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Some dominating results of the join and corona operations between discrete topological graphs

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Abstract

The dominating sets play an important role in applications of graph theory. In this field, some recent studies discussed the properties of the minimum dominating set (γ-set). The other type of study produces a topological space from the set of vertices or the set of edges of a graph. The previous paper introduces a new method to construct a graph G_τ from the topological space. In this paper, the dominating set and the domination number of G_τ are proved with their inverse. The domination number and inverse domination number of corona operation and join operation between two graphs are discussed.

Keywords: dominating set, inverse dominating set, corona operation, join operation
2020 MSC: 05C69

1 Introduction

In graph theory, the study of domination and dominating sets plays a prominent role. The topic has been extensively studied for more than 30 years due to its applications in several areas, such as wireless networks [19]. In any graph, a dominating set is a set of a vertices such that every vertex in the graph is dominated by at least one vertex from this set [20, 21, 22]. Let $G = (V,\ E)$ be a graph, where $V(G)$ is a set of all vertices of G, and $E(G)$ is a set of all edges of G. The degree of any vertex in $V(G)$ like v is the number of all edges incident on it. This graph without no isolated vertex and no pendant vertex such that degree of isolated vertex is 0, and degree of pendant vertex is 1. The subgraph of G is induced subgraph, if contains all vertices of $V(G)$, and all edges that incident to they. The domination number of a graph G is the minimum cardinality of a dominating set of G, denoted by $\gamma(G)$. We refer the reader to [1]-[18], [23]-[31] for more details in certain properties of a dominating set and domination number of a graph. The importance of domination in different applications gives rise to different types of dominance for this purpose. We denote $G \bigodot H$ to the corona of two graphs G and H which is a graph obtained by taking one copy of G and $|V(G)|$ copies of H, where the i^{th} vertex of G is adjacent to every vertex in the i^{th} copy of H. The join $G + H$ of two graphs $G(V_1, E_1)$ and $H(V_2, E_2)$ is a graph having the vertex set $V_1 \cup V_2$ and edge set $E_1 \cup E_2 \cup \{v_1v_2 : \forall\ v_1 \in V_1, v_2 \in V_2\}$.

In this paper, a new results of dominating set employed in the discrete topological graph G_τ. Some bounds for $\gamma(G_\tau \bigodot H_\tau)$ and $\gamma(G_\tau + H_\tau)$ are obtained and characterized. Also, $\gamma^{-1}(G_\tau \bigodot H_\tau)$ and $\gamma^{-1}(G_\tau + H_\tau)$ are obtained.

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2 Some properties of Topological Graph

In this section, many properties that proved by authors in [23] for the discrete topological graph G_τ are given.

Definition 2.1. [23] Let X be a non-empty set and τ be a discrete topological graph denoted by $G_\tau = (V,\ E)$ is a graph of the vertex set $V(G_\tau) = \{A;;\ A \in \tau \text{ and } A \neq \emptyset,\ X\}$ and the edge set defined by $E(G_\tau) = \{A\ B;;\ A \subseteq B \text{ and } A \neq B\}$.

Proposition 2.2. [23] Let G_τ be a discrete topological graph on X, where $|X| = 2$, then $G_\tau \equiv N_2$.

Proposition 2.3. [23] Let G_τ be a discrete topological graph on a non-empty set X, where $|X| = n$. Then $|G_\tau| = 2^n - 2$.

3 Main results:

In this section, the dominating set and domination number of corona operation and join operation between two graphs are discussed with their inverse.

Theorem 3.1. Let G_τ be a discrete topological graph on X, where $|X| = n$. Then, $\gamma(G_\tau) = 2$.

Proof . If $n = 2$, then according to Proposition 2.2, we have $G_\tau \equiv N_2$, it has two dominating vertices, therefore $\gamma(G_\tau) = 2$, see Fig 1.a. Now, if $n > 2$, each vertex that have singleton element say u is adjacent to some vertices that have two or more elements, where u is subset of them. Thus, the vertex u is a dominates all these vertices. Moreover, u^c is the complement of a vertex u that have $n-1$ elements adjacent with all vertices that are not adjacent to the vertex u, (because all vertices which are u is subset of them be not subset of u^c, also all vertices that subset of u^c then u is not subset of them). Thus, the vertex u^c dominates to all the vertices adjacent to it. Hence, $D = \{u, u^c\}$ dominating set of a graph G_τ. Now, to prove the set D is a minimum dominating set, suppose that D is not minimum dominating set. So, there exist minimum dominating set $\acute{D}$ such that $\left|\acute{D}\right| < |D|$ and $\acute{D} = \{w\}$ there exist a vertex say v such that $w \nsubseteq v$ then, $e = wv \notin E(G_\tau)$ and w is not dominates of v. Hence, $\acute{D}$ does not dominate all vertices of graph. Then, D is a minimum dominating set in G_τ. Therefore, $\gamma(G_\tau) = 2$, see Fig 1. □

Proposition 3.2. Let G_τ be a discrete topological graph on X, where $|X| = n$. Then, $\gamma^{-1}(G_\tau) = 2$.

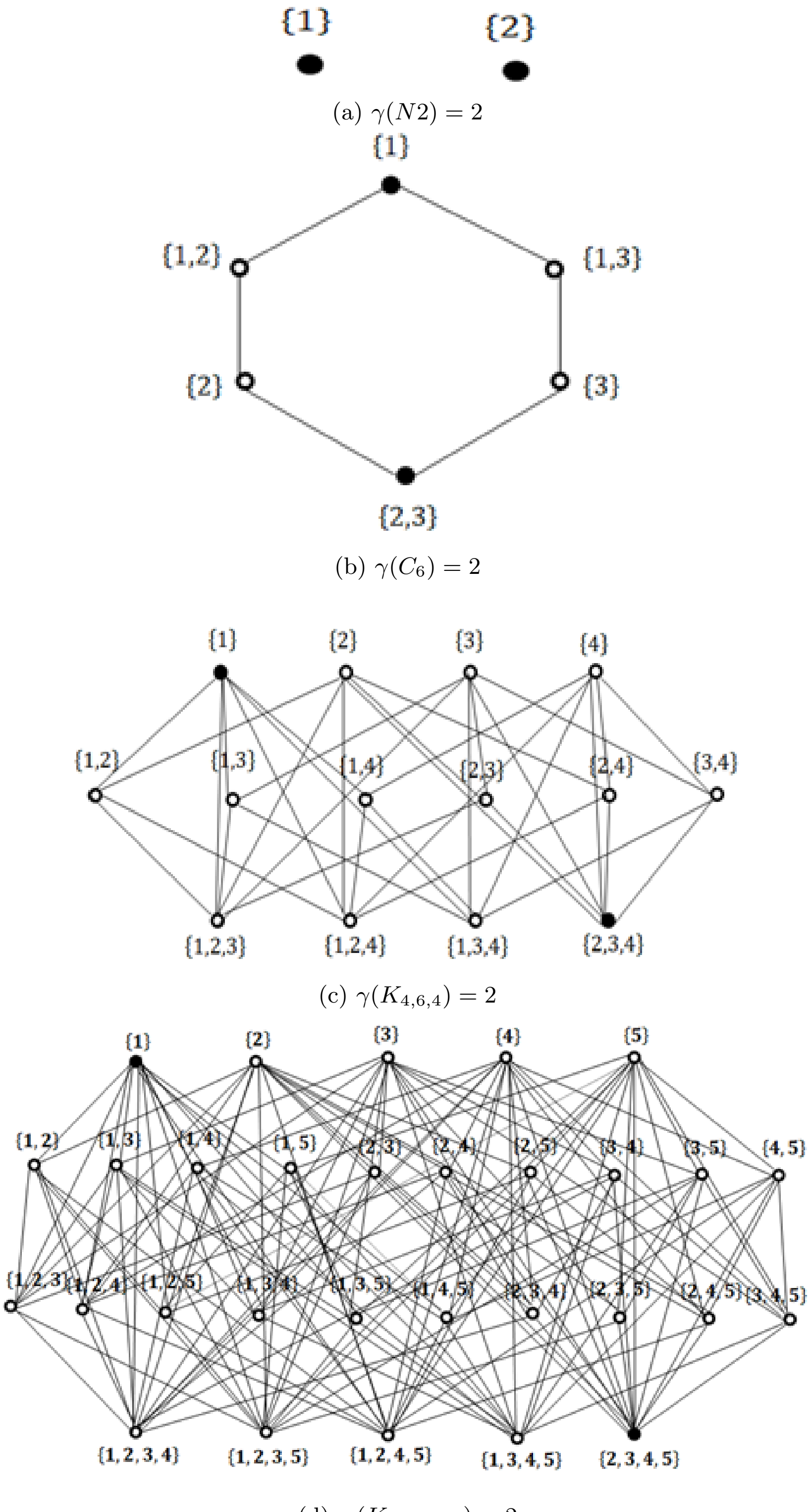
Proof . Let $D^{-1} = \{w, w^c\}$. By similar technique of Theorem 3.1, it implies that $D^{-1} = \{w, w^c\}$ is a minimum invers dominating set in a graph G_τ. Therefore, $\gamma^{-1}(G_\tau) = 2$, see Fig 2. □

Theorem 3.3. Let G_τ and H_τ be two a discrete topological graph on X and Y respectively, where $|X| = n$ and $|Y| = m$. Then, $\gamma(G_\tau \bigodot H_\tau) = 2^n - 2$.

Proof . Without loss of generality we may assume that $n \leq m$. Let D be a dominating set in the graph $G_\tau \bigodot H_\tau$. Also, in the graph $G_\tau \bigodot H_\tau$, each vertex in G_τ is adjacent to all vertices of one copy of H_τ. Let $u_i \in V(G_\tau)$. Then, u_i is adjacent with all vertices of i^{th} copy of H_τ and u_i dominates all vertices of one copy of H_τ. Thus, $u_i \in D$ and so $D = V(G_\tau)$. From Proposition 2.3, follows that $|D| = |V(G_\tau)| = 2^n - 2$. That is, D minimum dominating set in the graph $G_\tau \bigodot H_\tau$. Therefore, the number of minimum dominating set D is $2^n - 2$. Therefore, $\gamma(G_\tau \bigodot H_\tau) = 2^n - 2$, see Fig 3 and Fig 4. □

Theorem 3.4. Let G_τ and H_τ be a discrete topological graph on X and Y respectively, where $|X| = n$ and $|Y| = m$. Then, $\gamma^{-1}(G_\tau \bigodot H_\tau) = 2(2^n - 2)$.

Proof . Without loss of generality we may assume that $n \leq m$. Let D be a dominating set in the graph $G_\tau \bigodot H_\tau$ such that $D = V(G_\tau)$ and $\gamma(G_\tau \bigodot H_\tau) = 2^n - 2$ according to Theorem 3.3. By applying the definition of corona graph $G_\tau \bigodot H_\tau$. We deduced one copy of G_τ and n copies of H_τ, where the i^{th} vertex of G_τ is adjacent to every vertex in the i^{th} copy of H_τ. Now, let D^{-1} the inverse dominating set of a graph $G_\tau \bigodot H_\tau$ containing two vertices from every copy of a graph H_τ. Suppose that $D^{-1} = \{u\ ,\ u^c\}$, where u is a vertex have a singleton element and u^c is a vertex that have $n-1$ elements. That is, u and u^c are dominate all vertices of one copy of a graph H_τ and



(a) $\gamma(N2) = 2$

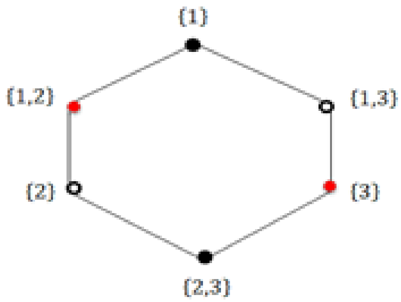
(b) $\gamma(C_6) = 2$

(c) $\gamma(K_{4,6,4}) = 2$

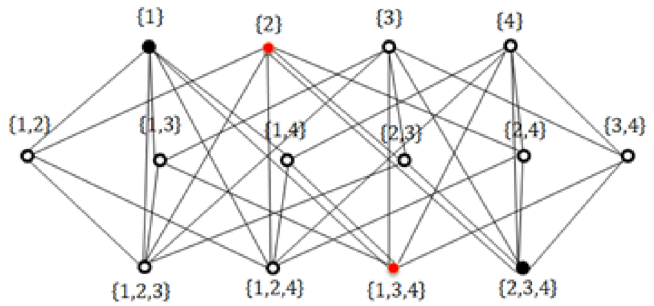
(d) $\gamma\left(K_{5,10,10,5}\right) = 2$

Figure 1: A minimum dominating set in a graph G_τ

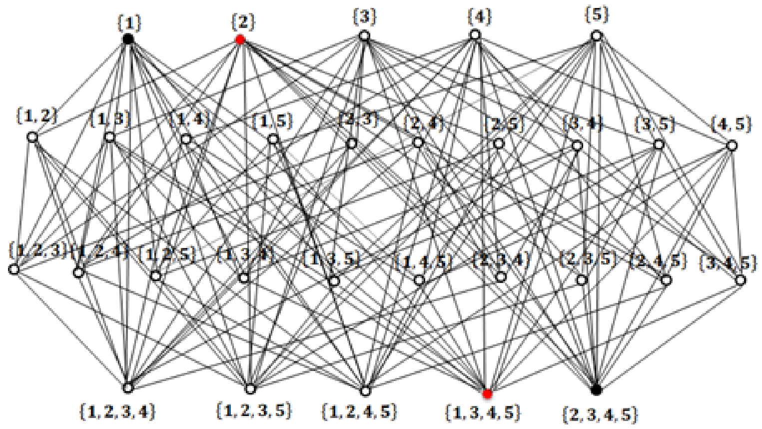
dominate one vertex from G_τ. Now, to prove that D^{-1} is a minimum inverse dominating set of a graph $G_\tau \bigodot H_\tau$. Let $\acute{D}$ be an inverse dominating set with order less than order D^{-1}. Then, there is one or more vertices which are not dominated by $\acute{D}$ in $G_\tau \bigodot H_\tau$ which is a contradiction. Thus, D^{-1} is an inverse dominating set of a graph $G_\tau \bigodot H_\tau$. Therefore, $\gamma^{-1}\left(G_\tau \bigodot H_\tau\right) = 2(2^n - 2)$, see Fig 5 and Fig 6. $\square$



(a) $\gamma^{-1}(C_6) = 2$



(b) $\gamma^{-1}(K_{4,6,4}) = 2$



(c) $\gamma^{-1}(K_{5,10,10,5}) = 2$

Figure 2: The inverse dominating set in a graph G_τ.

Theorem 3.5. Let G_τ and H_τ be a discrete topological graph on X and Y respectively, where $|X| = n$ and $|Y| = m$. Then, $\gamma(G_\tau + H_\tau) = 2$.

Proof . By similar technique of Theorem 3.1, if $n = 2$ the graph G_τ contains two dominate vertices. At the same time, the set of vertices in G_τ dominate all vertices of the graph H_τ according to the definition of join operation. Then, the number of minimum dominating set D is 2. Consequently, the minimum dominating set of a graph $(G_\tau + H_\tau)$ contains two elements. Thus, $\gamma(G_\tau + H_\tau) = 2$. Now if $n > 2$, then a minimum dominating set of graph G_τ contains two elements say u and u^c such that the vertex u have singleton element and the vertex u^c have $n - 1$ elements. Therefore, $\gamma(G_\tau + H_\tau) = 2$, see Fig 7. $\square$

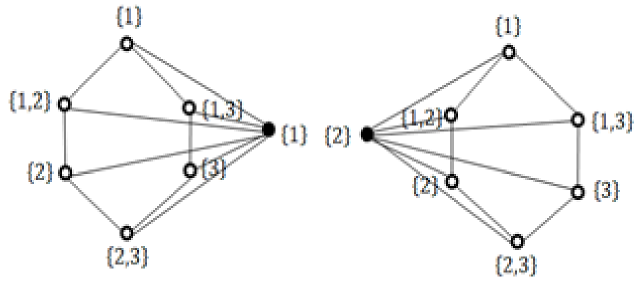


Figure 3: A minimum dominating set in $G_\tau \equiv N_2 \bigodot C_6$.

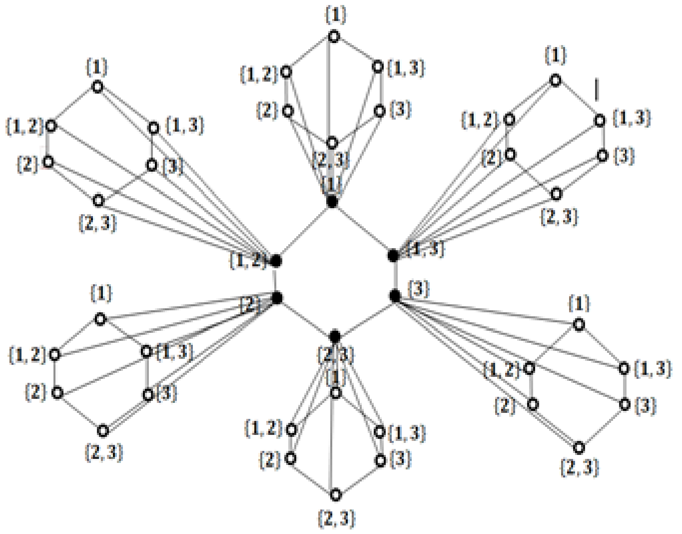


Figure 4: A minimum dominating set in $G_\tau \equiv C_6 \bigodot C_6$.

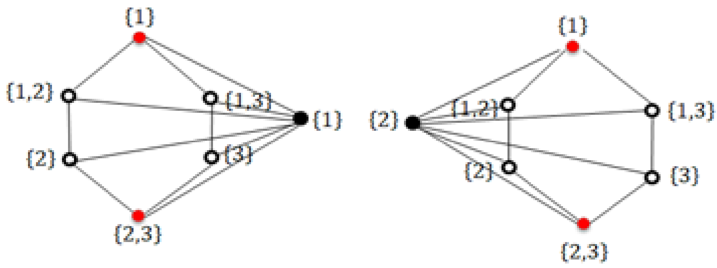


Figure 5: A minimum inverse dominating set in $N_2 \bigodot C_6$.

Proposition 3.6. Let G_τ and H_τ be a discrete topological graph on X and Y respectively, where $|X| = n$ and $|Y| = m$. Then, $\gamma^{-1}(G_\tau + H_\tau) = 2$.

Proof . In similar technique of Theorem 3.5, let $D^{-1} = \{w,\ w^c\}$ such that w have a singleton elements and w^c that have $n-1$ elements, these vertices dominate all vertices in a graph $G_\tau + H_\tau$. Therefore, $\gamma^{-1}(G_\tau + H_\tau) = 2$, see Fig 8. $\square$

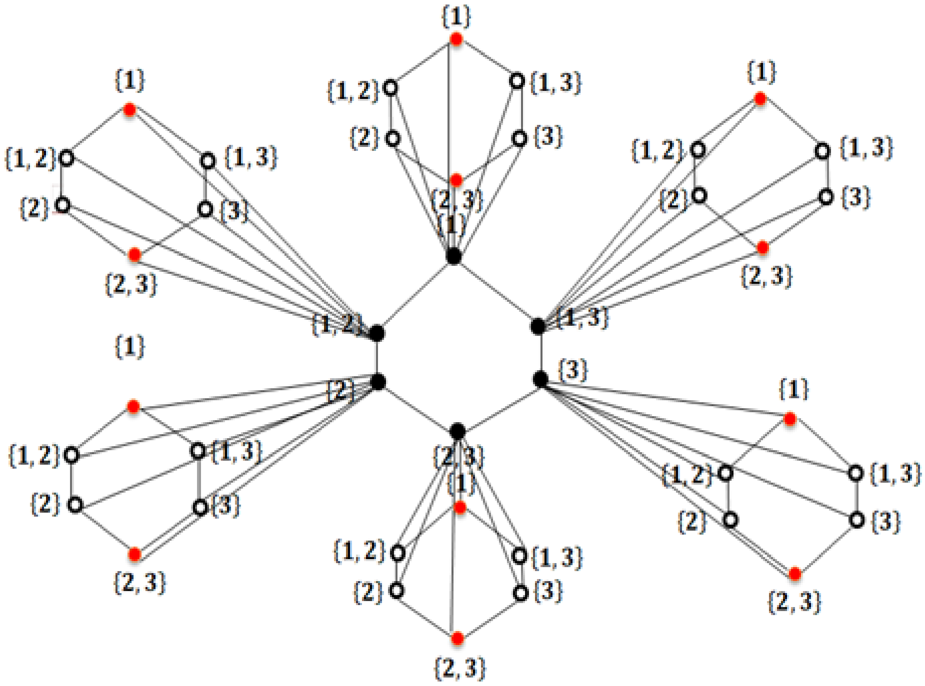
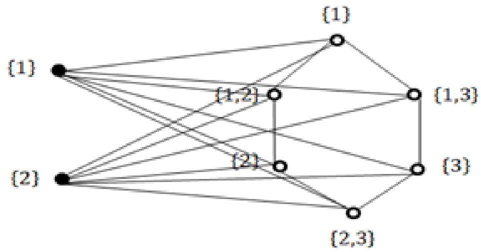
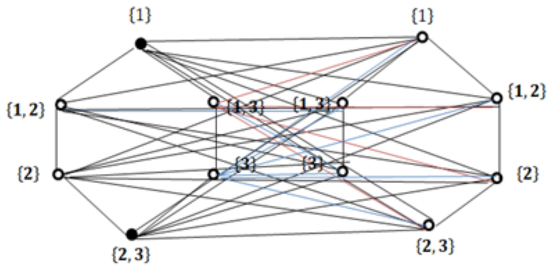


Figure 6: A minimum inverse dominating set in$C_6 \odot C_6$.



(a) A minimum dominating set in $N_2 + C_6$.

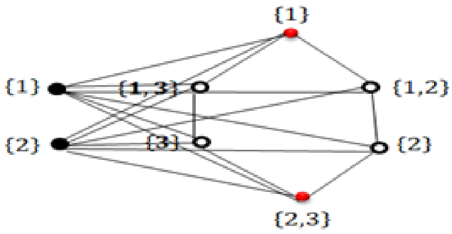


(b) A minimum dominating set in $C_6 + C_6$.

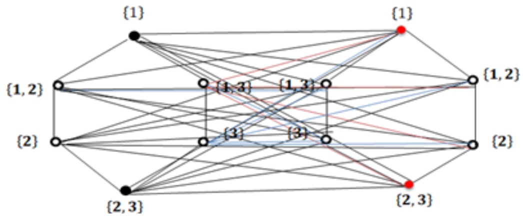
Figure 7: A minimum dominating set in $G_\tau + H_\tau$.

4 Conclusions

Several properties of domination are applied on the discrete topological graph. The general dominating set and its inverse are given. Also, the minimum domination number of corona and join operations of two discrete topological graphs are proved with their inverse.



(a) A minimum inverse dominating set in $N_2 + C_6$.



(b) A minimum inverse dominating set in $C_6 + C_6$.

Figure 8: A minimum inverse dominating set in $G_\tau + H_\tau$.

5 Open problems

Applying more dominating parameters on the discrete topological graph G_τ such that: bi-domination, total domination, independent domination, pitchfork domination, arrow domination and co-even domination.

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