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On the stability of an Euler Lagrange type cubic functional equation using the fixed point method

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Abstract

This paper establish the existence of solution, and the Hyers-Ulam-Rassias stability of an Euler Lagrange type cubic functional equation using the fixed point method.

Keywords: Hyers-Ulam-Rassias stability, Euler Lagrange type cubic functional equation, fixed point
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1 Introduction

S. M. Ulam raised a number of questions in his famous lecture of 1940 to the Mathematics Club of the University of Wisconsin, one of which may be put as follows.

Suppose G is a group and G' is a metric group and $f : G \longrightarrow G'$. Given $\varepsilon > 0$, does there exist a $\delta > 0$ such that $d\Big(f(xy),\ f(x)f(y)\Big) < \delta$ for all $x, y \in G$ implies there exists a homomorphism $L : G \longrightarrow G'$ such that

$$d\Big(f(x),\ L(x)\Big) < \varepsilon \qquad \text{for all } x \in G\ ?$$

If there exist such a homomorphism $L(xy) = L(x)L(y)$, then it is said to be stable. Similarly, if a functional equation is replaced by a functional inequality, then when can we assert that the solution of the functional inequality lie near the solution of the functional equation? The study of this problem led to the development of what is now known as the stability problems of functional equations.

D. H. Hyers [9] was the first to give an affirmative answer to the above problem, in 1941, with respect to addition in a Banach space. This result was extended by T. M. Rassias [18] in 1978 with an unbounded Cauchy difference, and later in 1993, T. M. Rassias and P. Šemrl [19] further extended by considering a monotonically increasing symmetric homogeneous function of degree p. In 1994, Găvruta [4] also further extended the result of T. M. Rassias. The problem with regards to multiplication was addressed by John A. Baker [2] in 1980, and Skof [20] first discussed the stability problem of a quadratic functional equation in 1983. Many researchers have contributed to the study of the stability problems of functional equations.

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2 Preliminaries

In 2002, Jun and Kim [10] introduced and obtained a general solution of the cubic functional equation

$$f(2x+y)+f(2x-y)=2f(x+y)+2f(2x-y)+12f(x). \tag{2.1}$$

and proved the Hyers-Ulam-Rassias stability.

In 2009, Eshaghi Gordji and Khodaei [6] introduced the functional equation

$$f(x+ky)+f(x-ky)=k^2\big[f(x+y)+f(x-y)\big]+2(1-k^2)f(x) \tag{2.2}$$

for fixed integers k with $k \neq 0, \pm 1$ and obtain its general solution and the generalized Hyers-Ulam-Rassias stability in quasi-Banach spaces.

In 2010, Jun et al. [12] introduced the functional equation

$$\begin{aligned} &f(x+ny)+f(x-ny)+f(nx) \\ &\qquad = n^2[f(x+y)+f(x-y)]+(n^3-2n^2+2)f(x) \end{aligned} \tag{2.3}$$

where $n \geq 2$, and established the equivalence of (2.3) and the cubic functional equation (2.1). It is interesting to note that if f is a solution of (2.3) then (2.3) can be reduced to (2.2). One may also refer to [5], [14], [15], etc. for more works on the stability problem of cubic functional equations.

In 1968, J. B. Diaz and B. Margolis [3] proved a fixed point result in a generalized complete metric space.

Definition 2.1. [3] Let X be a set. A function $d : X \times X \longrightarrow [0, \infty]$ is called a *generalized metric* on X if d satisfy the following conditions:

1. $d(x, y) = 0$ if and only if $x = y$;
2. $d(x, y) = d(y, x)$ for all $x, y \in X$;
3. $d(x, z) \leq d(x, y) + d(y, z)$ for all $x, y, z \in X$.

The pair (X, d) is then called a generalized metric space.

Let (X, d) be a generalized complete metric space. A mapping $T : X \longrightarrow X$ is said to be Lipschitzian [13] if there exists a constant $k \geq 0$ such that for all $x, y \in X$,

$$d\big(Tx, Ty\big) \leq kd(x, y).$$

The smallest number k for which the above relation holds true is called the Lipschitz's constant of J. A Lipschitzian with the Lipschitz's constant $k < 1$ is called a contraction mapping.

Theorem 2.2. [3] Let (X, d) be a complete generalized metric space and $T : X \longrightarrow X$ be a contraction mapping with the Lipschitz constant $\alpha < 1$. Then for each $x \in X$, either

$$d(T^n x, T^{n+1}x) = \infty \qquad \text{for all nonnegative integers } n,$$

or there exist a positive integer n_0 such that

1. $d(T^n x, T^{n+1}x) < \infty$ for all $n \geq n_0$;
2. The sequence $\{T^n x\}$ converges to a fixed point z^* of T;
3. z^* is the unique fixed point of T in the set
$$Y = \Big\{y \in X | d(T^{n_0}x, y) < \infty\Big\};$$
4. $d(y, z^*) \leq \frac{1}{1-\alpha} d(y, Ty)$ for all $y \in Y$.

Using this result, many authors have discussed the stability problems of different functional equations. For instance, one may refer to [8], [10], [12], [11], [16], [17], and the references therein.

3 Main results

In this section, we consider the Euler Lagrange type cubic functional equation

$$\begin{aligned} f(nx+y)+f(nx-y)+n\big[f(x+ny)+f(x-ny)\big] \\ = n(n^2+1)\big[f(x+y)+f(x-y)\big], \end{aligned} \tag{3.1}$$

$n \neq 0, \pm 1$ and prove the existence of solution of the functional equation, and obtain the Hyer's-Ulam-Rassias stability of the same using the fixed point method.

We first note that $f(x)=cx^3$ is a solution of (3.1) for all $c \in \mathbb{R}$. Also, if f is a solution of (3.1) then, putting $x=y=0$, $y=0$ and $x=0$ in (3.1), we get $f(0)=0$, $f(nx)=n^3 f(x)$ and $f(-y)=-f(y)$, respectively.

In light of this, we call the functional equation (3.1) an Euler Lagrange type cubic functional equation.

3.1 Existence of solution

Let X and Y be real linear spaces.

Theorem 3.1. If a mapping $f : X \longrightarrow Y$ is a solution of the functional equation (2.3), then f is a solution of the functional equation (3.1).

Proof . Let f be a solution of (2.3). Then, putting $x=y=0$, $y=0$ and $x=0$ in (2.3) we get $f(0)=0$, $f(nx)=n^3 f(x)$ and $f(-y)=-f(x)$, respectively.

Replacing x by nx in (2.3), we get

$$\begin{aligned} & n^3[f(x+y)+f(x-y)]+f(n^2x)=n^2[f(nx+y)+f(nx-y)] \\ & \qquad +n^3(n^3-2n^2+2)f(x) \\ \Longrightarrow \quad & n[f(x+y)+f(x-y)]+n^4 f(x)=[f(nx+y)+f(nx-y)] \\ & \qquad +n(n^3-2n^2+2)f(x) \\ \Longrightarrow \quad & n[f(x+y)+f(x-y)]=[f(nx+y)+f(nx-y)] \\ & \qquad +(2n-2n^3)f(x). \end{aligned} \tag{3.2}$$

Since $f(nx)=n^3 f(x)$, (2.3) can be rewritten as

$$f(x+ny)+f(x-ny)=n^2[f(x+y)+f(x-y)]+2(1-n^2)f(x). \tag{3.3}$$

Multiplying (3.3) by n and adding the resultant equation with (3.2), we get (3.1), showing that every cubic function is a solution of (3.1). □

Remark 3.2. It may be noted that a mapping f is a cubic mapping if and only if it is a solution of the functional equation (2.1) (refer [10]) or the functional equation (2.3) (refer [12]). It is interesting to prove the equivalence of the functional equation (3.1) with either (2.1) or (2.3).

Theorem 3.3. If $f : \mathbb{R} \longrightarrow \mathbb{R}$ is a solution of the functional equation (3.1) and is continuous at a point then f is continuous on $\mathbb{R}$ and $f(x)=f(1)x^3$ for all $x \in \mathbb{R}$.

Proof . Suppose that f is continuous at $\alpha \in \mathbb{R}$ and $\{x_k\}$ is a sequence in $\mathbb{R}$ with $x_k \longrightarrow 0$ as $k \to \infty$. Then for a fixed m and n,

$$f\big(n\alpha+mx_k\big)=n^3 f\left(\alpha+\frac{m}{n}x_k\right)=n^3 f(x_0)$$

as $n \to \infty$, showing that f is continuous at 0.

Now, replacing y with ny in (3.1), we get

$$n^3\big[f(x+y)+f(x-y)\big]+n\Big[f\big(x+n^2y\big)+f\big(x-n^2y\big)\Big]$$
$$=n(n^2+1)\big[f(x+ny)+f(x-ny)\big]$$

and replacing x with $x+y$ and y with $x-y$, respectively in the above, we get

$$8n^3\big[f(x)+f(y)\big]+n\Big[f\Big((n^2+1)x+(1-n^2)y\Big)+f\Big((1-n^2)x+(n^2+1)y\Big)\Big]$$
$$=(n^3+n)\Big[f\Big((n+1)x+(1-n)y\Big)+f\Big((1-n)x+(n+1)y\Big)\Big]$$

Substituting $x=\alpha$ and $y=x_k$ in the above equation, we get $f(x_k)\longrightarrow 0$ as $k\to\infty$. □

3.2 Stability

Considering X to be a real linear space and Y a complete linear metric space, we obtain a stability result of the functional equation (3.1) in the sense of Hyer's-Ulam-Rassias using the fixed point method.

Theorem 3.4. Let $\phi: X^2\longrightarrow[0,\infty)$ be a function such that a mapping $f: X\longrightarrow Y$ with the property $f(0)=0$ satisfies the inequality

$$\Big\|f(nx+y)+f(nx-y)+n[f(x+ny)+f(x-ny)] - n(n^2+1)[f(x+y)+f(x-y)]\Big\|\le\phi(x,y) \tag{3.4}$$

for all $x,y\in X$ and $n\neq 0,\pm1$. If $\psi: X\longrightarrow[0,\infty)$, defined by $\psi(x)=\phi\left(\frac{x}{n},0\right)$ for all $x\in X$, be such that there exists $\alpha<1$ with

$$\psi(x)\le\alpha n^3\psi\left(\frac{x}{n}\right) \tag{3.5}$$

for all $x\in X$, then there exists a unique Euler Lagrange type cubic function $C: X\longrightarrow Y$ such that

$$\big\|f(x)-C(x)\big\|\le\frac{\alpha}{1-\alpha}\psi(x)\qquad\forall\ x\in X. \tag{3.6}$$

Proof . Consider the set $S=\Big\{g\big|\ g:X\longrightarrow Y,\ g(0)=0\Big\}$. Define the generalized metric d on S by

$$d(g,h)=d_\psi(g,h):=\inf S_\psi(g,h),\qquad\forall\ g,h\in S$$
$$\text{where}\qquad S_\psi(g,h)=\Big\{\mu\in\mathbb{R}_+:\big\|g(x)-h(x)\big\|\le\mu\psi(x),\ \forall\ x\in X\Big\},$$

and $\inf S_\psi(g,h)=\infty$, if $S_\psi(g,h)=\emptyset$. Then (S,d) is complete. For, if $\{g_n\}$ is a Cauchy sequence in $\big(S,d\big)$, then for any $\varepsilon>0$ there exists a positive integer N such that

$$d\big(g_m(x),g_n(x)\big)<\varepsilon\qquad\text{if}\ \ m,n\ge N.$$

Since

$$d\big(g_m,g_n\big)=\inf S_\phi(g_m,g_n)$$
$$=\inf\Big\{\mu\in(0,\infty):\big\|g_m(x)-g_n(x)\big\|\le\mu\phi(x,0),\ \forall\ x\in X\Big\}<\varepsilon,$$

there exists $\lambda\in(0,\varepsilon)$ such that

$$\big\|g_m(x)-g_n(x)\big\|\le\mu\phi(x,0)<\varepsilon\phi(x,0)\qquad\forall\ x\in X. \tag{3.7}$$

Hence $\{g_n(x)\}$ is a Cauchy sequence in Y for all $x\in X$. Since Y is complete, for every $x\in X$, there exists $g(x)\in Y$ such that

$$\lim_{n\to\infty}g_n(x)=g(x)\qquad\text{and}\qquad g(0)=0.$$

Thus, $g \in S$ and from (3.7), for all $x \in X$, we have

$$\begin{aligned} & \big\|g_n(x) - g(x)\big\| \le \varepsilon\phi(x,0) && \forall\, n \in \mathbb{N} \\ \Longrightarrow \quad & d(g_n, g) = \inf S_\phi(g_n, g) \le \varepsilon && \forall\, n \in \mathbb{N} \\ \Longrightarrow \quad & \lim_{n\to\infty} g_n = g, \end{aligned}$$

showing that (S, d) is complete.

Define the mapping $T : S \longrightarrow S$ by

$$T\left(g(x)\right) := \frac{1}{n^3} g(nx) \qquad \forall\, g \in S \text{ and } \forall\, x \in X.$$

We observe that for all $g, h \in S$, if $\mu \in S_\psi(g,h)$ and $\mu < \lambda$, then

$$\begin{aligned} & \big\|g(x) - h(x)\big\| \le \mu\psi(x) \le \lambda\psi(x) && \forall\, x \in X \\ \Longrightarrow \quad & \lambda \in S_\psi(g,h). \end{aligned}$$

Also, we note that, for all $g, h \in S$ and $\lambda \in (0, \infty)$,

$$d(g,h) = \inf S_\psi(g,h) < \lambda.$$

This implies there exists $\mu \in S_\psi(g,h)$, such that $\mu < \lambda$.

Therefore, $\lambda \in S_\psi(g,h)$, by the above observation and hence

$$\begin{aligned} & \big\|g(x) - h(x)\big\| \le \lambda\psi(x) \qquad \forall\, x \in X \\ \Longrightarrow \quad & \left\|\frac{1}{n^3} g(nx) - \frac{1}{n^3} h(nx)\right\| \le \frac{1}{n^3}\lambda\psi(nx) \qquad \forall\, x \in X \\ \Longrightarrow \quad & \left\|\frac{1}{n^3} g(nx) - \frac{1}{n^3} h(nx)\right\| \le \lambda\alpha\psi(x) \qquad \forall\, x \in X, \qquad \text{by (3.5)} \\ \Longrightarrow \quad & d(Tg, Th) \le \lambda\alpha, \qquad \text{for every } \ \lambda \in (0,\infty). \end{aligned}$$

Hence $d(Tg, Th) \le \alpha d(g,h)$, $\forall\, g, h \in S$ and we have thus shown that T is a contraction mapping on S with Lipschitz's constant α.

Putting $y = 0$ in (3.4), we have

$$\begin{aligned} & \big\|2f(nx) - 2n^3 f(x)\big\| \le \phi(x, 0), \qquad \forall\, x \in X, \\ \Longrightarrow \quad & \left\|f(x) - \frac{1}{n^3} f(nx)\right\| \le \frac{1}{2n^3}\psi(nx) \le \frac{\alpha}{2}\psi(x), \end{aligned} \tag{3.8}$$

for all $x \in X$, that is,

$$d\big(f, Tf\big) \le \alpha < \infty.$$

Now, using Theorem 2.2, we get the following.

1. There exists a fixed point C of T in S such that

$$C(x) = \lim_{k\to\infty} \frac{f(n^k x)}{n^{3k}}, \qquad \forall\, x \in X.$$

Substituting x and y by $n^k x$ and $n^k y$, respectively in (3.4) and dividing by n^{3k}, we have

$$\begin{aligned} \frac{1}{n^{3k}} \Big\| & f(n^k(nx+y)) + f(n^k(nx-y)) + n[f(n^k(x+ny)) \\ & + f(n^k(x-ny))] - n(n^2+1)[f(n^k(x+y)) + f(n^k(x-y))]\Big\| \\ & \le \frac{1}{n^{3k}}\phi(n^k(x,y)). \end{aligned}$$

Taking the limits as $k \to \infty$, we get

$$\Big\|C(nx+y)+C(nx-y)+n[C(x+ny)+C(x-ny)] - n(n^2+1)[C(x+y)+C(x-y)]\Big\| \le \lim_{k\to\infty}\frac{\phi(x,y)}{n^{3k}} = 0.$$

That is,

$$C(nx+y)+C(nx-y)+n[C(x+ny)+C(x-ny)] - n(n^2+1)[C(x+y)+C(x-y)] = 0.$$

Thus the function $C: X \longrightarrow Y$ satisfy the cubic equation (3.1) and hence C is a cubic function.
Also, by inequality (3.8), we obtain

$$\begin{aligned}\Big\|T^k f(x) - T^{k+1} f(x)\Big\| &= \frac{1}{n^{3k}}\Big\|f(n^k x) - \frac{1}{n^3} f(n^{k+1}x)\Big\| \le \frac{\alpha}{2n^{3k}}\psi(n^k x)\\ &\le \frac{\alpha}{2n^{3k}}(\alpha n^3)\psi(n^{k-1}x)\dots \qquad \text{by (3.5)}\\ &\le \frac{\alpha}{2n^{3k}}(\alpha n^3)^k\psi(x) = \frac{\alpha^{k+1}}{2}\psi(x)\end{aligned}$$

for all $x \in X$ and $k \in \mathbb{N}$, that is,

$$d\left(T^k f,\ T^{k+1} f\right) \le \alpha^{k+1} < \infty, \qquad \text{for all } \ k \in \mathbb{N}.$$

2. There exists $n_0 \in \mathbb{N}$ such that the mapping C is the unique fixed point of T in the set $E = \Big\{g \in S : d\left(T^{m_0} f, g\right) < \infty\Big\}$. Hence we have

$$d\left(T^{m_0} f, C\right) < \infty.$$

Since

$$d\left(f, T^{m_0} f\right) \le d\left(f, Tf\right) + d\left(Tf, T^2 f\right) + \cdots + d\left(T^{m_0-1} f, T^{m_0} f\right) < \infty,$$

we have $f \in E$ and therefore,

$$d\left(f, C\right) \le d\left(f, T^{m_0} f\right) + d\left(T^{m_0} f, C\right) < \infty.$$

Hence we have

$$\big\|f(x) - C(x)\big\| \le \mu\psi(x) \qquad \text{for all } \ x \in X.$$

3. $d(f,C) \le \frac{1}{1-\alpha} d\left(f, Tf\right)$.
But since $d\left(f, Tf\right) \le \alpha$, it follows that

$$d\left(f, C\right) \le \frac{\alpha}{1-\alpha},$$

which implies the inequality (3.6).

Thus the theorem is proved. □

Corollary 3.5. Let $\theta > 0$ and $0 \le p < 3$. If $f : X \longrightarrow Y$ be a mapping satisfying

$$\Big\|f(nx+y)+f(nx-y)+n[f(x+ny)+f(x-ny)] - n(n^2+1)[f(x+y) + f(x-y)]\Big\| \le \theta\big(\|x\|^p + \|y\|^p\big)$$

for all $x, y \in X$, then there exists a unique cubic mapping $C : X \longrightarrow Y$ such that

$$\big\|f(x) - C(x)\big\| \le \frac{1}{2^{3-p}-1}\theta\|x\|^p, \qquad \forall\ x \in X.$$

Proof . The proof follows from Theorem 3.4 by taking

$$\phi(x,y) = \theta\big(\|x\|^p + \|y\|^p\big)$$

for all $x, y \in X$. We can then choose $\alpha = 2^{-(3-p)}$ to get the desired result. □

4 Some research problems

Recently, Eshaghi Gordji et al. [7] introduced the notion of orthogonal sets (one may refer to [1]) and gave a generalization of Banach fixed point theorem in this setting. In [1], the authors proved a generalization of Theorem 2.2 in the setting of orthogonal sets.

For C^*-algebras A and B, let a mapping $f : A \longrightarrow B$ be called a cubic $*$–homomorphism if f satisfy (3.1) such that $f(x^*) = f(x)^*$ and $f(xy) = f(x)f(y)$ for all $x, y \in A$.

Problem 1. Can the stability and hyperstability problems for the cubic $*$–homomorphism be addressed in orthogonally Lie C^*-algebras using orthogonal fixed point theorems?

In [8], the authors introduced the functional equation

$$f(x+y) + 2f\left(\frac{x}{2}+y\right) + 2f\left(\frac{x}{2}-y\right) = 2f(x) + 5f(y). \tag{4.1}$$

The function $f(x) = c\|x^2\|$, $c \in \mathbb{R}$ is a solution of the functional equation (4.1) in an inner product space if the inner product of x and y, $(x, y) = 0$. In light of which they termed the functional equation (4.1) as orthogonally quadratic equation.

Problem 2. Can we find an orthogonal set on which the functional equation (2.2) is equivalent to the cubic functional equation (2.3)?

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