	9.4636	4.019951	243.7283	0.529613874

The data was analyzed according to the usual spatial adjacency matrix W_{ij}^{Adj} with dimension (85×85) by using the greatest possibility method in estimating the parameters of the general spatial regression model (SAC) in formula (4.1) and also under the Rook criterion, as these parameters were tested using the F test and extracting the significant value of the test and comparing it With the significance level of 0.05, so we note that the F-test value equals 14.74235 and its P-Value is (0.217185) which is less than the significance level 0.05. This indicates that the differences are significant, that is, there is at least one of the explanatory variables (age rate, tumor size Treatment, hormone, immunity) have a significant effect on the Y-dependent variable (the number of infected), It is also noted that the value of the coefficient of the value of the coefficient of determination R² is 0.288089, which indicates that 28% of the differences in the number of injured people under spatial influences are caused by the explanatory variables.

After a statistical comparison of MAPE it was found that the modified Rook was the best.

Table 4: shows the real and estimated values of the dependent variable Y of using the usual spatial adjacency matrix W_{ij}^{Adj} under the Rook.

	Y	Y_hat		Y	Y_hat		Y	Y_hat		Y	Y_hat
1	17	34.17303	23	25	59.79909	45	50	92.26278	67	8	32.55898
2	51	75.51326	24	24	48.06132	46	49	99.49724	68	3	14.11808
3	40	71.8199	25	42	78.50752	47	202	406.3909	69	14	34.83737
4	25	37.22444	26	40	85.45901	48	94	174.6795	70	42	60.44528
5	21	49.87278	27	246	487.7528	49	38	70.86553	71	39	77.84974
6	45	76.80437	28	76	128.8662	50	8	25.18588	72	18	33.26249
7	184	321.0572	29	19	42.65768	51	1	16.67734	73	42	80.0406
8	59	113.5154	30	26	43.36621	52	4	16.70021	74	25	56.51341
9	26	53.5577	31	94	127.475	53	4	24.13065	75	124	248.7111
10	27	46.48807	32	59	112.6325	54	3	18.38655	76	43	96.32475
11	45	74.36824	33	29	55.99358	55	29	72.22987	77	21	45.08611
12	24	50.19739	34	19	44.26188	56	49	92.59345	78	11	36.00717
13	49	80.07474	35	16	34.82892	57	241	480.3856	79	18	29.11628
14	43	88.13064	36	20	47.52983	58	86	159.2277	80	19	31.91561
15	35	64.23056	37	14	43.43479	59	33	65.11568	81	17	43.01045
16	40	80.30701	38	80	182.9124	60	12	31.25255	82	10	36.7171
17	34	80.14413	39	25	54.19868	61	21	48.93049	83	9	27.29746
18	217	458.7071	40	7	24.39865	62	24	51.87698	84	36	79.8131
19	77	147.4183	41	28	56.7478	63	25	55.00797	85	28	46.13841
20	22	45.15236	42	42	85.91037	64	28	71.35829			
21	10	24.85871	43	38	61.58434	65	118	247.1861			
22	20	47.80571	44	47	87.79591	66	30	61.11845			

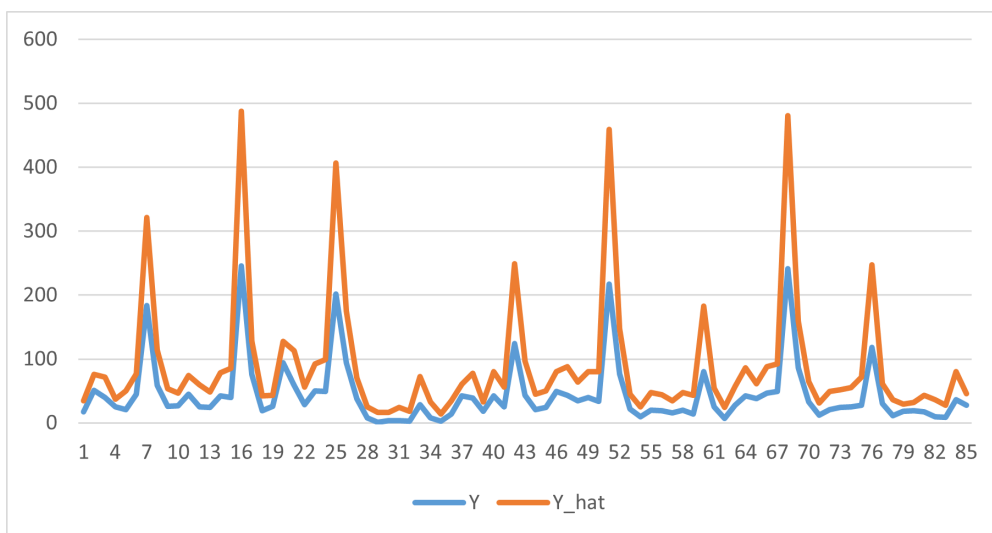


Figure 3: shows the real and estimated values of the dependent variable (Y).

9. Conclusions

1. It was found that the value of the F-test of the general spatial regression model(SAC) when using the regular and modified spatial adjacency matrixes (W_{ij} , W_{ij}^{Adj}) has significant differences, meaning that there is at least one explanatory variable that has a significant effect on the dependent variable Y.
2. It was shown through the graph that the estimated values of the dependent variable Y when using the regular and modified spatial adjacency matrixes (W_{ij} , W_{ij}^{Adj}) are close to the real values of the same variable.
3. Show by the value of the coefficient of determination R^2 when using the matrix $M = W_{ij}$ that there are 36% of the differences And when using the W_{ij}^{Adj} matrix, the percentage of differences 28% that occurred for the dependent variable Y, and this is caused by the explanatory variables.

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