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A new way to obtain fixed point functions using the grey wolf optimizer algorithm

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Abstract

In this paper, we introduce a new iterative method for finding the fixed point of a nonlinear function. In fact, we want to offer a new way to obtain the fixed point of various functions using the Grey Wolf Optimizer algorithm. This method is new and very efficient for solving a nonlinear equation. We explain this method with three benchmark functions and compare results with other methods, such as ALO, MVO, MFO and SCA.

Keywords: Meta-heuristic algorithms , Fixed point problems, Grey Wolf Optimizer Algorithm, Bisection algorithm
2020 MSC: 54H25, 47H10, 47N10

1 Introduction

Obtaining the roots of equations, especially nonlinear equations, is one of the most important topics in engineering and basic sciences. For this sake, many researchers have checked this problem for some years [35, 43, 47, 48, 37, 38].

Meta-heuristic or meta-heuristic or meta-heuristic algorithms are a type of random algorithms that are used to find the optimal answer. Optimization methods and algorithms are divided into two categories: exact algorithms and approximate algorithms.

Optimization becomes one of the main and vital issues of human beings. Hence, various optimization algorithms with metaheuristics perspective have been introduced. The Gray Wolf Optimization Algorithm (GWO) is one of the newest meta-innovative algorithms introduced by Mr. Mirjalili and his colleagues in 2014.

This algorithm belongs to the group of collective algorithms and many other meta-heuristic algorithms are inspired by nature and are based on the structure of the intelligence chain that models the social behavior of gray wolves during hunting.

In implementing the GWO algorithm, four types of gray wolves, such as alpha, beta, delta, and omega, are used to model the hierarchy of gray wolf leadership, with three main steps for hunting, including: search for prey, siege of prey, and Attacks on prey are executed. The results of various benchmark functions indicate the optimal performance of the algorithm compared to similar algorithms.

Well-known population- based meta-heuristic algorithms include evolutionary algorithms (genetic algorithm) [12], ant colony optimization (ACO) [10, 5], bee colony(BC) [45], particle swarm optimization method (PSO) [15], forest

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optimization algorithm (FO) [31], Battle royale optimization algorithm (BRO) [31], runner- root algorithm(RRA) [22], intelligent water drops algorithm (IWD) [34], Artificial Bee Colony algorithm(ABC) [14, 13] Firefly Algorithm(FA) [41], Differential evolution (DE) algorithms [17], biogeography based optimization (BBO) algorithm [20].

In recent years, new meta-heuristic algorithms have been developed with respect to living organisms in nature (inspired by nature), the most famous of which are the Gray Wolf Optimization Algorithm(GWO) [9], the Dragonfly algorithm (DA) [23], the Flower Pollination Optimization Algorithm (FPA) [1], Whale optimization Algorithm (WOA) [27], Grasshopper Optimisation Algorithm (GOA) [33], social spider algorithm (SSA) [6], Sine Cosine Algorithm (SCA) [26], Multi-Verse Optimizer algorithm (MVO) [28], Moth-flame optimization algorithm (MFO) [24], Ant Lion Optimizer algorithm (ALO) [25], Emperor Penguins Colony algorithm [11] and so on [4, 46, 2, 8, 7, 16, 42, 30, 29].

In this paper, we introduce a novel iterative method that obtain the fixed point of various functions using the Grey Wolf Optimizer algorithm.

In Sect. 2, the Grey Wolf Optimizer Algorithm is explained and fixed point problem is illustrated. Also suggested method illustrated in Sect.3. Section 4 measures the resolution of the offered method by different methods on several functions. Also, the result is available at Sect. 5.

2 Preliminaries

In the present section, the Bisection method and the Grey Wolf Optimizer Algorithm is explained and fixed point problem is illustrated.

2.1 The Gray Wolf Optimizer Algorithm

The Gray Wolf Algorithm is a metaheuristic algorithm inspired by the hieratical hierarchical structure and social behavior of gray wolves while hunting. This algorithm is population-based, has a simple process, and can be easily generalized to large-scale problems. Gray wolves are thought of as apex predators, which are at the top of the food chain pyramid. Gray wolves prefer to live in a group, each group has an average of 5-12 members. All members of this group have a very precise hierarchy of social domination and have specific tasks. In each herd of wolves there are 4 degrees to hunt, which is modeled as a pyramidal structure as shown below. Wolves are called alpha group wolves,

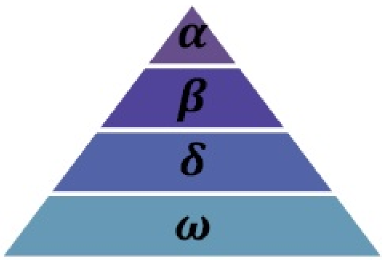

Figure 1: Model pyramidal structure

which can be male or female. These wolves dominate the herd Beta wolves: Help alpha wolves in the decision-making process and are also prone to be chosen instead. Delta Wolves: Lower than beta wolves and includes older wolves, predators and baby care wolves.

Omega Wolves: The lowest rank in the hierarchy that has the least rights over the rest of the group. After all, they eat and do not participate in the decision-making process.

In explaining and teaching the gray wolf algorithm, we can say that this algorithm includes 3 main steps:

1- Tracking and tracking (hunting and tracking)

2- Approaching, encircling (looping) around the prey and misleading it until it stops moving (Pursing and encircling)

3- Attacking hunting

In this paper, the hierarchical structure and social behavior of wolves during the hunting process are mathematically modeled and used to design an algorithm for optimization.

Optimization is done using alpha, beta and delta wolves. A wolf is assumed to be the main alpha of the algorithm, a wolf beta and a delta are involved, and the rest of the wolves follow them. Gray wolves have the ability to estimate the hunting position. To model this process, see the following steps: In the initial search, we have no idea about the hunting position. Alpha, beta, and delta wolves are thought to have better first-hand knowledge of hunting position (optimal answer point). In Gray Wolf Optimizer (GWO), we consider the most appropriate solution as alpha, and the second and third appropriate solutions are named beta and delta, respectively. The rest of the solutions are considered omega. Hunting operations are usually led by Alpha.

Beta and delta wolves may occasionally hunt. In the mathematical model of gray wolf hunting behavior, we assumed that alpha, beta, and delta had better knowledge of the potential prey position. The first three solutions are best stored, and the other agent is required to update their positions according to the position of the best search agents.

2.2 Definition the fixed point

In mathematics, a fixed point (invariant point) of a function isa point that is mapped to itself by the function. In other word ,a number c is a fixed point for a given function g if $g(c) = c$. A set of fixed points is sometimes called a fixed set. An iterative method for solving equation $g(x) = x$ is the recursive relation $x_{i+1} = g(x_i)$, $i = 0, 1, 2, \ldots$ with some initial guess x_0. The algorithm stops when one of the following stopping criterion is met:

- $D1$: total number of iterations is N, for some N, fixed a priori.
- $D2$: $|x_{i+1} - x_i| < \epsilon$ for some ϵ, fixed a priori.

This procedure is shown in figure 2.

Repeat

1. Give the transcendental equation $x = g(x)$,
2. Give initial guess in interval $[a, \ b]$,
3. Do $x_{i+1} = g(x_i)$

until (D_1 or D_2 is true)

Figure 2: Fixed Point iteration scheme

3 The Gray Wolf Optimizer Algorithm for Solving Fixed Point of Functions

At present part, we present one modern repetitious procedure to gain the solution estimation of a fixed point question as $g(x) = x$. We describe a function $f(x) = g(x) - x$. Accordingly the question of discovering the fixed points of $g(x)$ is decreased to discovering the roots of $f(x)$. We subsequent describe a function $h(x) = |f(x)|$. The question of discovering the roots of $f(x)$ is better decreased to discovering an x that minimizes $h(x)$. The opinion where is that for obtaining the half point of the distance I to begin with one volunteer answer, GWO algorithm is utilized to impute one superior estimation and determined a distance $I_k = [a_k, b_k]$ one volunteer solution x_k is calculated utilizing the GWO algorithm. If $f(x_k) = 0$ we are accomplished, again calculate one modern distance I_{k+1} into I_k pertaining against whether $f(x_k).f(a_k) < 0$ or $f(x_k).f(b_k) < 0$.

4 Implement Methods on Various Functions

In this section, we illustrate our algorithm with some examaples and compare the results with other evolutionary optimization algorithms such as ALO, MVO, MFO and SCA.

4.1 Introducing Different Functions

Introducing different functions

$$
\begin{array}{lll}
g_1(x) : \dfrac{x^2}{4000} - \cos(x) + 1 = x & ; & x \in [-20, +20] \\
g_2(x) : 10 + x^2 - 10\cos(2\pi x) = x & ; & x \in [-20, 1) \\
g_3(x) : 20 + e - 20e^{-0.2\sqrt{x^2}} - e^{\cos(2\pi x)} = x & ; & x \in [1, 21]
\end{array}
$$

Results for the three functions are shown in Table 1 and Their diagrams are also shown in Figures 3-5. Figures 3 to 5 show Diagram of the recovery process of the g_1 to g_3 functions by the GWO algorithm in (a) and diagram of the finding of the fixed point of the g_1 to g_3 functions by the GWO algorithm in (b).

Table 1: The outcomes acquired toward any function via ALO, MVO, SSA, SCA and BMFO

algorithm	Components	$g_1(x)$	$g_2(x)$	$g_3(x)$
ALO	Error	$1.22E-09$	$5.06E-10$	$1.49E-10$
	X-best	$2.08E+01$	$5.04E-03$	$1.49E-10$
	mean(e)	$8.57E-05$	$1.91E-06$	$1.01E-04$
	std(e)	$1.44E-03$	$4.69E-05$	$2.83E-04$
MFO	Error	$1.07E-14$	$1.00E-06$	$3.33E-269$
	X-best	$1.99E+01$	$1.00E+00$	$3.33E-269$
	mean(e)	$2.93E-04$	$1.19E-04$	$1.55E-05$
	std(e)	$4.38E-03$	$2.65E-03$	$1.83E-04$
MVO	Error	$1.70E-05$	$5.33E-07$	$1.26E-07$
	X-best	$2.01E+01$	$5.04E-03$	$1.26E-07$
	mean(e)	$1.40E-03$	$1.89E-06$	$2.13E-04$
	std(e)	$3.59E-03$	$3.24E-05$	$1.17E-03$
SCA	Error	$5.60E-05$	$2.17E-109$	$7.97E-128$
	X-best	$1.99E+01$	$2.17E-109$	$7.97E-128$
	mean(e)	$2.98E-04$	$9.39E-06$	$5.48E-05$
	std(e)	$1.38E-03$	0.000289475	$1.35E-03$
GWO	Error	$1.72E-06$	$9.90E-07$	$0.00E+00$
	X-best	$2.08E+01$	$-9.90E-07$	$0.00E+00$
	mean(e)	$1.78E-04$	$8.38E-06$	$3.58E-05$
	std(e)	$1.32E-03$	0.000233549	0.000940455

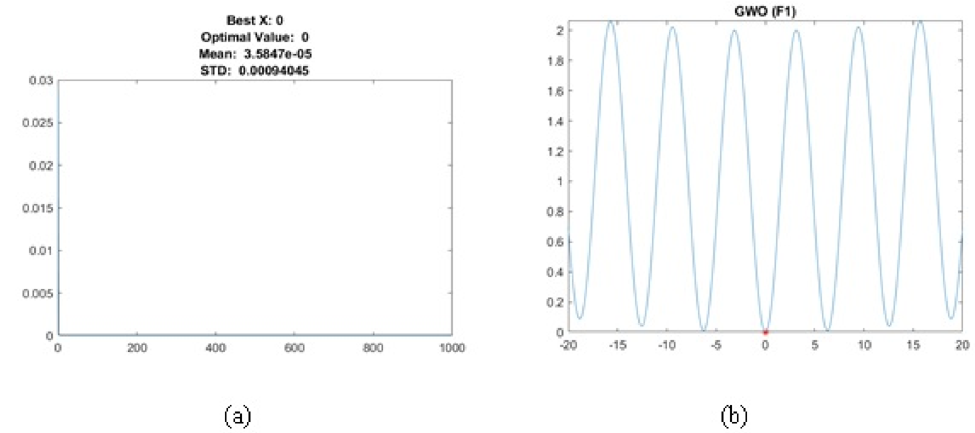

Figure 3: Diagram of the recovery process of the g_1 function by the GWO algorithm in (a) and diagram of the finding of the fixed point of the g_1 function by the GWO algorithm using the intersection of the diagram $g_1(x) = x$ in (b)

5 Conclusion

In this paper, we introduce a novel iterative method for finding a fixed point of a function g in a real interval $[a, b] \subseteq \mathbb{R}$ by using Gray Wolf Optimizer Algorithm. If the function g is hard, it is sometimes difficult to determine suitable initial value close to the location of a fixed point. Derivative method (find the derivative of $g(x) - x$ and find its root) is also sometimes not useful for various reasons like the derivative may not exist, the derivative is hard to compute or finding the root of a derivative itself may be difficult. GWO algorithm helps in finding a good initial value and the proposed method does away with the need to compute the derivative. Our proposed algorithm is easy to use and reliable. As comparison with other algorithm shows, the accuracy of our proposed method also is good.

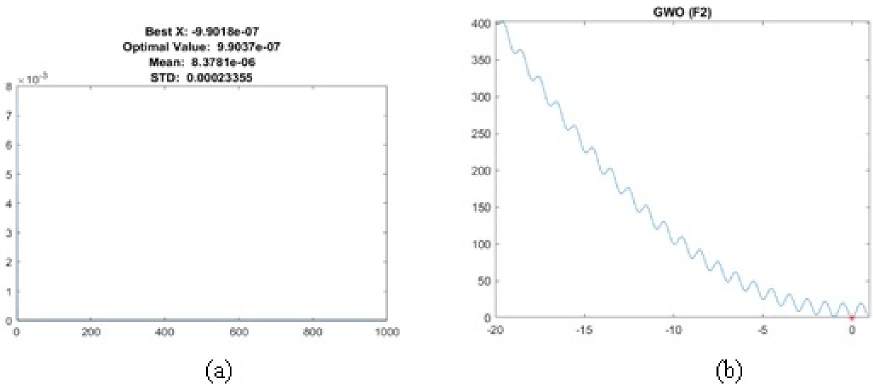

Figure 4: Diagram of the recovery process of the g_2 function by the GWO algorithm in (a) and diagram of the finding of the fixed point of the g_2 function by the GWO algorithm using the intersection of the diagram $g_2(x) = x$ in (b)

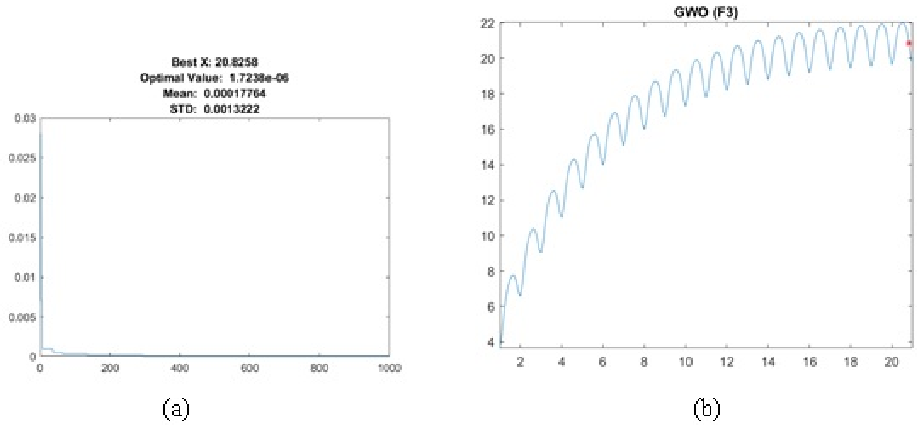

Figure 5: Diagram of the recovery process of the g_3 function by the GWO algorithm in (a) and diagram of the finding of the fixed point of the g_3 function by the GWO algorithm using the intersection of the diagram $g_3(x) = x$ in (b)

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