



Improved optimality checkpoint for decision making by using the sub-triangular form

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Abstract

Decision-making in Operations Research is the main point in various studies in our real-life applications. However, these different studies focus on this topic. One drawback some of their studies are restricted and have not addressed the nature of values in terms of imprecise data (ID). This paper thus deals with two contributions. First, decreasing the total costs by classifying sub-sets of costs. Second, improving the optimality solution by the Hungarian assignment approach. This newly proposed method is called fuzzy sub-Triangular form (FS-TF) under ID. The results obtained are exquisite as compared with previous methods including, robust ranking technique, arithmetic operations, magnitude ranking method and centroid ranking method. This current novelty offers an effective tool to accesses solving the ID to solve assignment problems.

Keywords: Assignment problems, Decision making, Imprecise data, Optimality check Point, Sub-Triangular form.

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1. Introduction

The assignment problems help in picking the project and matching with an appropriate manager which leads to an efficient outcome [2]. There are a huge of literature reviews which considered related to classical assignment problems. Most of the resources are focused on minimizing time and costs. One of these methods [7] used assignment problems by the machine learning approach. Another researcher [15] suggests an algorithm showing the number of vehicles needed to achieve comparable performance in the assignment problem. While [8] formulate a dual approach to multi-dimensional assignment problems.

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As well known that Assignment problems appear in various fields such as engineering, transportation, and industrial management [3]. Therefore, to solve this type of problem, the decision values of the model must be fixed values. But to model in our real-life applications and to perform computation we must deal with uncertain environments and inexactness [17]. These ambiguity and inexactness are due to values inaccuracy, simplification of models, variations of the values in the system, computational errors, etc. Consequently, we cannot successfully use conventional assignment problems. Hence, the use of fuzzy assignment problems is more appropriate under imprecise data [5]. Recently studies considered developing a new approach to solving solid assignment problems under an intuitionistic fuzzy environment [6, 9]. Despite, most recent studies confirm to convert the ID into crisp data. This (ID) leads to obtaining approximate solutions with an extension according to different types of assignment approaches.

Therefore, to fill the gap for this work it must be an attempt to deal with the proposed new method based on classifying these ID by using the sub-Trident form through the newly proposed method fuzzy sub-Triangular form (FS-TF) to achieve better conventional data. Therefore, different types of ID can be discovered in a wide variety of optimality and decision planning to the industrial management.

The structure design for this work as follows: Section 2 discuss the definition and mathematical formulation of the proposed fuzzy sub-Triangular form, Next, the types of assignment problem within the industrial scope, along with their methods, are presented in Section 3. This section is divided according to two parts; (i) proposed a new method (FS-TF) and (ii) improving the optimality through the Hungarian assignment approach. Section 4 showed the comparisons of the results between existing methods and the proposed method. Finally, the conclusions, future direction, and possible way of solution approaches are offered in Section 5.

2. Outline methodology of the FS-TF

The assignment problem is a special application of general linear programming models. Assume we have n projects and m persons. We know the cost performing of projects to assigning individual persons. The basic target in the assignment problems is based on two particular terms. First, is to minimize the total cost ($\widetilde{C}_{o_{ij}}$). Second, the number of rows are equals to the number of columns ($n \times m$). The form of assignment problem under imprecise data is as follows [11].

$$\min Z(\tilde{x}) = \sum_{i=1}^n \sum_{j=1}^m (\widetilde{C}_{o_{ij}}) \tilde{z}_{ij}; \text{ where; } i = 1, 2, \dots, n; j = 1, 2, \dots, m \quad (2.1)$$

Const.

$$\sum_{i=1}^n \tilde{z}_{ij} = 1, \quad i = 1, 2, \dots, n$$

$$\sum_{j=1}^m \tilde{z}_{ij} = 1, \quad j = 1, 2, \dots, m$$

$$\tilde{z}_{ij} \in \{0 \text{ or } 1\}$$

$$\text{Where; } \tilde{z}_{ij} = \begin{cases} 1 & \text{if the } i^{\text{th}} \text{ projects is assigned the } j^{\text{th}} \text{ person} \\ 0 & \text{otherwise} \end{cases}$$

On the other hand, the structures of the assignment model under (ID) can be designed as in Table 1.

Table 1: The Structure of the Assignment Model with ID

No.	1	2	...j...	n
1	$(\widetilde{C}o_{11})$	$(\widetilde{C}o_{12})$... $(\widetilde{C}o_{1j})$...	$(\widetilde{C}o_{1n})$
2	$(\widetilde{C}o_{21})$	$(\widetilde{C}o_{22})$... $(\widetilde{C}o_{2j})$...	$(\widetilde{C}o_{2n})$
⋮				
i	$(\widetilde{C}o_{i1})$	$(\widetilde{C}o_{i2})$... $(\widetilde{C}o_{ij})$...	$(\widetilde{C}o_{in})$
⋮				
m	$(\widetilde{C}o_{m1})$	$(\widetilde{C}o_{m2})$... $(\widetilde{C}o_{mj})$...	$(\widetilde{C}o_{mn})$

Based on the proposed method named fuzzy sub-Triangular form (FS-TF) Figure 1 shows the procedure of this newly proposed method [12].

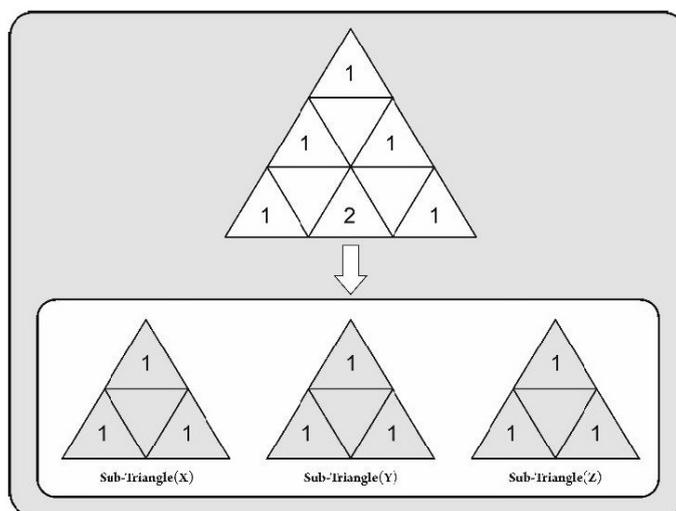


Figure 1: The Procedure of Sub-Trident Form

$$a_1 = P(X) = \frac{a + b}{2}, \quad b_1 = P(Y) = \frac{a + b}{2}, \quad c_1 = P(Z) = \frac{a + b}{2} \tag{2.2}$$

$$a_2 = P(X) = \frac{a + b}{2}, \quad b_2 = P(Y) = \frac{a + 2b}{3}, \quad c_2 = P(Z) = \frac{2a + b}{3} \tag{2.3}$$

$$a_3 = P(X) = \frac{a + 2b}{2}, \quad b_3 = P(Y) = \frac{2a + b}{3}, \quad c_3 = P(Z) = \frac{a + b}{2} \tag{2.4}$$

It can be noticed above there is three sub-Trident levels in this proposed method through Equations ((2.2), (2.3) and (2.4)). Then, we have the following fuzzy sub-Triangular [1].

$$FST_{r(a,b,c)} = (a_a, b_b, c_c) \tag{2.5}$$

Hence;

$$a_a = \frac{a_1 + a_2 + a_3}{3}, \tag{2.6}$$

$$b_b = \frac{b_1 + b_2 + b_3}{3}, \tag{2.7}$$

$$c_c = \frac{c_1 + c_2 + c_3}{3}, \tag{2.8}$$

Where,

$P(X) = a_a$: denotes the sub-Triangle of (X) .

$P(Y) = b_b$: denotes the sub-Triangle of (Y) .

$P(Z) = c_c$: denotes the sub-Triangle of (Z) .

This procedure follows to obtain the Trident form as.

$$FST_{ri} = \frac{1}{3}(a_a^{\frac{1}{3}} + b_b^{\frac{1}{3}} + c_c^{\frac{1}{3}}) \tag{2.9}$$

Then we utilize the Hungarian approach to access the optimality cost after converting the impression from the data and obtain the crisp data. Thus, we can be described by the following steps were applied to the $(n \times m)$ cost matrix to find the optimal assignment [18].

- Step 1:** Select the smallest value in each row and subtract it from all values in the row.
- Step 2:** Select the smallest value in each column and subtract it from all values in the column.
- Step 3:** A minimum value of lines drawn to cover all zero in the cost matrix.
- Step 4:** Check for optimality, If the minimum value is an optimal assignment. Or, if lines are less than n , then the optimal value is not yet possible. Hence, go to the next step.
- Step 5:** Determine the smallest value entry not covered by any line. Subtract this entry from each uncovered row, and then add it to each covered column. Go to Step 3.

3. Application of the proposed method to the assignment problem

In this section, we consider the procedure of the proposed method (FS-TF) with assignment issues under the same ID in the existing model. Hence, four rows representing the projects ($P_{ro1}, P_{ro2}, P_{ro3}$, and P_{ro4}). In contrast, four columns representing person managers ($P_{er1}, P_{er2}, P_{er3}$, and P_{er4}). Hence, the Triangular costs matrix is given by.

Table 2: The Triangular Costs Matrix

No.	P_{ro1}	P_{ro2}	P_{ro3}	P_{ro4}
P_{er1}	(1,5,9)	(3,7,11)	(7,11,5)	(2,6,10)
P_{er2}	(4,8,12)	(1,5,9)	(4,9,13)	(2,6,10)
P_{er3}	(0,4,8)	(3,7,11)	(6,10,14)	(3,7,11)
P_{er4}	(6,10,14)	(0,4,8)	(4,8,12)	(-1,3,7)

The values in Table 2 show the extracting Triangular values by using the proposed method (FS-TF) starting from Equations (2.6),(2.7),(2.8),(2.9) to obtain different crisp values represents the costs matrix. Thus, with the assistance of scientific workplace version 5.5 [13].

Proceeding with the Hungarian approach try to get close to the minimum cost by improving a candidate solution iteratively as described in Section 2. Hence, Tables 4 and 5 shows the results of Hungarian steps.

Table 3: The Results of the Proposed Method (FS-TF)

No.	P_{ro1}	P_{ro2}	P_{ro3}	P_{ro4}
P_{er1}	<u>1.453</u>	1.707	2.073	1.585
P_{er2}	1.804	<u>1.453</u>	1.860	1.585
P_{er3}	<u>1.205</u>	1.707	1.998	1.707
P_{er4}	2.154	1.205	1.804	<u>0.995</u>

Table 4: Applying the Hungarian Approach

No.	P_{ro1}	P_{ro2}	P_{ro3}	P_{ro4}
P_{er1}	0.0	0.254	0.620	<u>0.132</u>
P_{er2}	<u>0.351</u>	0.0	<u>0.407</u>	0.132
P_{er3}	0.0	0.502	0.793	0.502
P_{er4}	1.003	<u>0.210</u>	0.809	0.0

Table 5: The Results of the Hungarian Approach

No.	P_{ro1}	P_{ro2}	P_{ro3}	P_{ro4}
P_{er1}	0.0	0.044	0.213	0.0
P_{er2}	0.0	0.0	0.0	0.0
P_{er3}	0.0	0.292	0.386	0.370
P_{er4}	0.652	0.0	0.402	0.0

Hence, the fuzzy optimal total cost obtained from Table 3 is represented as; $(\tilde{C}_{14} + \tilde{C}_{23} + \tilde{C}_{31} + \tilde{C}_{42})$ can be obtained recalling into Equation (2.1). Thus, the $P_{er1} \rightarrow P_{ro4}$, $P_{er2} \rightarrow P_{ro3}$, $P_{er3} \rightarrow P_{ro1}$, $P_{er4} \rightarrow P_{ro2}$. Then, the minimum cost are obtain by computing the values according to the initial Table 2; $(2, 6, 10) + (4, 9, 13) + (0, 4, 8) + (0, 4, 8) = (6, 33, 39)$

\therefore optimality = 5.855 unit.

4. Comparison of the results between the modified methods and the proposed method

To show the difference in terms of efficiency scores between the output (i.e. Total cost) of our modified model (FS-TF) and the existing models including Kalaiarasi [4], Thakur [16], Selvi [14], and Mary [10]. The results are reported in Table 6.

Table 6: Comparison of the results of optimality total costs

Total cost	The efficiency of the Existing Models				The efficiency of the
	Kalaiarasi [4]	Singh [16]	Selvi [14]	Mary [10]	Proposed Method (FS-TF)
	23.00	22.75	8.50	7.76	<u>5.855</u>

Thus, Table 6 demonstrate the efficiency resulting obtained from the proposed method (FS-TF) is better than the previous imprecise models to convert the (ID) into accurate crisp data under the same Hungarian approach. It can also observe the efficiency of this method is easy to check point optimality equal to 5.855 unit, is adequate and accurate results to locate the triple values as illustrated in Table 2. Furthermore, this proposed method characteristics to accept negative values as can be seen in the cell a44. Hence, this proposed method assessments to classify the results according to three partitions (recalling into Equations (2.6),(2.7),(2.8)), respectively.

5. Conclusions

Every process of decision-making can be represented as a result of a final choice of output can be represented as an action or as an opinion of choice. The proposed method (FS-TF) provides two criteria. The first one is efficient in obtaining accurately crisp values. And, the second one is for an effective way to accesses minimizing total cost. Moreover, the results of comparison based on the total cost obtained are more applicable than previous methods. This in turn provides as good opportunity to decide the manger's which leads to maximizing profits for industrial companies. For future work, more types of membership functions can be capture using integer and negative (ID).

References

- [1] J. Chen, T. Theeramunkong, T. Supnithi and X. Tang, *Knowledge and Systems Sciences*, 18th International Symposium, KSS 2017, Bangkok, Thailand, Springer, 2017.
- [2] A.M. Coroiu, *Fuzzy methods in decision making process-A particular approach in manufacturing systems*, In IOP Conf. Ser.: Materials Sci. Eng. 95(1) (2015) 012154.
- [3] T.G. Crainic, M. Gendreau and J.Y. Potvin, *Intelligent freight-transportation systems: Assessment and the contribution of operations research*, Transport. Res. Part C: Emerg. Technol. 17(6) (2009) 541–557.
- [4] K. Kalaiarasi, S. Sindhu and M. Arunadevi, *Optimization of fuzzy assignment model with triangular fuzzy numbers using robust ranking technique*, Int. J. Innov. Sci. Engg. Technol. 1(3) (2014) 10–15.
- [5] M. Khalid, M. Sultana and F. Zaidi, *New improved one's assignment method*, Appl. Math. Sci. 8(84) (2014) 4171–4177.
- [6] P.S. Kumar, *Developing a new approach to solving solid assignment problems under an intuitionistic fuzzy environment*, Int. J. Fuzzy Syst. Appl. 9(1) (2020) 1–34.
- [7] M. Lee, Y. Xiong, G. Yu and G.Y. Li, *Deep neural networks for linear sum assignment problems*, IEEE Wireless Commun. Lett. 7(6) (2018) 962–965.
- [8] J. Li, T. Kirubarajan, R. Tharmarasa, D. Brown and K.R. Pattipati, *A dual approach to multi-dimensional assignment problems*, J. Global Optim. 81 (2021) 691—716.
- [9] M. Mammadova and Z. Jabrayilova, *Application of fuzzy optimization method in decision-making for personnel selection*, Intell. Cont. Autom. 5(4) (2014) 190–204.
- [10] R.Q. Mary and D. Selvi, *Solving fuzzy assignment problem using centroid ranking method*, Int. J. Math. Appl. 6(3) (2018) 9–16.
- [11] K. Prabakaran and K. Ganesan, *The fuzzy Hungarian method for solving intuitionistic fuzzy traveling salesman problem*, J. Phys.: Conf. Ser. 1000(1) (2018) 012008.
- [12] A.P. Prakash and M.G. Lakshmi, *Sub-trident ranking using fuzzy numbers*, Int. J. Math. Appl. 4(2) (2016) 143–150.
- [13] Scientific Workplace, *Mathematical Scientific Soft-Ware*, <https://www.mackichan.com/index.html?products/downloadreq55.html> mainFrame, (2015).
- [14] D. Selvi, R.Q. Mary and G. Velammal, *Method for solving fuzzy assignment problem using magnitude ranking method*, Int. J. Appl. Adv. Sci. Res. (2017) 16–20.
- [15] A. Simonetto, J. Monteil and C. Gambella, *Real-time city-scale ridesharing via linear assignment problems*, Transport. Res. Part C: Emerg. Technol. 101 (2019) 208–232.
- [16] J.P. Singh and N.I. Thakur, *A novel method to solve assignment problems in a fuzzy environment*, Industrial Eng. Lett. 5(2) (2015) 31–35.
- [17] S. Suprasongsin, V.N. Huynh and P. Yenradee, *A three-dimensional fuzzy linguistic evaluation model*, J. Adv. Comput. Intell. Intell. Inf. 22(5) (2018) 767–776.