Int. J. Nonlinear Anal. Appl. 14 (2023) 1, 965–977
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/ijnaa.2022.25176.2941

Fixed point results for generalized (α, ψ, φ)-Geraghty contraction in b-metric spaces

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(Communicated by Ali Farajzadeh)

Abstract

In this paper, first, we introduce (α, ψ, φ)-Geraghty and generalized (α, ψ, φ)-Geraghty contraction mappings in b-metric spaces and then we prove the existence and uniqueness of fixed point after exploring all the conditions which guarantee the existence of fixed point. Our results extend and generalize related fixed point results in the existing literature. We also provide examples in support of our main findings.

Keywords: α-admissible mapping, generalized (α, ψ, φ)-Geraghty Contraction, b-Metric Spaces
2020 MSC: 55M20

1 Introduction

The family of contractive mappings in different spaces is a great interest and has already been studied in the literature since long time. Their application is also studied by many authors, see [3, 16]. In [12] Geraghty introduced the concept of Geraghty contraction mapping in metric spaces and proved fixed point theorem for that mapping. Afterwards, a number of authors generalized his work, see [1, 2, 4, 5, 6, 8, 10, 11, 13, 14, 15]. We focus on the work of Karapinar et al. [15]. He introduced the notion of φ-Geraghty and Ciric type φ-Geraghty contractive mappings in complete metric spaces and proved the existence and uniqueness of fixed points. Inspired and motivated by the work of Karapinar et al. [15] the main purpose of this paper is to establish fixed point results for (α, ψ, φ)-Geraghty and generalized (α, ψ, φ)-Geraghty contraction mappings in the setting of b-metric spaces.

2 Preliminaries

We need the following symbols and class of functions to prove certain results of this section:

1. $\mathbb{R}^+ = [0, \infty)$;
2. $\mathbb{R}$ is the set of all real numbers;
3. $\mathbb{N}$ is the set of all natural numbers;
4. $\Psi = \{\psi \colon \mathbb{R}^+ \to \mathbb{R}^+$, such that, ψ is continuous, strictly increasing, $\psi(x + y) = \psi(x) + \psi(y)$ and $\psi(0) = 0$ };

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Received: November 2021 *Accepted:* August 2022

5. $\Theta = \{\theta\colon \mathbb{R}^+ \to [0,1)$, such that, $\theta(t_n) \to 1 \Rightarrow t_n \to 0$, as $n \to \infty\}$;
6. $\Theta' = \{\theta\colon \mathbb{R}^+ \to [0,1)$, such that $\limsup_{n\to\infty} \theta(t_n) = 1 \Rightarrow \lim_{n\to\infty} t_n = 0\}$;
7. $\Theta_s = \{\theta\colon \mathbb{R}^+ \to [0,\frac{1}{s})$, such that, $\theta(t_n) \to \frac{1}{s} \Rightarrow t_n \to 0$ as $n \to \infty$ for $s \geq 1\}$;
8. $\Theta'_s = \{\theta\colon \mathbb{R}^+ \to [0,\frac{1}{s})$, such that, $\limsup_{n\to\infty} \theta(t_n) = \frac{1}{s} \Rightarrow \lim_{n\to\infty} t_n = 0\}$.

Definition 2.1. (See [9]) Let X be a nonempty set and $s \geq 1$ be a given real number. A function $d : X \times X \to \mathbb{R}^+$ is said to be a b-metric if and only if for all $x, y, z \in X$, the following conditions are satisfied

1. $d(x,y) = 0$ if and only if $x = y$;
2. $d(x,y) = d(y,x)$;
3. $d(x,z) \leq s[d(x,y) + d(y,z)]$.

The pair (X,d) is called a b-metric space.

Definition 2.2. (See [12]) Let (X,d) be a metric space. An operator $T : X \to X$ is called a Geraghty contraction if there exists a function $\theta \in \Theta$ which satisfies for all $x, y \in X$ the condition

$$d(Tx,Ty) \leq \theta(d(x,y))d(x,y).$$

Theorem 2.3. (See [12]) Let (X,d) be a complete metric space. If $T\colon X \to X$ is a Geraghty contraction mapping, then T has a unique fixed point.

Theorem 2.4. (See [18]) Let (X,d) be a complete metric space and $T : X \to X$. Assume that there exists a $\varphi\colon \mathbb{R}^+ \to \mathbb{R}^+$ satisfying:

1. $\varphi(t) < t$ for any $t \in \mathbb{R}^+$;
2. for any $\varepsilon > 0$, there exists $\delta > 0$ such that $\varepsilon < t < \varepsilon + \delta \Rightarrow \varphi(t) \leq \varepsilon$;
3. $d(Tx,Ty) \leq \varphi(L(x,y))$, for all $x, y \in X$; where

$$L(x,y) = \max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{[d(x,Ty) + d(y,Tx)]}{2}\right\}.$$

Then T has a unique fixed point.

Definition 2.5. (See [17]) Let $\alpha : X \times X \to \mathbb{R}^+$ be a function. A mapping $T : X \to X$ is said to be α-admissible, if for all $x, y \in X$, $\alpha(x,y) \geq 1$ implies $\alpha(Tx,Ty)) \geq 1$.

Recently, Karapinar et al. [15] introduced the notion of φ-Geraghty and Ćirić type φ-Geraghty contractive mappings in complete metric spaces and proved the existence and uniqueness of fixed points.

Definition 2.6. Let X be a nonempty set. A function $f : X \to \mathbb{R}^+$ is called upper semicontinuous at a point $\bar{x} \in X$ if for every $\epsilon > 0$, there exists a $\delta > 0$ such that $f(x) - f(\bar{x}) < \epsilon$ for all $x \in X$ with $|x - \bar{x}| < \delta$.

Definition 2.7. (See [7]) Let X be a b-metric space and $\{x_n\}$ be a sequence in X, we say that

1. $\{x_n\}$ is b-converges to $x \in X$ if $d(x_n, x) \to 0$ as $n \to \infty$.
2. $\{x_n\}$ is a b-Cauchy sequence if $d(x_n, x_m) \to 0$ as $n, m \to \infty$.
3. (X,d) is b-complete if every b-Cauchy sequence in X converges to a point in X.

Definition 2.8. (See [15]) Suppose that $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is a function and $\theta \in \Theta$. A self-mapping T on a metric space (X,d) is called φ-Geraghty contraction if it satisfies the following conditions:

1. $\varphi(t) < t$ for any $t \in (0,\infty)$;
2. For any $\varepsilon > 0$, there is a $\delta > 0$ such that $\varepsilon < t < \varepsilon + \delta \Rightarrow \varphi(t) \leq \varepsilon$;
3. $d(Tx,Ty) \leq \theta(d(x,y))\varphi(d(x,y))$.

Theorem 2.9. (See [15]) Let (X, d) be a complete metric space. If a self-mapping $T : X \to X$ forms a φ-Geraghty contraction, then T has a unique fixed point.

Definition 2.10. (See [15]) Suppose that $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is a function and $\theta \in \Theta'$. A self-mapping T on a metric space (X, d) is called *Ćirić* type φ-Geraghty contraction if it satisfies the following conditions:
(φ_0) φ is upper semicontinuous;
(φ_1) $\varphi(t) < t$ for any $t \in \mathbb{R}^+$;
(φ_2) For any $\varepsilon > 0$, there exists a $\delta > 0$ such that $\varepsilon < t < \varepsilon + \delta$ implies $\varphi(t) \leq \varepsilon$;
(φ_3') $d(Tx, Ty) \leq \theta(L(x, y))\varphi(L(x, y))$; for all x, y in X, where

$$L(x, y) = \max\left\{d(x, y), d(x, Tx), d(y, Ty), \frac{[d(x, Ty) + d(y, Tx)]}{2}\right\}.$$

Theorem 2.11. (See [15]) Let (X, d) be a complete metric space. If a self- mapping $T : X \to X$ forms a Ćirić type φ-Geraghty contraction, then T has a fixed point.

Remark 2.12. By (φ_1), It is easy to see that (φ_2) is equivalent to the following:

$$(\varphi_2') \text{ For any } \varepsilon > 0, \text{ there exists a } \delta > 0 \text{ such that } t < \varepsilon + \delta \Rightarrow \varphi(t) \leq \varepsilon.$$

Indeed, if $0 < t \leq \varepsilon$ from (φ_1), we have $\varphi(t) < t \leq \varepsilon$.

3 Main Results

In this section, we introduce (α, ψ, φ)-Geraghty and generalized (α, ψ, φ)-Geraghty contraction mappings in the setting of b-metric spaces and prove fixed point results for the mappings introduced.

Definition 3.1. Let (X, d) be a b-metric space, $T : X \to X$ and $\alpha : X \times X \to \mathbb{R}^+$. A mapping T is said to be (α, ψ, φ)-Geraghty contraction mapping if there exists $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$, $\psi \in \Psi$ and $\theta \in \Theta_s$ satisfies for all $x, y \in X$ the following conditions:

1. $\varphi(t) < t$ for any $t \in \mathbb{R}^+$;
2. For any $\varepsilon > 0$, there exists a $\delta > 0$ such that

$$\varepsilon < t < \varepsilon + \delta \Rightarrow \varphi(t) \leq \varepsilon;$$

3. $\alpha(x, y)\psi(sd(Tx, Ty)) \leq \theta(\psi(d(x, y)))\varphi(\psi(d(x, y)))$.

Theorem 3.2. Let (X, d) be a complete b-metric space and $T : X \to X$. Suppose the following conditions hold:

1. T is an α-admissible mapping;
2. T is an (α, ψ, φ)-Geraghty contraction mapping;
3. There exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$.

Then T has a unique fixed point.

Proof . By (3) above, there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$. Define a sequence $\{x_n\}$ in X by

$$x_n = Tx_{n-1},$$

for $n \in \mathbb{N}$. Suppose that $x_{n_0} = x_{n_0+1}$ for some $n_0 \in \mathbb{N}$. Since $Tx_{n_0} = x_{n_0+1} = x_{n_0}$, the point x_{n_0} forms a fixed point of T. From now on we suppose that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N} \cup \{0\}$. Since T is α-admissible, we have

$$\alpha(x_0, Tx_0) = \alpha(x_0, x_1) \geq 1 \Rightarrow \alpha(Tx_0, Tx_1) = \alpha(x_1, x_2) \geq 1.$$

Continuing in this manner, we get $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \geq 0$. By the properties of ψ, φ_1, θ and φ_3, we have the following.

$$\begin{aligned}\psi(d(x_{n+1}, x_{n+2})) &= \psi(d(Tx_n, Tx_{n+1}))\\ &\leq \psi(sd(Tx_n, Tx_{n+1}))\\ &\leq \alpha(x_n, x_{n+1})\psi(sd(Tx_n, Tx_{n+1}))\\ &\leq \theta(\psi(d(x_n, x_{n+1})))\varphi(\psi(d(x_n, x_{n+1})))\\ &< \frac{1}{s}\varphi(\psi(d(x_n, x_{n+1}))))\\ &\leq \varphi(\psi(d(x_n, x_{n+1})))\\ &< \psi(d(x_n, x_{n+1})).\end{aligned}$$

Therefore, $d(x_{n+1}, x_{n+2}) < d(x_n, x_{n+1})$ for all $n \geq 0$. Hence, the non-negative sequence $\{d(x_n, x_{n+1})\}$ is non-increasing in $\mathbb{R}^+$. Accordingly, it is convergent to some real number $l \geq 0$. We claim $l = 0$. We suppose on the contrary that $l > 0$. Hence, we have $0 < l < d(x_n, x_{n+1})$ for all $n \geq 0$. Set $\varepsilon = l$. From (φ_2'), there exists $\delta > 0$ such that $t < \varepsilon + \delta \Rightarrow \varphi(t) \leq \varepsilon$. On the other hand from definition of ε, we can choose $n_0 \in \mathbb{N}$ such that $\varepsilon < d(x_{n_0}, x_{n_0+1}) < \varepsilon + \delta$ and by the properties of ψ, φ_2, φ_3, and θ, we have

$$\psi(\varepsilon) < \psi(d(x_{n_0}, x_{n_0+1})) < \psi(\varepsilon) + \psi(\delta) \Rightarrow \varphi(\psi(d(x_{n_0}, x_{n_0+1}))) \leq \psi(\varepsilon).$$

We have also

$$\varepsilon < d(x_{n_0+2}, x_{n_0+3}) < d(x_{n_0+1}, x_{n_0+2}) = d(Tx_{n_0}, Tx_{n_0+1}),$$

which implies

$$\begin{aligned}\psi(\varepsilon) &< \psi(d(x_{n_0+2}, x_{n_0+3}))\\ &< \psi(sd(x_{n_0+1}, x_{n_0+2}))\\ &= \psi(sd(Tx_{n_0}, Tx_{n_0+1}))\\ &\leq \alpha(x_{n_0}, x_{n_0+1})\psi(sd(Tx_{n_0}, Tx_{n_0+1}))\\ &\leq \theta(\psi(d(x_{n_0}, x_{n_0+1})))\varphi(\psi(d(x_{n_0}, x_{n_0+1})))\\ &< \frac{1}{s}\varphi(\psi(d(x_{n_0}, x_{n_0+1})))\\ &\leq \frac{1}{s}\psi(\varepsilon) \leq \psi(\varepsilon),\end{aligned}$$

which is a contradiction. Hence

$$l = \lim_{n\to\infty} d(x_n, x_{n+1}) = 0. \tag{3.1}$$

Now, we prove the sequence $\{x_n\}$ is Cauchy. We fix $\varepsilon_1 > 0$. Then by (φ_2'), there exists a $\delta_1 > 0$ such that $t < \varepsilon_1 + \delta_1 \Rightarrow \varphi(t) \leq \varepsilon_1$. Without loss of generality, we assume $\delta_1 < \varepsilon_1$. Due to (3.1) there exists $n_0 \in \mathbb{N}$ such that

$$d(x_n, x_{n+1}) < \frac{\delta_1}{s}, \text{ for all } n \geq n_0, \tag{3.2}$$

which implies $\psi(d(x_n, x_{n+1}) < \psi(\frac{\delta_1}{s})$ or $\psi(sd(x_n, x_{n+1})) < \psi(\delta_1)$. By induction, we show that for any fixed $k \geq n_0$

$$d(x_k, x_{k+l}) < \varepsilon_1 + \delta_1, \text{ for all } l \in \mathbb{N}. \tag{3.3}$$

The inequality trivially holds for $l = 1$ by (3.2). Now, we assume that (3.3) is satisfied for some $j \in \mathbb{N}$ and now, we show that it holds for $l = j + 1$.

From the triangle inequality and properties of ψ, φ_2, φ_3, and θ, we get

$$\begin{aligned}
\psi(d(x_k,x_{k+j+1})) &\leq \psi(s[d(x_k,x_{k+1})+d(x_{k+1},x_{k+j+1})]) \\
&= \psi(sd(x_k,x_{k+1}))+\psi(sd(x_{k+1},x_{k+j+1})) \\
&= \psi(sd(x_k,x_{k+1}))+\psi(sd(Tx_k,Tx_{k+j})) \\
&\leq \psi(sd(x_k,x_{k+1}))+\alpha(x_k,x_{k+j})\psi(sd(Tx_k,Tx_{k+j})) \\
&\leq \psi(sd(x_k,x_{k+1}))+\theta(\psi(d(x_k,x_{k+j})))\varphi(\psi(d(x_k,x_{k+j}))) \\
&< \psi(sd(x_k,x_{k+1}))+\frac{1}{s}\varphi(\psi(d(x_k,x_{k+j}))) \\
&< \psi(\delta_1)+\frac{1}{s}\varphi(\psi(d(x_k,x_{k+j}))) \\
&\leq \psi(\delta_1)+\frac{1}{s}\psi(\varepsilon_1) \\
&\leq \psi(\delta_1)+\psi(\varepsilon_1) \\
&= \psi(\varepsilon_1+\delta_1).
\end{aligned}$$

Thus, we have

$$d(x_k,x_{k+j+1})<\varepsilon_1+\delta_1.$$

Consequently (3.3) holds for $l=j+1$. Hence, we drive that $d(x_k,x_{k+l})<\varepsilon_1+\delta_1$ for all $k\geq n_0$ and $l\geq 1$. Since ε_1 is arbitrary, we conclude that

$$\lim_{m,n\to\infty} d(x_n,x_m)=0.$$

Thus, the sequence $\{x_n\}$ is Cauchy. Since (X,d) is complete, there exists $u\in X$ such that $x_n\to u$ as $n\to\infty$.
Claim $Tu=u$. Arguing by contradiction, we assume that $Tu\neq u$. So there exists $r>0$ such that $d(u,Tu)=r>0$. Since $x_n\to u$, we can choose $n_0\in\mathbb{N}$ such that $d(x_n,u)<\frac{r}{2s}$ for all $n\geq n_0$. Then, by the properties of ψ, φ_1, φ_3, and θ, we get the following

$$\begin{aligned}
\psi(r) &= \psi(d(u,Tu)) \\
&\leq \psi(sd(u,x_{n+1})+sd(x_{n+1},Tu)) \\
&= \psi(sd(u,x_{n+1}))+\psi(sd(Tx_n,Tu)) \\
&\leq \psi(sd(u,x_{n+1}))+\alpha(x_n,u)\psi(sd(Tx_n,Tu)) \\
&\leq \psi(sd(u,x_{n+1}))+\theta(\psi(d(x_n,u)))\varphi(\psi(d(x_n,u))) \\
&< \psi(sd(u,x_{n+1}))+\frac{1}{s}\varphi(\psi(d(x_n,u))) \\
&< \psi(\frac{r}{2})+\frac{1}{s}\psi(d(x_n,u)) \\
&< \psi(\frac{r}{2})+\psi(\frac{r}{2s}) \\
&\leq \psi(\frac{r}{2})+\psi(\frac{r}{2}) \\
&= \psi(\frac{r}{2}+\frac{r}{2})=\psi(r),
\end{aligned}$$

which is a contradiction. Thus $Tu=u$, that is, u is a fixed point of T. Next, we show uniqueness. Suppose that $v\neq u$ is another fixed point of T. Then, by the properties of ψ, φ_2, φ_3 and θ, we have

$$\begin{aligned}
\psi(d(u,v)) &= \psi(d(Tu,Tv)) \\
&\leq \psi(sd(Tu,Tv)) \\
&\leq \alpha(u,v)\psi(sd(Tu,Tv)) \\
&\leq \theta(\psi(d(u,v)))\varphi(\psi(d(u,v))) \\
&< \frac{1}{s}\varphi(\psi(d(u,v))) \\
&\leq \varphi(\psi(d(u,v)))<\psi(d(u,v)).
\end{aligned}$$

That implies $d(u,v)<d(u,v)$, which is a contradiction. Hence $u=v$. □

Remark 3.3. By taking $s = 1$, $\alpha(x, y) = 1$ and $\psi(t) = t$ in Theorem 3.2, we get Theorem 2.9 in [15].

Thus Theorem 3.2 generalizes Theorem 2.9 in [15]. Now, we give an example in support of Theorem 3.2.

Example 3.4. $X = [0, \frac{3}{4}] \cup \{1\}$ be endowed with the b-metric $d : X \times X \to \mathbb{R}^+$ defined by

$$d(x, y) = (x - y)^2,$$

for all $x, y \in X$. Then (X, d) is a complete b-metric space with $s = 2$. Let $T : X \to X$ be defined by

$$T(x) = \begin{cases} \frac{x}{4} & \text{if } x \in [0, \frac{3}{4}], \\ \frac{1}{8} & \text{if } x = 1. \end{cases}$$

Define $\alpha : X \times X \to \mathbb{R}^+$, $\theta : \mathbb{R}^+ \to [0, \frac{1}{2})$, $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ and $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ as

$$\alpha(x, y) = \begin{cases} \frac{3}{2} & \text{if } (x, y) \in [0, \frac{3}{4}], \\ 0 & \textit{otherwise.} \end{cases},$$

$\theta(t) = \frac{3}{8}$; $\psi(t) = \frac{t}{4}$, and $\varphi(t) = \frac{t}{2}$.

1. Now, we show that T is an α-admissible mapping. If $x, y \in [0, \frac{3}{4}]$, then $\alpha(x, y) > 1, Tx \leq \frac{3}{4}$ and $Ty \leq \frac{3}{4}$. By the definition of α, it follows that $\alpha(Tx, Ty) > 1$. Therefore, T is an α-admissible mapping.

2. We show that T is an (α, ψ, φ)-Geraghty contraction mapping.

Case I: For $x, y \in [0, \frac{3}{4}]$, we have

$$\begin{aligned} \alpha(x, y)\psi(sd(Tx, Ty)) &= \frac{3}{4}(Tx - Ty)^2 \\ &= \frac{3}{64}(x - y)^2 \\ &\leq \frac{3}{64}(x - y)^2 \\ &= \theta(\psi(d(x, y)))\varphi(\psi(d(x, y))). \end{aligned}$$

Case II: If $x \in [0, \frac{3}{4}]$ and $y = 1$, we have

$$\alpha(x, y)\psi(sd(Tx, Ty)) = 0 \leq \theta(\psi(d(x, y)))\varphi(\psi(d(x, y))).$$

Case III: If $x = y = 1$, we have

$$\alpha(x, y)\psi(sd(Tx, Ty)) = 0 \leq \theta(\psi(d(x, y)))\varphi(\psi(d(x, y))).$$

3. Further for $x \in [0, \frac{3}{4}]$, we have $\alpha(x, Tx) \geq 1$.
Therefore, from 1, 2 and 3 all the conditions of Theorem 3.2 are satisfied and T has a unique fixed point $u = 0$.

Definition 3.5. Let (X, d) be a b-metric space, $T : X \to X$ and $\alpha : X \times X \to \mathbb{R}^+$. A mapping T is said to be a generalized (α, ψ, φ)-Geraghty contraction mapping, if there exists $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$, $\psi \in \Psi$ and $\theta \in \Theta'_s$ satisfies for all $x, y \in X$ the following conditions:
(φ_0) φ is upper semicontinuous;
(φ_1) $\varphi(t) < t$ for any $t \in \mathbb{R}^+$;
(φ_2) For any $\varepsilon > 0$, there exists a $\delta > 0$ such that $\varepsilon < t < \varepsilon + \delta \Rightarrow \varphi(t) \leq \varepsilon$;
(φ'_3) $\alpha(x, y)\psi(sd(Tx, Ty)) \leq \theta(\psi(L(x, y)))\varphi(\psi(L(x, y)))$, where

$$L(x, y) = max\{d(x, y), d(x, Tx), d(y, Ty), \frac{(d(x, Ty) + d(y, Tx))}{2s}\}.$$

Theorem 3.6. Let (X, d) be a complete b-metric space and $T : X \to X$.
Suppose the following conditions hold

1. T is an α-admissible mapping;

2. T is a generalized, (α, ψ, φ)-Geraghty contraction mapping;
3. There exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$.

Then T has a unique fixed point.

Proof . Define a sequence $\{x_n\}$ in X by

$$x_n = Tx_{n-1}$$

for $n \in \mathbb{N}$. Suppose that $x_{n_0} = x_{n_0+1}$ for some $n_0 \in \mathbb{N}$. Since $Tx_{n_0} = x_{n_0+1} = x_{n_0}$ the point x_{n_0} forms a fixed point of T and the proof is complete.

From now on we suppose that $x_n \neq x_{n-1}$ for all $n \in \mathbb{N}$. Since T is an α-admissible, by (3) we have $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \geq 0$. Thus, we have $d(x_n, x_{n+1}) > 0$ and consequently $L(x_n, x_{n+1}) > 0$. By (φ_3') together with the properties of ψ, φ_1, and θ, we have

$$\begin{aligned}
\psi(d(x_{n+1}, x_{n+2})) &= \psi(d(Tx_n, Tx_{n+1})) \\
&\leq \psi(sd(Tx_n, Tx_{n+1})) \\
&\leq \alpha(x_n, x_{n+1})\psi(sd(Tx_n, Tx_{n+1})) \\
&\leq \theta(\psi(L(x_n, x_{n+1})))\varphi(\psi(L(x_n, x_{n+1}))) \\
&< \frac{1}{s}\varphi(\psi(L(x_n, x_{n+1}))) \\
&\leq \varphi(\psi(L(x_n, x_{n+1}))) < \psi(L(x_n, x_{n+1})).
\end{aligned}$$

So, we obtain

$$\psi(d(x_{n+1}, x_{n+2})) < \psi(L(x_n, x_{n+1})), \tag{3.4}$$

where

$$\begin{aligned}
L(x_n, x_{n+1}) &= max\left\{d(x_n, x_{n+1}), d(x_n, Tx_n), d(x_{n+1}, Tx_{n+1}), \frac{(d(x_n, Tx_{n+1}) + d(x_{n+1}, Tx_n))}{2s}\right\} \\
&= \max\left\{d(x_n, x_{n+1}), d(x_n, x_{n+1}), d(x_{n+1}, x_{n+2}), \frac{(d(x_n, x_{n+2}) + d(x_{n+1}, x_{n+1}))}{2s}\right\} \\
&= \max\left\{d(x_n, x_{n+1}), d(x_{n+1}, x_{n+2}), \frac{d(x_n, x_{n+2})}{2s}\right\}.
\end{aligned}$$

Using the triangle inequality, we have

$$\begin{aligned}
\frac{d(x_n, x_{n+2})}{2s} &\leq \frac{s[d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2})]}{2s} \\
&= \frac{d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2})}{2} \\
&\leq \max\Big\{d(x_n, x_{n+1}), d(x_{n+1}, x_{n+2})\Big\}.
\end{aligned}$$

Consequently, we drive that

$$L(x_n, x_{n+1}) = \max\{d(x_n, x_{n+1}), d(x_{n+1}, x_{n+2})\},$$

and (3.4) becomes

$$\psi(d(x_{n+1}, x_{n+2})) < \psi\Big(max\{d(x_n, x_{n+1}), d(x_{n+1}, x_{n+2})\}\Big). \tag{3.5}$$

The case where

$$\max\{d(x_n, x_{n+1}), d(x_{n+1}, x_{n+2})\} = d(x_{n+1}, x_{n+2}),$$

is impossible due to (3.5). Accordingly, we have

$$max\{d(x_n, x_{n+1}), d(x_{n+1}, x_{n+2})\} = d(x_n, x_{n+1}),$$

and by (3.5), we get

$$\psi(d(x_{n+1}, x_{n+2})) < \psi(L(x_n, x_{n+1})) = \psi(d(x_n, x_{n+1})),$$

for all $n \in \mathbb{N} \cup \{0\}$. That implies

$$d(x_{n+1}, x_{n+2}) < d(x_n, x_{n+1}), \text{ for all } n \in \mathbb{N} \cup \{0\}. \tag{3.6}$$

Hence, the non-negative sequence $\{d(x_n, x_{n+1})\}$ is non-increasing in $\mathbb{R}^+$. Accordingly, it is convergent to some real number $\mu \geq 0$. We claim that $\mu = 0$. Assume to the contrary that $\mu > 0$. We note that $\mu < d(x_n, x_{n+1})$ for all $n \in \mathbb{N} \cup \{0\}$. Set $\mu = \varepsilon > 0$. Then by (φ_2'), there exist $\delta > 0$ such that $t < \varepsilon + \delta \Rightarrow \varphi(t) \leq \varepsilon$. On the other hand, for sufficiently large $n \in \mathbb{N}$, we have

$$0 < \varepsilon < L(x_n, x_{n+1}) = d(x_n, x_{n+1}) < \varepsilon + \delta.$$

Using the properties of ψ, θ, φ_2', φ_3', and (3.6), we get

$$\begin{aligned}
0 < \psi(\varepsilon) &< \psi(d(x_{n+1}, x_{n+2})) \\
&= \psi(d(Tx_n, Tx_{n+1})) \\
&\leq \psi(sd(Tx_n, Tx_{n+1})) \\
&\leq \alpha(x_n, x_{n+1})\psi(sd(Tx_n, Tx_{n+1})) \\
&\leq \theta(\psi(L(x_n, x_{n+1})))\varphi(\psi(L(x_n, x_{n+1}))) \\
&< \frac{1}{s}\varphi(\psi(L(x_n, x_{n+1}))) \\
&\leq \frac{1}{s}\psi(\varepsilon) \leq \psi(\varepsilon),
\end{aligned}$$

which is a contradiction. Thus, we have

$$\mu = \limsup_{n\to\infty} d(x_n, x_{n+1}) = 0. \tag{3.7}$$

Now, we show that $\{x_n\}$ is a Cauchy sequence. Let $\varepsilon_1 > 0$ be fixed, then there exist $\delta_1 > 0$ which satisfies the following.

$$t < \varepsilon_1 + 2\delta_1 \Rightarrow \varphi(t) \leq \varepsilon_1. \tag{3.8}$$

From (3.7), we can choose $k \in \mathbb{N}$ large enough to satisfy $d(x_k, x_{k+1}) < \frac{\delta_1}{s}$. We want to show inductively that

$$d(x_k, x_{k+l}) < \varepsilon_1 + \delta_1, \tag{3.9}$$

for all $k \in \mathbb{N}$. We assume that $\delta_1 < \varepsilon_1$. We have already proved for $k = 1$. So, we suppose that (3.9) is satisfied for some $j \geq 1 \in \mathbb{N}$. For $l = j + 1$, we get

$$\begin{aligned}
L(x_k, x_{k+j}) &= \max\left\{d(x_k, x_{k+j}), d(x_k, Tx_k), d(x_{k+j}, Tx_{k+j}), \frac{d(x_k, Tx_{k+j}) + d(x_{k+j}, Tx_k)}{2s}\right\} \\
&= \max\left\{d(x_k, x_{k+j}), d(x_k, x_{k+1}), d(x_{k+j}, x_{k+j+1}), \frac{d(x_k, x_{k+j+1}) + d(x_{k+j}, x_{k+1})}{2s}\right\} \\
&\leq \max\Big\{d(x_k, x_{k+j}), d(x_k, x_{k+1}), d(x_{k+j}, x_{k+j+1}), \\
&\qquad \frac{s[d(x_k, x_{k+j}) + d(x_{k+j}, x_{k+j+1})] + s[d(x_k, x_{k+1}) + d(x_k, x_{k+j})]}{2s}\Big\} \\
&= \max\left\{d(x_k, x_{k+j}), d(x_k, x_{k+1}), d(x_{k+j}, x_{k+j+1}), \frac{2d(x_k, x_{k+j}) + d(x_{k+j}, x_{k+j+1}) + d(x_k, x_{k+1})}{2}\right\}
\end{aligned}$$

$$
\begin{aligned}
&< \max\left\{\varepsilon_1+\delta_1, \frac{\delta_1}{s}, \frac{\delta_1}{s}, \varepsilon_1+\delta_1+\frac{\delta_1}{2s}+\frac{\delta_1}{2s}\right\} \\
&< \max\{\varepsilon_1+\delta_1, \delta_1, \varepsilon_1+2\delta_1\} = \varepsilon_1+2\delta_1.
\end{aligned}
$$

So, we have

$$
L(x_k, x_{k+j}) < \varepsilon_1 + 2\delta_1. \tag{3.10}
$$

Then, by (3.8), (3.10), and the properties of ψ, θ, and φ_3', we obtain

$$
\begin{aligned}
\psi(d(x_k, x_{k+j+1})) &\le \psi(s[d(x_k, x_{k+1}) + d(x_{k+1}, x_{k+j+1})]) \\
&= \psi(sd(x_k, x_{k+1})) + \psi(sd(x_{k+1}, x_{k+j+1})) \\
&= \psi(sd(x_k, x_{k+1})) + \psi(sd(Tx_k, Tx_{k+j})) \\
&\le \psi(sd(x_k, x_{k+1})) + \alpha(x_k, x_{k+j})\psi(sd(Tx_k, Tx_{k+j})) \\
&\le \psi(sd(x_k, x_{k+1})) + \theta(\psi(L(x_k, x_{k+j})))\varphi(\psi(L(x_k, x_{k+j}))) \\
&< \psi(sd(x_k, x_{k+1})) + \frac{1}{s}\varphi(\psi(L(x_k, x_{k+j}))) \\
&\le \psi(sd(x_k, x_{k+1})) + \frac{1}{s}\psi(\varepsilon_1) \\
&< \psi(\delta_1) + \psi(\varepsilon_1) = \psi(\delta_1 + \varepsilon_1).
\end{aligned}
$$

That implies $d(x_k, x_{k+j+1}) < \delta_1 + \varepsilon_1$. Consequently (3.9) holds for $l = j+1$. Hence, $d(x_k, x_{k+l}) < \delta_1 + \varepsilon_1 < 2\varepsilon_1$ for all $k \in \mathbb{N}$ and $l \ge 1$, which means

$$
\limsup_{n,m\to\infty} d(x_n, x_m) = 0.
$$

Hence, the sequence $\{x_n\}$ is Cauchy. Since (X, d) is complete, there exist $u \in X$ such that $x_n \to u$ as $n \to \infty$. As a next step, we shall show that $Tu = u$. Suppose on the contrary, that $Tu \ne u$, there exists $r > 0$ such that $r = d(u, Tu) > 0$. Note that, due to the fact that the sequence $\{x_n\}$ is convergent to u, we can choose $n_0 \in \mathbb{N}$ such that $d(u, x_n) < \frac{r}{2s}$ for all $n \ge n_0$. So, we have the following estimation for $n \ge n_0$.

$$
\begin{aligned}
L(x_n, u) &= \max\left\{d(x_n, u), d(x_n, Tx_n), d(u, Tu), \frac{d(x_n, Tu) + d(u, Tx_n)}{2s}\right\} \\
&= \max\left\{d(x_n, u), d(x_n, x_{n+1}), d(u, Tu), \frac{d(x_n, Tu) + d(u, x_{n+1})}{2s}\right\} \\
&\le \max\left\{d(x_n, u), s[d(x_n, u) + d(u, x_{n+1})], d(u, Tu), \frac{s[d(x_n, u) + d(u, Tu)] + d(u, x_{n+1})}{2s}\right\} \\
&< \max\left\{\frac{r}{2s}, r, r, \frac{\frac{r}{s} + sr + \frac{r}{2s}}{2s}\right\} = r.
\end{aligned}
$$

It yields that

$$
\lim \sup_{n,m\to\infty} L(x_n, u) = r.
$$

By using the triangle inequality together with the properties of φ_3', ψ , and θ, we drive that

$$
\begin{aligned}
0 < \frac{\psi(r)}{s} \le \psi(r) &= \psi(d(u, Tu)) \\
&\le \psi(s[d(u, x_{n+1}) + d(x_{n+1}, Tu)]) \\
&= \psi(sd(u, x_{n+1})) + \psi(sd(Tx_n, Tu)) \\
&\le \psi(sd(u, x_{n+1})) + \alpha(x_n, u)\psi(sd(Tx_n, Tu)) \\
&\le \psi(sd(u, x_{n+1})) + \theta(\psi(L(x_n, u)))\varphi(\psi(L(x_n, u))) \\
&< \psi(sd(u, x_{n+1})) + \frac{1}{s}\varphi(\psi(L(x_n, u))).
\end{aligned}
$$

Letting $n \to \infty$ in the above inequality, together with the properties of θ, φ_0, and φ_1, we get

$$\begin{aligned}
0 < \frac{\psi(r)}{s} \leq \psi(r) &= \psi(d(u,Tu)) \\
&\leq \limsup_{n\to\infty}[\psi(sd(u,x_{n+1})) + \theta(\psi(L(x_n,u)))\varphi(\psi(L(x_n,u)))] \\
&= \limsup_{n\to\infty}\psi(sd(u,x_{n+1})) + \limsup_{n\to\infty}\theta(\psi(L(x_n,u))).\limsup_{n\to\infty}\varphi(\psi(L(x_n,u))) \\
&\leq \limsup_{n\to\infty}\theta(\psi(L(x_n,u))).\varphi(\psi(r)) \\
&< \frac{1}{s}\varphi(\psi(r)) \\
&< \frac{1}{s}\psi(r).
\end{aligned}$$

Thus, $\limsup_{n\to\infty}\theta(\psi(L(x_n,u))) = \frac{1}{s}$. Since $\theta \in \Theta'_s$, we have $\limsup_{n\to\infty}\psi(L(x_n,u)) = \psi(r) = 0 \Leftrightarrow r = 0$. Accordingly, we have $d(u,Tu) = r = 0$, that is, u is a fixed point of T.

Now, we show uniqueness. Suppose $v \neq u$ is another fixed point of T. We get $L(u,v) = d(u,v)$ and by using the properties of φ'_3, ψ, ψ_1 , and θ, we have

$$\begin{aligned}
0 < \psi(d(u,v)) &= \psi(d(Tu,Tv)) \\
&\leq \psi(sd(Tu,Tv)) \\
&\leq \alpha(u,v)\psi(sd(Tu,Tv)) \\
&\leq \theta(\psi(L(u,v)))\varphi(\psi(L(u,v))) \\
&< \frac{1}{s}\varphi(\psi(d(u,v))) \\
&\leq \varphi(\psi(d(u,v))) < \psi(d(u,v)).
\end{aligned}$$

So $d(u,v) < d(u,v)$ which is a contradiction. Hence, $u = v$. Therefore, T has a unique fixed point. □

Remark 3.7. By taking $s = 1$, $\alpha(x,y) = 1$ and $\psi(t) = t$ in Theorem 3.6, we get Theorem 2.11 in [15].

Thus Theorem 3.6 generalizes Theorem 2.11 in [15]. Now, we give two examples in support of Theorem 3.6.

Example 3.8. Let $X = \mathbb{R}^+$ be endowed with the b-metric $d : X \times X \to \mathbb{R}^+$ defined by

$$d(x,y) = (x-y)^2$$

for all $x, y \in X$. Then (X,d) is a complete b-metric space with $s = 2$. Let $T : X \to X$ be defined by

$$T(x) = \begin{cases} \frac{1-x^2}{16} & \text{if } x \in [0,1] \\ 4x & \textit{otherwise.} \end{cases}$$

Define $\alpha : X \times X \to \mathbb{R}^+$, $\theta : \mathbb{R}^+ \to [0, \frac{1}{2})$,$\psi : \mathbb{R}^+ \to \mathbb{R}^+$ and $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ by

$$\alpha(x,y) = \begin{cases} \frac{5}{4} & \text{if } (x,y) \in [0,1], \\ 0 & \textit{otherwise.} \end{cases};$$

$\theta(t) = \frac{15}{64}$; $\psi(t) = \frac{t}{2}$ and $\varphi(t) = \frac{t}{4}$.

1. We show that T is an α-admissible mapping. If $x, y \in [0,1]$, then $\alpha(x,y) > 1$, $Tx \leq 1$ and $Ty \leq 1$. By definition of α, it follows that $\alpha(Tx,Ty) > 1$. If $x, y \in (1,\infty)$, we have $\alpha(x,y) = 0 < 1$. Again, for $x \in [0,1]$ and $y \in (1,\infty)$, we have $\alpha(x,y) = 0 < 1$. Therefore T is an α-admissible mapping.

2. Now, we show that T is a generalized (α,ψ,φ)-Geraghty contraction mapping.

Case I: If $x, y \in [0,1]$, we have

$$\begin{aligned}
\alpha(x,y)\psi(sd(Tx,Ty)) &= \frac{5}{4}(Tx-Ty)^2 \\
&= \frac{5}{1024}(x^2-y^2)^2 \\
&= \frac{5}{1024}(x-y)^2(x+y)^2 \\
&\leq \frac{5}{256}(x-y)^2 \\
&= \frac{5}{32}\frac{(x-y)^2}{4} \\
&= \theta(\psi(d(x,y)))\varphi(\psi(d(x,y))) \\
&\leq \theta(\psi(L(x,y)))\varphi(\psi(L(x,y))).
\end{aligned}$$

Case II: If $x, y \in (1,\infty)$, we have

$$\alpha(x,y)\psi(sd(Tx,Ty)) = 0 \leq \theta(\psi(L(x,y)))\varphi(\psi(L(x,y))).$$

Case III: If $x \in [0,1]$ and $y \in (1,\infty)$, we have,

$$\alpha(x,y)\psi(sd(Tx,Ty)) = 0 \leq \theta(\psi(L(x,y)))\varphi(\psi(L(x,y))).$$

3. Further for $x \in [0,1]$, we have $\alpha(x,Tx) \geq 1$.

Therefore, from 1, 2 and 3 all the conditions of Theorem 3.6 are satisfied and T has a unique fixed point $u = \sqrt{65}-8$.

Example 3.9. Let $X = \{a_1, a_2, a_3, a_4\}$ and $d : X \times X \to \mathbb{R}^+$ defined by:
$d(a_1,a_2) = d(a_2,a_1) = 1$, $d(a_3,a_4) = d(a_4,a_3) = 10$, $d(a_1,a_4) = d(a_4,a_1) = d(a_2,a_4) = d(a_4,a_2) = 6$, $d(a_1,a_3) = d(a_3,a_1) = d(a_2,a_3) = d(a_3,a_2) = 8$, $d(a_i,a_i) = 0$, for any $i = 1,2,3,4$. It is easy to see that the pair (X, d) forms a b-metric space with $s = \frac{6}{5}$. Assume $T : X \to X$, $\alpha : X \times X \to \mathbb{R}^+$, $\theta : \mathbb{R}^+ \to [0, \frac{5}{6})$, $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ and $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ be defined by

$$Ta_1 = Ta_2 = a_1, Ta_3 = Ta_4 = a_2;$$

for any $i, j = 1,2,3,4$

$$\alpha(a_i,a_j) = \begin{cases} \frac{3}{2} & \text{if } i \neq j, \\ 1 & \text{if } i = j. \end{cases};$$

$$\varphi(t) = \begin{cases} \frac{t}{2} & \text{if } t \in [0,2) \\ \frac{3}{2} & \text{if } t \in [2,\infty). \end{cases};$$

$\theta(t) = \frac{1}{2}$ and $\psi(t) = \frac{t}{3}$.

1. We show that T is an α-admissible mapping.

For $i \neq j$ we have $\alpha(a_i,a_j) = \frac{3}{2} \geq 1$, which implies $\alpha(Ta_i,Ta_j) \geq 1$. If $i = j = 1$ we have $\alpha(a_i,a_j) = 1 \geq 1$, which implies $\alpha(Ta_i,Ta_j) = 1 \geq 1$.

Also, again if $i = j = 2,3,4$ we have $\alpha(a_i,a_j) = 1 \geq 1$ implies $\alpha(Ta_i,Ta_j) \geq 1$. Therefore T is an α-admissible mapping.

2. Now we show that T is a generalized, (α,ψ,φ)-Geraghty contraction mapping. On the other hand, because $d(Ta_1,Ta_2) = d(Ta_3,Ta_4) = 0$ and (φ_3') is obviously satisfied, relevant for our study is only the set $\{(a_1,a_3),(a_1,a_4),(a_2,a_3),(a_2,$
For this reason, we consider the following cases:

Case I: If $x = a_1$, $y = a_3$, then

$$\begin{aligned}
\alpha(a_1,a_3)\psi(sd(Ta_1,Ta_3)) &= \alpha(a_1,a_1)\beta(a_3,a_2)\psi(sd(a_1,a_2)) = \frac{3}{5} \\
&\leq \theta(\psi(L(a_1,a_3))).\varphi(\psi(L(a_1,a_3))) = \frac{3}{4}.
\end{aligned}$$

Case II: If $x = a_1$, $y = a_4$, then

$$\begin{aligned}\alpha(a_1, a_4)\psi(sd(Ta_1, Ta_4)) &= \alpha(a_1, a_1)\beta(a_4, a_2)\psi(sd(a_1, a_2)) = \frac{3}{5}\\ &\leq \theta(\psi(L(a_1, a_4))) \cdot \varphi(\psi(L(a_1, a_4))) = \frac{3}{4}.\end{aligned}$$

Case III: If $x = a_2$, $y = a_3$, then

$$\begin{aligned}\alpha(a_2, Ta_2)\beta(a_3, Ta_3)\psi(sd(Ta_2, Ta_3)) &= \alpha(a_2, a_1)\beta(a_3, a_2)\psi(sd(a_1, a_2)) = \frac{3}{5}\\ &\leq \theta(\psi(L(a_2, a_3)))\varphi(\psi(L(a_2, a_3))) = \frac{3}{4}.\end{aligned}$$

Case IV: If $x = a_2$, $y = a_4$, then

$$\begin{aligned}\alpha(a_2, Ta_2)\beta(a_4, Ta_4)\psi(sd(Ta_2, Ta_4)) &= \alpha(a_2, a_1)\beta(a_4, a_2)\psi(sd(a_1, a_2)) = \frac{3}{5}\\ &\leq \theta(\psi(L(a_2, a_4))).\varphi(\psi(L(a_2, a_4))) = \frac{3}{4}.\end{aligned}$$

3. Further for $x = a_2$, we have $\alpha(a_2, Ta_2) = \alpha(a_2, a_1) = \frac{3}{2} \geq 1$.

Thus, from 1, 2 and 3 all the conditions of Theorem 3.6 are satisfied. Moreover, $u = a_1$ is a unique fixed point of T.

4 Conclusion

Karapinar et al. [15] established fixed point theorems for φ-Geraghty and Ćirić type φ-Geraghty contractive mappings in complete metric spaces and proved the existence and uniqueness of fixed points. In this paper, we introduce (α, ψ, φ)-Geraghty and generalized (α, ψ, φ)-Geraghty contraction mappings in b-metric spaces and prove the existence and uniqueness of fixed point for the mappings introduced. Our results extend and generalize related fixed point results in the literature, in particular that of Karapinar et al. [15]. We have also provide examples to support our main results.

Acknowledgments

The authors would like to thank the College of Natural Sciences, Jimma University for funding this research work.

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