

Calculation reduced transition probabilities (BE2 \downarrow) for two holes in ^{64}Ni within modified surface-delta interaction

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Abstract

Reduced electric quadrupole transition probabilities (BE2 \downarrow) in the mixed configuration of ^{64}Ni with two holes have been calculated within the nuclear shell model. In the present work modified surface delta interaction MSDI within the model space ($1P_{3/2} 0f_{5/2}$) has been used for two holes neutrons. The closed nuclear core is represented by the Ni-66 nucleus. We have used a theoretical study to find a relationship between the semi-classical coupling angle $\theta_{a,b}$ and the energy levels at different orbital within (hole-hole) configuration. We observed good agreement between theoretical energy levels with experimental data, new values have been specified for both the excited energy levels and the reduced electric quadrupole transition probabilities (BE2 \downarrow), these values are considered as a proposal, that grows theoretical understanding, of the energy levels and the expected transition probabilities through, this work.

Keywords: Modified surface delta interaction, ^{64}Ni , Reduced Transition Probabilities (BE2) \downarrow

1. Introduction

Nickel-64 nucleus has been the original subject of theoretical attention and restorative experimental since they lie far from the stability valley, thus allowing to recognize the evolution of the shell structure when approaching the neutron drip line. This has result in a major number of experiments in this, part [32, 28, 33, 25, 41, 31] directing explore the evolution of the single particle orbitals theoretically describing the spectroscopic properties of these nuclei the shell model with various two-body

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effective interactions has been widely used recently. Through the last years, shell model effective interactions without any experimental modifications have proved to be able to describe with remarkable accuracy the spectroscopic properties of nuclei in several mass regions [6, 7, 8]. Having nuclear shell model SM in notion, Talmi applied SDI the surface delta interaction to estimate feature of nuclear cases with few particle on a closed core [13]. It uses presumption, initially there exists an inert core of (closed shell), which acts within forces on to valence particles; another presumption, there exists a residual-interaction by means of 2-body forces acting between the valence particles. Schiffer [29] opinion the nuclei where 2 (hole, particle) there exists an inert core model of close shell, which acts with central forces upon valence nucleons; another, there exists a residual interaction caused by two-body forces acting between the valence nucleons. Schiffer [29] considered only those nuclei where two (particle or hole) are present in addition to closed shell, and move in the orbits j_a and j_b of a self-consistent field. Schiffer pointed out the universal behavior of the effective interaction in terms of the angle between the angular momenta of the interacting nucleons, property which was later shown to be related to the short-range character of the effective interaction [38, 39]. The angle between the protons and neutrons (holes, particles) spin vector j_a and vector j_b were determined in Ref. [4, 34]. Several of theoretic study explaining nuclear SM, by using MSDI [20, 30, 27]. J. Kostensalo and J.Suhonen [23] clarified a calculation of the properties pairing interaction for (even-even) reference nuclei in the mass region $A = 50$ to 102. K. Kaki [24] studied the elastic scattering for 48–82 Ni. In Ref. [22] explain beta decay of 62,64,68 Ni. In Ref. [35] explain optical model Ni isotopes. In the recent years, we studied the energy levels for different states within shell model [17, 14], the excitation energies for state hole–hole [15], particle–hole [13, 16] and particle–particle [18, 19] by using MSDI and surface delta interaction. Previous studies encourage the aim of the present study by application of MSDI predicting low-lying levels structure of ^{64}Ni nuclei. We have used a theoretical process to find link between the classical coupling angle and energy levels at different orbital and determined the reduced electric quadruple transition probabilities (BE2 ↓).

2. Theory

Among the hypothesis of the shell model, there is a remaining interaction which are resulting between two valence nucleons, defining as the collision force between the nucleons which occurs as a cause of the perturbation in the Hamiltonian operation and equal to the sum of two basic parts. This is clarified as the following [2, 5]:

$$H = \sum_{k=l} H_0 + \sum_{k \leq l} V_{kl} \quad (1)$$

Where $\sum_{k < l} V_{kl}$ is expressed on residual 2-body interactions and can be re-written as:

$$\sum_{k < l} V_{kl} = \sum_{IM} \sum_{j_a \geq j_b} \sum_{j_c \geq j_d} \langle j_a j_b | V | j_c j_d \rangle_I a_{IM}^+(j_a j_b) a_{IM}^+(j_c j_d) \quad (2)$$

If δ_j is the energy of single particle state j relative to the closed core and $\langle j_a j_b | V | j_c j_d \rangle = V_{abab}^{IT}$ is the matrix element [26] the interaction energy when the 2 particle couple to angular momentum I the energy of this state relative to the closed shell

$$\langle H \rangle = 2\delta_j + \langle j_a j_b | V | j_c j_d \rangle \quad (3)$$

In order to depict the low-lying states, one should consider a situation where several single-particle levels are the basis; if there are two states denoted by $|j_a j_b IM\rangle$ and $|j_c j_d IM\rangle$ then their energies

with respect to the (core) are given by [26, 3].

$$\langle H \rangle_{11} = \delta_{ja} + \delta_{jb} + V_{abab}^{IT} \tag{4}$$

$$\langle H \rangle_{22} = \delta_{jc} + \delta_{jd} + V_{abab}^{IT} \tag{5}$$

$$\langle H \rangle_{12} = \langle H \rangle_{21} = V_{abab}^{IT} \tag{6}$$

To calculate the matrix element by using MSDI potential for the residual nucleon -nucleon interaction [13, 15].

$$V_{a,b} = -4\pi A_T \delta \Omega_{a,b} \delta(\hat{r}(a) - R_0) \delta(\hat{r}(b) - R_0) + B \tau_a \tau_b \tag{7}$$

The $\hat{r}(a), \hat{r}(a)$ are the position of vectors, R_0 represent nucleus radius [11, 9, 19] A_T represent strength the interaction .The magnitude, $B \tau_a \tau_b$ represent o amount for the split between the levels with various angular momentum. $V_{a,b}$ is represent MSDI. Matrix element of is written [26, 21, 12].

$$\begin{aligned} V_{abab}^{IT} = & -\frac{A_T}{2(2I+1)} \times \sqrt{\frac{(2j_a+1)(2j_b+1)(2j_c+1)(2j_d+1)}{(1+\delta_{ab})(1+\delta_{cd})}} \\ & \times [(-1)^{l_a+l_b+j_c+j_d} h_I(j_a j_b) h_I(j_c j_d)] [1 - (-1)^{l_c+l_d+I+T}] \\ & - [k_I(j_a j_b) k_I(j_c j_d)] [1 + (-1)^T] + \{[2T(T+1) - 3]B + C\} \delta_{a,c} \delta_{b,d} \end{aligned} \tag{8}$$

Where it is $h_I(j_a j_b) = \langle j_b - \frac{1}{2} j_a \frac{1}{2} | I 0 \rangle, k_I(j_a j_b) = \langle j_a \frac{1}{2} j_b \frac{1}{2} | I 1 \rangle$; Where $\langle | \rangle$ is the Clebsh-Gordon coefficients

The behavior of (2h) nuclei is very much the same that nuclei except that the single particle energy $e_{h(hole)} = -e_{p(particle)}$ [13, 16, 3] The comporment of diagonal ion 2 - body matrix element as a function of the angular momentum I of (hole - hole) case is special when their data are plot in advantage way The major source of facts about total angular momentum and parity is the study of electromagnetic transfers, because electromagnetic interaction is a well know phenomenon different nuclear forces [26, 3]. The survey of gamma radiation emission has become associated with nuclear interactions, internal transformation, and nuclear decay. The decay rate of the emission gamma ray photon that results from the multipolar transitions from the initial state I_i to the final state I_f .The reduced transition probabilities can be calculated from the following equation [26].

$$\begin{aligned} B(EL : I_i \rightarrow I_f) = & \frac{1}{(2I_i + 1)} \sum_{M_i M_f M} |\langle \psi_{I_f M_f} | T_{LM}^{(E)} | \psi_{I_i M_i} \rangle|^2 \\ & \left(\frac{2I_f + 1}{2I_i + 1} \right) |\langle \psi_{I_f} || T_L^{(E)} || \psi_{I_i} \rangle|^2 \end{aligned} \tag{9}$$

where I_i and I_f represents the angular momentum of the initial and final states while $T_{LM}^{(E)}$ represents the multipolar electric effect .Gamma decay is expressed in Weisskopf units, and in order to judge whether the transition is relatively weak or strong, the B value of the single particle is often given in Weisskopf units in practical values. This unit is an estimate of the B value of the single particle (proton or neutron), and the following equations show how it depends on the mass number [26, 36].

$$B_\omega(E\lambda) = \left(\frac{1}{4\pi} \right) \left[\frac{3}{(3+\lambda)} \right]^2 (1.2A^{\frac{1}{3}})^{2\lambda} e^2 f m^{2\lambda} \tag{10}$$

where λ is the gamma ray photon of the transition, and A is the mass number. The most commonly used equations are

$$B_{\omega}(E_1) = 0.0645A^{\frac{2}{3}}e^2fm^2$$

$$B_{\omega}(E_2) = 0.0594A^{\frac{4}{3}}e^2fm^4$$

when hole - hole in orbits j_a and j_b with $I = j_a + j_b$ one can write [40].

$$I^2 = (j_a + j_b)^2 = j_a^2 + j_b^2 + 2 \times (j_a j_b) \cos \theta_{a,b} \quad (11)$$

$\theta_{a,b}$ is the angle between the vectors j_a and j_b . Since the length of vector j is given by $\sqrt{j(j+1)}$ one obtains from eq (12) in a classical picture [10]. Where $\theta_{a,b}$ is the angle between the vectors j_a and j_b . Since the length of vector j is given by $\sqrt{j(j+1)}$ one obtains from eq (12) in a classical picture [10].

$$\cos \theta_{a,b} = \frac{I(I+1) - j_a(j_a+1) - j_b(j_b+1)}{2\sqrt{j_a(j_a+1)j_b(j_b+1)}} \quad (12)$$

The I -dependence of the matrix element V_{abab}^{IT} can thus be plotted as a function of the angle $\theta_{a,b}$. The radial overlaps of the particle orbits for light nuclei differ from those for heavy nuclei. The proton -neutron configurations correspond to nucleon pair having mixed isospin and one find [15, 16].

$$E_I(p, n) = \frac{1}{2} \{ (V_{abab}^{IT})_{T_I=1} + (V_{abab}^{IT})_{T_I=0} \} \quad (13)$$

Plotting the excitation energy of these states as a function of the corresponding angle $\theta_{a,b}$ determined as specified by Eq.(12). For neutron and proton in various orbits the absolute value of average two body energy is given by [19]:

$$\bar{E} = \left| \sum_I (2I+1) E_I \left\{ \sum_I (2I+1) \right\}^{1/2} \right| \quad (14)$$

With E_I defined by Eq. (13).

3. Results and discussion

Some properties of nuclear structure for ground bands of ^{64}Ni nuclei have been calculated using MSDI. Hole nucleons of these nucleus are distributed in: $(1P_{3/2}0f_{5/2})$ model space. In these calculations MSDI have been utilized to estimate the energy levels and classical coupling angles $\theta_{a,b}$ for 2 neutrons hole. Thus, ^{64}Ni nuclei are discussed the following. There are 2 neutrons hole less than closed shell $N = 38$. In this case, the original MSDI Hamiltonian is adjusted to properly the ground state energy. It is found that the acceptance with the experimental value is very good. In order to involve the neutrons contribution, configurations mixing between the orbit is applied. The spectrum of this nucleus was calculated by using Eq.(4,5,6 and 8). Energy levels can be obtained by using the single particle energy [1]. Where: $\delta_{0f_{5/2}} = 8.8061$ MeV and $\delta_{1P_{3/2}} = 9.1162$ MeV. Table 1 show a comparison between a theoretical, Theor.Res. and experimental, Exp.Res., excitation energies, MeV, for ^{64}Ni nucleus by using Model Space $(1P_{3/2}0f_{5/2})$. From table 1 it can be concluded the

model space considered predict results in agreement with the observed data especially at low-excited energy levels. The MSDI calculations of the energies, and parity are in good agreement with the experimental values [37]. Good agreement between theoretical energy levels with experimental data. The new energy levels, which are expected for this nucleus in the states $(4_1^+, 2_2^+$ and 1_1^+ were not well established experimentally. Plotting excitation energy of these states as a function of the identical angle determined according to Eq. (12). Table 2 show (according to states of angular momentum I , all possible cases of the classical coupling angle data within configuration $(1P_{3/2}0f_{5/2})$ Table 3. show reduced transition probability $B(E2; \downarrow)$ for ^{64}Ni nucleus with the experimental value [37]. Fig. 1 show the behaviour for even states $j_a + j_b + I = even$ and odd states $j_a + j_b + I = odd$ of effective interaction for ^{64}Ni . Curvature is a measure of a short- range attractive force. The highest spin I values correspond to least possible angle $\theta_{a,b}$ and contrariwise. The energy levels seem to follow 2-universal functions which depend on the classical- coupling angles $\theta_{a,b}$.

Table 1: Comparison between a theoretical, Theor.Res. and experimental, Exp.Res., excitation energies, MeV, for ^{64}Ni nucleus

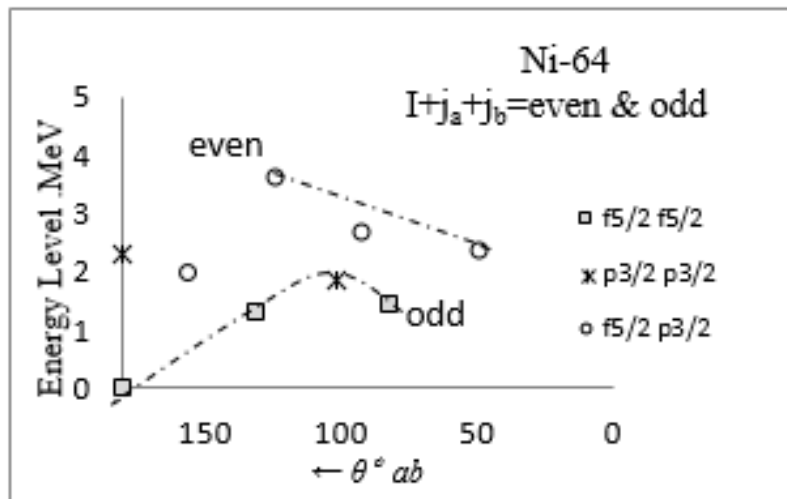
I^π	$\frac{Energy}{(MeV)}$	$\frac{Energy}{(MeV)}$	I^π
	Theor. Res.	Exp. Res. [37]	
0_1^+	0.000	0.000	$0+$
2_1^+	1.3351	1.3457	$2+$
4_1^+	1.4448
2_2^+	1.849
1_1^+	1.9809
0_2^+	2.2968	2.2765	$(2)+$
4_2^+	2.3946	2.6101	4^+
3_1^+	2.7170	2.867	$(0)+$
2_3^+	3.647	3.6478	2^+

Table 2: According to states of angular momentum I , all possible cases of the classical coupling angle data.

I^π	Configuration ^{64}Ni	state	$\theta_{a,b}^\circ$
0_1^+	$(\frac{5}{2}, \frac{5}{2}, \frac{5}{2}, \frac{5}{2})$	Odd	180
2_1^+	$(\frac{5}{2}, \frac{5}{2}, \frac{5}{2}, \frac{5}{2})$	Odd	131.0823
4_1^+	$(\frac{5}{2}, \frac{5}{2}, \frac{5}{2}, \frac{5}{2})$	Odd	81.78679
2_2^+	$(\frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2})$	Odd	101.537
1_1^+	$(\frac{5}{2}, \frac{5}{2}, \frac{3}{2}, \frac{3}{2})$	Odd	156.4218
0_2^+	$(\frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2})$	Odd	180
4_2^+	$(\frac{5}{2}, \frac{5}{2}, \frac{3}{2}, \frac{3}{2})$	Even	49.10661
3_1^+	$(\frac{5}{2}, \frac{5}{2}, \frac{3}{2}, \frac{3}{2})$	Odd	92.50139
2_3^+	$(\frac{5}{2}, \frac{5}{2}, \frac{3}{2}, \frac{3}{2})$	Even	124.5668

Table 3: Reduced transition probability $B(E2; \downarrow)$ for ^{64}Ni nucleus with the experimental value [37]

Transition n levels	Theoretical $B(E2), e^2 fm^4$	Experimental $B(E2), e^2 fm^4$
$2_1^+ \rightarrow 0_1^+$	117.6924	117.6863 ± 37.96
$2_2^+ \rightarrow 0_1^+$	52.59697	...
$2_3^+ \rightarrow 0_1^+$	11.21293	...
$2_3^+ \rightarrow 0_2^+$	240.6839	...
$4_1^+ \rightarrow 2_1^+$	79.15135	101.7417 ± 16.7039
$4_2^+ \rightarrow 2_1^+$	79.15135	212.5947 ± 16.7039
$4_2^+ \rightarrow 2_2^+$	194.7856	...
$3_1^+ \rightarrow 2_1^+$	66.05897	...
$3_1^+ \rightarrow 2_2^+$	68.96398	...
$1_1^+ \rightarrow 2_1^+$	22.02019	...
$1_1^+ \rightarrow 2_2^+$	11.54148	...

Figure 1: The value of theoretical energy levels for states as (hole -hole) a function of the classical coupling angles $\theta_{a,b}$

4. Conclusion

The current theoretical calculations give the data of the energy levels, by using the MSDI potential for Ni nuclei. The shell model used in the present study predicts most of the energy levels and their systematics with their total angular momentum. Comparing the experimental results with the shell-model calculations shows that the level structures exhibit mainly the single-particle character. The most important results of calculations can be summarized as follows:

- The agreement between theoretical and experimental levels is satisfactory for excitation energies and transition probabilities $B(E2; \downarrow)$. New values for $B(E2; \downarrow)$ not indicated by the experimental data.
- We observe the energy levels appear to follow two overall functions which depend on the classical coupling angles but are unconstrained of angular momentum I . The minimum angular momentum I values correspond to maximum angle and vice versa.

- The theoretical calculations in the nuclear shell model by using the MSDI reasonably agree with the experimental data. This indicates that the shell model is very good to describe the nuclear structure for 2-hole in Ni nuclei

References

- [1] G. Audi, A.H. Wapstr and C. Thibault, *The AME2003 atomic mass evaluation:(II). Tables, graphs and references*, Nuclear Phys. A 729 (2003) 337–676.
- [2] S. Bacca, *Complete identification of states in ^{208}Pb below $E_x = 6.2$ Mev*, Eur. Phys. J. Plus. 131 (2016) 107–119.
- [3] P.J. Brussaard and P.W.M. Glauddemans, *Shell-Model Applications in Nuclear Spectroscopy*, North-Holland, Amsterdam, 197.
- [4] E. Caurier, *The shell model as a unified view Of nuclear structure*, Rev. Mod. Phys. 77 (2005) 427–480.
- [5] A.S. Changizi and Ch. Qi, *Odd-even staggering in neutron drip line nuclei*, Nuclear Phys. A 951 (2016) 97–115.
- [6] L. Coraggioet, A. Covello, A. Gargano and N. Itaco, *Proton-neutron multiplets in exotic ^{134}Sb : testing the shell-model effective interaction*, Phys. Rev. C. 73(031302) (2006).
- [7] L. Coraggioet, A. Covello, A. Gargano and N. Itaco, *Low-momentum nucleon-nucleon interactions and shell-model calculations*, Phys. Rev. C 75(024311) (2007).
- [8] L. Coraggioet, A.Covello, A. Gargano and N. Itaco, *Similarity of nuclear structure in the ^{132}Sn and ^{208}Pb regions:proton-neutron multiplets*, Phys. Rev. C 80(2) (2009).
- [9] A. Faessler and A. Plastino, *Model applications in spectroscopy*, Z. Phys. 203 (1967) 333–443.
- [10] A. Faessler and A. Plastino, *The surface delta interaction in the transuranic nuclei*, Z. Phys. 203(4) (1967) 333–345.
- [11] H.T. Fortune, R. Sherr, *$2p_{3/2}$ strength in $^{40,41}\text{Sc}$ and the $^{39}\text{Ca}(p,b)$ reaction rate*, Phys. Review C 65(067301) (2002).
- [12] W.M. Glaudemans, *Two-body matrix elements from a modified surface delta interaction*, Nucl. Phys. A 102 (1967) 593–600.
- [13] D.N. Hameed and A.K. Hasan, *Energy levels Of isobaric nuclei (^{16}N , ^{16}F) within the modified surface delta-interaction model*, Ukrain. J. Phys. 63 (2018) 579–590.
- [14] D.N. Hameed and A.K. Hasan, *Determining the excitation energies of ^{68}Ni nucleus a function of the coupling angle by means of modified surface delta*, J. Phys. Conf. Ser. 1963 (012062) (2021) 1–9.
- [15] D.N. Hameed and A.K. Hasan, *The relationship between the energy levels And semi-classical coupling angle $\theta_{1,2}$ for ^{48}Sc , ^{54}Co nuclei using pandya transformation Indian*, Indian J. Phys. 95 (2021) 1833–1836.
- [16] D.N. Hameed and A.K. Hasan, *Energy levels of nuclei ^{40}Sc and ^{40}K as a function of semi-classical coupling angle $\theta_{1,2}$ within the modified surface delta-interaction*, Nucl. Phys. At. Energy 20(2) (2019) 146–152.
- [17] A.K. Hasan, *Shell model calculations for $^{18,19,20}\text{O}$ isotopes by using usda and usdb interactions*, Ukr. J. Phys. 63 (2018) 3–9.
- [18] A.K. Hasan and A.R.H. Subber, *Level structure of ^{210}Po by means Of surface delta interaction*, Turk. J. Phys. 37 (2013) 348–350.
- [19] A.K. Hasan and D.N. Hameed, *Energy levels of ^{50}Ca nucleus as a function of the classical coupling angle within MSDI*, Neuro Quantol. 19(5) (2021) 61–67.
- [20] A. Heusler, *Complete identification of states in ^{208}Pb below $E_x = 6.2$ MeV*, Phys. Rev. C 93(054321) (2016).
- [21] A. Jasielska and S.Wiktor, *Shell model interaction*, Acta Phys. Polon. B 7(2) (1976) 333–342.
- [22] J. Jing Liu, *RA beta decay of nuclides ^{56}Fe , ^{62}Ni , ^{64}Ni and ^{68}Ni in the crust of magnetars*, RRA. 11 (2016) 174–178.
- [23] K. Joel and S. Jouni, *Spin-multipole nuclear matrix elements in the Pn quasiparticle random-phase approximation:implications for B and B β Half-Lives*, Phys. Rev. C 95(014322) (2017).
- [24] K. Kaki, *Neutron density distributions analyzed in terms of relativistic impulse approximation for nickel isotopes*, Int. J. Modern Phys. E 24 (2015) 315–515.
- [25] K. Kaneko, T. Mizusaki, Y. Sun and S. Tazaki, *Systematical shell-model calculation in the pairing-plus-multipole Hamiltonian with a monopole interaction for the p f $5/2$ g $9/2$ shell*, Phys. Rev. C 92(044331) (2015) 67–89.
- [26] R.D. Lawson, *Theory of The Nuclear Shell Model*, Clarendon Press, Oxford, 1980.
- [27] F.A. Majeed and S. M. Obaid, *Nuclear structure study of $^{22,24}\text{Ne}$ and ^{24}Mg nuclei*, Rev. Mex. Fís. 65 (2019) 159–167.
- [28] N.S. Martorana, G. Cardella, E.G.Lanza , L. Acosta and M.V. Andrés, *On the nature of the Pygmy Dipole Resonance in ^{68}Ni* , Il Nuovo Cimento C 41 (2018) 1–4.

- [29] A. Molinari, M. Johnson, H.A. Bethe and W.M. Alberico, *Effective two-body interaction in simple nuclear spectra*, Nucl. Phys. A 239 (1975) 45–60.
- [30] E. Mraheemet, *The effects Of core polarisation On some even–even sd-shell nuclei using michigan three-range yukawa and modified surface delta interactions*, Pramana J. Phys. 23 (2019) 345–355.
- [31] P. Papakonstantinou, A. Panagiota, H. Hergert and R. Roth, *Isoscalar and neutron modes in the E 1 spectra Of ni isotopes and The relevance Of shell effects and the continuum*, Phys. Rev. C 92(034311) (2015) 234–245.
- [32] J. Piekarewicz, *Nuclear breathing mode in neutron-rich nickel isotopes: sensitivity to the symmetry energy and the role of the continuum*, Phys. Rev. C 91(1) (2015).
- [33] B. Pritychenko, M. Birch, B. Singh and M. Horoi, *Tables of E2 transition probabilities from the first 2+ states in even-even nuclei*, Atom. Data and Nucl Data Tables 107 (2016) 1–139.
- [34] J.P. Schiffer, *He spectra of near-magic odd-odd nuclei and the effective interaction*, Ann. Phys. 66 (1971) 798–800.
- [35] S. Shim, *Excitation energies means Of modified*, J. Korean Phys. Soc. 73 (2018) 1631–1641.
- [36] S. Sharma and A.J. Obaid, *Mathematical modelling, analysis and design of fuzzy logic controller for the control of ventilation systems using MATLAB fuzzy logic toolbox*, J. Interdiscip. Math., 23(4) (2020) 843–849.
- [37] B. Singh and C. June, *Nuclear data sheets for A = 64*, Nucl. Data Sheets 178 (2021) 41–537.
- [38] P. Van Isacker, *Geometry of shell-model matrix elements*, EPJ Web Conf. 78(03004) (2014).
- [39] P. Van Isacker, *A geometry for the shell model*, EPJ Web Conf. 178(05002) (2018).
- [40] P. Van Isacker and A.O. Macchiavelli, *Geometry of the shears mechanism in nuclei*, Phys. Rev. C 87(061301) (2013).
- [41] D. Wójcik, A. Trzcinska, E. Piasecki, M. Kisielinski, M. Kowalczyk, W. Cichocka and H. Zhang, *Transfer cross sections at near-barrier energy for the $^{24}\text{Mg}+^{90,92}\text{Zr}$ systems*, Acta Phys. Polon. B. 49(3) (2018) 3–8.