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On neutrosophic semi-regularization topological spaces

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Abstract

In this work, the idea of neutrosophic semi-regularization of neutrosophic topology is shown, as well as some of its characteristics. We show that for any neutrosophic set in neutrosophic topological space is a neutrosophic regular generalized α-closed set in (Ψ, τ) if and only if it is neutrosophic regular generalized closed set in (Ψ, τ^{α}), where τ^{α} is the family of all neutrosophic α-open sets in (Ψ, τ).

Keywords: neutrosophic points, neutrosophic rg-closed, neutrosophic regular generalized α-closed, neutrosophic regular generalized closed.
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1 Introduction

As an elaboration of Zadeh's fuzzy sets [31] from 1965 and Atanassav's intuitionistic fuzzy sets [5] from 1983, Smarandache has proposed and described neutrosophic sets (NSs). Three values represent A (NS): truth (memberships), indeterminacy, and falsity (non-memberships). Salama and Alblowi [28] proposed the new concept of neutrosophic topological space (NTS) in 2012, which had only been examined recently. Arokiarani M et al. looked at various concepts like neutrosophic (/regular/semi) closed sets in 2017 [4]. Rao and Srinivasa [26] then looked into the concept of a neutrosophic per-closed set. In 2018, Ebenanjar M et al. described neutrosophic b-clsed in (NTS) [7]. In 2020, the concept of a neutrosophic bg-closed set is introduced and investigated in (NTS) [24]. Non-classical spaces are used to study the expansion of some topological sets, such as soft sets [11, 8, 1, 12, 9], fuzzy sets [14, 15, 16, 17, 2], nano sets [18], permutation sets [19, 20, 30, 21, 22, 23], and others [27, 13]. To investigate our non-classical expansion, we'll use the concept of neutrosophic. The main purpose of this work is to consider and discussed new classes of neutrosophic topological spaces is called neutrosophic semi-regularization space, as well as some of its characteristics. We show that for any neutrosophic set in neutrosophic topological space is a neutrosophic regular generalized α-closed set in (Ψ, τ). if and only if it is neutrosophic regular generalized closed set in (Ψ, τ^{α}). , where τ^{α} is the family of all neutrosophic α-open sets in (Ψ, τ).

2 Preliminaries

In this section, we'll go through the background information are referred from the references [7, 10, 25, 3, 6, 29].

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Definition 2.1. Let $\Psi = \varphi$, then $K = \{\langle \varepsilon, \gamma_K(\varepsilon), \rho_K(\varepsilon), r_K(\varepsilon)\rangle : \varepsilon \in \Psi\}$ is said to be neutrosophic set (NS) , if $\gamma_K(\varepsilon), \rho_K(\varepsilon)$ *and* $r_K(\varepsilon)$ are the degrees of membership, indeterminacy and non-membership, according $\forall \varepsilon \in \Psi$ to K . Also, if $H = \{\langle \varepsilon, \gamma_H(\varepsilon), \rho_H(\varepsilon), r_H(\varepsilon)\rangle : \varepsilon \in \Psi\}$ is (NS). Then

(1) $K \subseteq H$ if and only if $\gamma_K(\varepsilon) \le \gamma_H(\varepsilon),\ \rho_K(\varepsilon) \ge \rho_H(\varepsilon)$ *and* $r_K(\varepsilon) \ge r_H(\varepsilon)$,
(2) $K \cap H = \{\langle \varepsilon, \min\{\gamma_K(\varepsilon), \gamma_H(\varepsilon)\}, \max\{\rho_K(\varepsilon), \rho_H(\varepsilon)\}, \max\{r_K(\varepsilon), r_H(\varepsilon)\}\rangle : \varepsilon \in \Psi\}$
(3) $K^c = \{\langle \varepsilon, r_K(\varepsilon), 1 - \rho_K(\varepsilon), \gamma_K(\varepsilon)\rangle : \varepsilon \in \Psi\}$,
(4) $K \cup H = \{\langle \varepsilon, \max\{\gamma_K(\varepsilon), \gamma_H(\varepsilon)\}, \min\{\rho_K(\varepsilon), \rho_H(\varepsilon)\}, \min\{r_K(\epsilon), r_H(\varepsilon)\}\rangle : \varepsilon \in \Psi\}$

Definition 2.2. Let $\tau = \{T_i | i \in I\}$ be a family of neutrosophic sets (NSs) in Ψ. Then (Ψ, τ) is said to be neutrosophic topological space (NTS) if and only if τ such that:

(1) 1_N , $0_N \in \tau$, where $0_N = \{\langle \varepsilon, (0, 1, 1)\rangle : \varepsilon \in \Psi\}$ *and* $1_N = \{\langle \varepsilon, (1, 0, 0)\rangle : \varepsilon \in \Psi\}$.
(2) $T_i \cap T_j \in \tau,\ \forall T_i, T_j \in \tau$,
(3) $\cup_{i\in\delta} T_i \in \tau$ for any $\delta \subseteq I$. In other side, we say T_i is neutrosophic open set (NOS) and T_i^c is neutrosophic closed set (NCS) if $T_i \in \tau$.

Definition 2.3. The neutrosophic closure of K is the intersection of all neutrosophic closed sets containing K and is denoted by $cl^N K$. The neutrosophic interior of K is the union of all neutrosophic open set is contained in K and is denoted by $int^N K$. Similarly , we define neutrosophic regular closure, neutrosophic α-closure, neutrosophic pre-closure, neutrosophic semi closure, neutrosophic b-closure and neutrosophic semi preopen closure of the neutrosophic set K of a (NTS) Ψ and are denoted by $rcl^N K, \alpha cl^N K, pcl^N K, scl^N K, bcl^N K$ *and* $spcl^N K$ respectively. The family of all neutrosophic α-open (resp. neutrosophic semi-open, neutrosophic preopen, neutrosophic semi-preopen, neutrosophic - open, neutrosophic regular open) sets in a $(NTS)(\Psi, \tau)$ is denoted by τ^α (*resp.* $NSO(\Psi, \tau), NPO(\Psi, \tau), NSPO(\Psi, \tau)$, $NBO(\Psi, \tau), NRO(\Psi, \tau)$). The complement of the neutrosophic α-open , neutrosophic semi-open, neutrosophic pre-open, neutrosophic semi-preopen, neutrosophic -open, neutrosophic regular open are their respective neutrosophic α-closed, neutrosophic semi-closed, neutrosophic preclosed, neutrosophic semi-preclosed, neutrosophic -closed, neutrosophic regular closed.

Definition 2.4. A $(NS)K$ in a $(NTS)(\Psi, \tau)$ is said to be

(1) a neutrosophic generalized closed set(NgCS) in Ψ if $cl^N K \tilde{\subseteq} H$ whenever $K \subseteq H$ and H is (NOS) in Ψ.
(2) a neutrosophic semi open set $(NSOS)$ if $K \subseteq cl^N(int^N K)$
(3) a neutrosophic regular open set $(NROS)$ if $K = int^N(cl^N K)$
(4) a neutrosophic α-open set $(N\alpha OS)$ if $K \subseteq int^N(cl^N(int^N K))$
(5) a neutrosophic b-open set $(NbOS)$ if $K \subseteq cl^N(int^N K) \cup int^N(cl^N K)$
(6) a neutrosophic semi preopen or neutrosophic β-open set $(N\beta OS)$ if $K \subseteq cl^N(int^N(cl^N K))$
(7) a neutrosophic pre-open set $(NPOS)$ if $K \subseteq int^N(cl^N K)$
(8) a neutrosophic regular generalized closed set $(NrgCS)$ in a (NTS) (Ψ, τ) if $cl^N K \subseteq H$ whenever $K \subseteq H$ and $H \in NRO(\Psi, \tau)$.
(9) a neutrosophic pre generalized closed set $(NPgCS)$ in a (NTS) (Ψ, τ) if $pcl^N K \subseteq H$ whenever $K \subseteq H$ and $H \in NPO(\Psi, \tau)$.

Remark 2.5.

(1) In Definition 2.4, the complement of each (NS) for (1,9,8) is (NOS) and they are referred by $(NgOS), (NrgOS)$ and $(NPgOS)$, respectively.
(2) In Definition 2.4, the complement of each (NS) for (2,3,4,5,6,7) is (NCS) and they are referred by $(NSCS), (NRCS)$, $(N\alpha CS), (NbCS), (N\beta CS)$ *and* $(NPCS)$, respectively.

Lemma 2.6. In a (NTS) we have the following:

(i) Every $(NROS)$ is (NOS).
(ii) Every (NOS) *is* $(N\alpha OS)$.
(iii) Every $(N\alpha OS)$ *is both* $(NSOS)$ *and* $(NPOS)$.
(iv) Every $(NSOS)$ *and every* $(NPOS)$ *is* $(N\beta OS)$.

Theorem 2.7. In a (NTS) , every $(NbOS)$ is $(N\beta OS)$ and every $(NbCS)$ is $(N\beta CS)$.

Theorem 2.8. In a (NTS)

(i) Every $(NPOS)$ is $(NbOS)$.
(ii) Every $(NSOS)$ is $(NbOS)$.

Remark 2.9. By 2.6, 2.7 and 2.8, we consider that for each (NS) K in a $(NTS)(\Psi,\tau)$. Then satisfies the following:

(i) $spcl^N K \subseteq bcl^N K \subseteq scl^N K \subseteq \alpha cl^N K \subseteq cl^n K \subseteq rcl^N K$.
(ii) $spcl^N K \subseteq bcl^N K \subseteq pcl^N K \subseteq \alpha cl^N K \subseteq cl^n K \subseteq rcl^N K$.

3 Neutrosophic Semi-Regularization of Neutrosophic Topology

In this section, we introduced neutrosophic semi-regularization spaces and study some their properties.

Definition 3.1. Let (Ψ,τ) be a (NTS), then the family of neutrosophic regular open sets forms a base for a smaller neutrosophic topology τ on Ψ called the neutrosophicsemi-regularization of τ.

Remark 3.2. It is clearly for any $(NTS)(\Psi,\tau)$ we have: $NSO(\Psi,\tau^\alpha) = NSO(\Psi,\tau)$ The following remark is very useful in the sequel

Proposition 3.3. If $K \in NSO(\Psi,\tau^\alpha)$, then $\tau^\alpha - cl^N K = \tau - cl^N K = \tau_s - cl^N K$.

Proof . We need only to show that $\tau_s - cl^N K \subseteq \tau^\alpha - cl^N K$ for $K \in NSO(\Psi,\tau)$. Let m be a neutrosophic point such that $m \notin \tau^\alpha - cl^N K$. Then there exists a $B \in t^\alpha$ such that $m \in B$ and $K \cap B = \varphi$. This implies that $\tau - int^N B \cap \tau - int^N K = \varphi$ and $\tau - cl^N(\tau - int^N B) \cap \tau - int^N K = \varphi$. Consequently $\tau - int^N(\tau - cl^N(\tau - int^N B)) \cap \tau - int^N K = \varphi$ and $\tau - int^N(\tau - cl^N(\tau - int^N B)) \cap \tau - cl^N(\tau - int^N K) = \varphi$. Since $K \in NSO(\Psi,\tau)$, $K \subseteq \tau - cl^N(\tau - int^N K)$. This implies that $\tau - int^N(\tau - cl^N(\tau - int^N B)) \cap K = \varphi$. Since $B \in t^\alpha$, $m \in \tau - int^N(\tau - cl^N(\tau - int^N B))$. Hence, $m \notin \tau^\alpha - cl^N K$, and the proof is complete. □

Corollary 3.4. Let (Ψ,τ) be a (NTS), then $\tau_s = (\tau^\alpha)_s$.

Proof . Since every $(NRCS)$ precisely $(NSOS)$, it follows from Remark 3.2 and Proposition 3.3 that $NRO(\Psi,\tau) = NRO(\Psi,\tau^\alpha)$. That means $NRC(\Psi,\tau) = NRC(\Psi,\tau^\alpha)$. This implies $\tau_s = (\tau^\alpha)_s$. □

Corollary 3.5. If K is a (NS) in $(NTS)(\Psi,\tau)$, then

(a) $\tau^\alpha - int^N(\tau^\alpha - cl^N K) = \tau - int^N(\tau - cl^N K)$.
(b) $\tau^\alpha - cl^N(\tau^\alpha - int^N(\tau^\alpha - cl^N K)) = \tau - cl^N(\tau - int^N(\tau - cl^N K)$.
(c) $\tau - cl^N(\tau - int^N(\tau - cl^N K)) \subseteq \tau^\alpha - cl^N K$.

Proof .

(a) From Remark 3.2, it follows that $NSC(\Psi,\tau^\alpha) = NSC(\Psi,\tau)$.By proposition 3.3, $\tau^\alpha - int^N B = \tau - int^N B$ for each $B \in NSC(\Psi,\tau)$, so that $\tau^\alpha - int^N(\tau^\alpha - cl^N K) = \tau - int^N(\tau^\alpha - cl^N K)$. Since $\tau - int^N(\tau^\alpha - cl^N K) = \tau - int^N(\tau - cl^N K)$, we conclude that $\tau^\alpha - int^N(\tau^\alpha - cl^N K) = \tau - int^N(\tau - cl^N K)$.
(b) This follows from (a) and proposition 3.3.
(c) This is an immediate consequence of (b).

□

Lemma 3.6. If K is a $(NS)(\Psi,\tau)$, then $\tau^\alpha - int^N(\tau^\alpha - cl^N K) = int^N(cl^N K)$.

Proof . This follows from Corollary 3.5. □

Lemma 3.7. Let K be a (NS) in $(NTS)(\Psi,\tau)$. Then $K \in NRO(\Psi,\tau)$ if and only if $K \in NRO(\Psi,\tau^\alpha)$

Proof . This is an immediate consequence of Lemma 3.6. □

Theorem 3.8. A $(NS)K\ in\ (NTS)(\Psi,\tau)\ is\ (Nrg\alpha CS)\ in\ )\Psi,\tau$ if and only if $K\ is\ (NrgCS)$ in the $(NTS)(\Psi,\tau^{\alpha})$

Proof . Necessity. Suppose that $K\ is\ (Nrg\alpha CS)\ in\ (\Psi,\tau)$. Let $K\subseteq B\ and\ B\in NRO(\Psi,\tau^{\alpha})$. Let us refer to $\alpha cl^{N}K\ in\ (\Psi,\tau^{\alpha})$ by $\alpha^{\tau}cl^{N}K$. Then by Lemma 3.7, $B\in NRO(\Psi,\tau)$ and we have $\alpha^{\tau}cl^{N}K=\alpha cl^{N}K\subseteq B$. Therefore, K is $(NrgCS)\ in\ (\Psi,\tau^{\alpha})$.

Sufficiency, suppose that $K\ is\ (NrgCS)\ in\ (\Psi,\tau^{\alpha}).K\subseteq B$ and $B\in NRO(\Psi,\tau)$. By Lemma 3.7, $B\in NRO(\Psi,\tau^{\alpha})$, and hence, $\alpha cl^{N}K=\alpha^{\tau}cl^{N}K\subseteq B$. Therefore, K is $(NrgCS)\ in\ (\Psi,\tau)$. □

4 Conclusion

In this article, we look at the concept of neutrosophic semi-regularization of neutrosophic topology and discover a number of intriguing characteristics. Finally, we hope that this article is only the beginning of new classes of functions between two neutrosophic semi-regularization spaces, additional theoretical research will be required to examine the relationships between them.

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