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Acceptance single sampling plan using generalized intuitionistic fuzzy number

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Abstract

In the present paper, the acceptance single sampling plan is developed by the generalized intuitionistic fuzzy numbers. The α_1-cut and α_2-cut sets of the generalized intuitionistic fuzzy numbers are applied to construct the acceptance single sampling plan. We also investigate the operating characteristic curve, where the parameter is considered the generalized intuitionistic fuzzy number. The (α_1, α_2)-cut set of generalized intuitionistic fuzzy operating characteristics is constructed. The bands are represented with upper and lower bounds instead of curves for operating characteristics and evaluated in detail. Finally, the numerical example is given to illustrate the proposed approach.

Keywords: Generalized intuitionistic fuzzy numbers, (α_1, α_2)-cut set, Binomial distribution, OC bands
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1 Introduction

The sampling plans are significant devices to distinguish the quality of constructed outcomes and products in many fields, including industry, engineering, and transportation. The decision of reliable and higher-quality product selection could be adopted through a statistical sampling approach in decision-making analysis. In sampling methods, the assessment system based on the acceptance or reject many manufactured products is commonly called acceptance sampling; for more details, see Montgomery [18]. The acceptance sampling plans (ASPs) design controlling of a small set of components from a consignment within a certain plan to attain a given output quality level with the least cost according to time, effort and destruction to the examined components.

Statistical quality control (SQC) is one of the substantial topics of the quality management system, where the ASP is the main index of SQC. The classical SQC is based on precise information with precise data and precise parameters. But in practice, we are dealing with situations in which some of these components cannot be measured and recorded precisely. Therefore, to overcome this problem, some researchers used fuzzy sets (FSs) theory in the acceptance sampling plan. Baloui Jamkhaneh et al. [6, 7, 9, 10] designed the acceptance double sampling plan, acceptance single sampling plan (SSP) based on Poisson distribution, acceptance SSP based on Binomial distribution, and acceptance SSP with inspection errors, under the fuzzy number of the fraction of defective cases. Turanoğlu et al. [23] analyzed the acceptance of single and double sampling plans when the parameters N, p, n and c are fuzzy numbers. Baloui Jamkhaneh et al. [8] provided the average outgoing quality and average total inspection for the SSP, such that the proportion nonconforming was a triangular fuzzy number.

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Aslam and June [3] proposed the double ASP under the generalized logistic lifetime distribution for products, where shape parameters are known. They provided the operating characteristic curve to several ratios of the true median life to the proposed life.

Khan et al. [16] considered the Birnbaum-Saunders distribution in acceptance sampling under the fuzzy environment, where the rate of imperfect cases is assumed fuzzy with the Birnbaum-Saunders distribution. They investigated the treatment of curves based on several compositions of parameters of Birnbaum-Saunders distribution.

By considering the intuitionistic fuzzy sense, Rasheed et al. [19] went further in survey sampling to tackle the problem of vagueness in the respondent's mind, whereas the credit of the survey sampling is relevant to the accuracy of the data. They focused on the intuitionistic fuzzy aggregative investment profit proportion to provide the best constructing facility position, where respondents declared both membership and non-membership functions, with hesitancy level and reduced the vagueness in the respondent's thought.

Recently, Aslam et al. [4] focused on the improvement of the group-sampling plan based on time-truncated tests of Weibull distribution under the neutrosophic statistics and extended the single group-sampling and double group-sampling plans based on the neutrosophic statistics. They compared the efficiency of both group-sampling plans under the neutrosophic statistical interval and the crisp group-sampling plan under classical statistics, which leads to the privilege of the proposed plan over the crisp method of competitive sampling plans.

The implication of the chain sampling plan is developed by Baloui Jamkhaneh and Sadeghpour Gildeh [11], where the rate of damaged products is considered a trapezoidal fuzzy number for more broad application. Afterward, Baloui Jamkhaneh and Sadeghpour Gildeh [12] applied the fuzzy acceptable quality level and fuzzy lot tolerance percent defective numbers and introduced the sequential sampling plan. Afshari et al. [1] focused on the fuzzy fraction of defective cases and provided a fuzzy multiple deferred state (FMDS) sampling plan by attribute so that the suggested plan is extended to the imperfect inspection states. Afshari et al. [2] proposed the FMDS sampling plan by attribute based on the fuzzy probability theory when the rate of defective components is ambiguous. Khan et al. [17] investigated fuzzy ASP for the transmuted Weibull distribution.

Intuitionistic fuzzy sets (IFSs) theory, defined by Atanassov et al. [5], is a useful tool in modeling real data, wherein hesitation between belongingness and non-belongingness cannot be ruled out. IFSs have been extensively applied to consider uncertainty in observation and parameters. Following that, IFSs were considered to analyze the sampling plan by many researchers. Isik and Kaya [15] designed single and double ASPs based on IFSs and derived the main characteristics of ASPs, including the acceptance probability, average sample number, average total inspection, and average outgoing quality for Poisson and Binomial distributions.

Shabani and Baloui Jamkhaneh [22] introduced a new generalized intuitionistic fuzzy number (GIFN_B) based on the generalization of the IFS and Baloui Jamkhaneh [14] defined some operators according to the generalized intuitionistic fuzzy (GIF) sets. Recently, Roohanizadeh et al. [20] and Roohanizadeh et al. [21] considered the system reliability of Pareto distribution with generalized intuitionistic fuzzy numbers. The main objective of this paper is to design the SSP, in which the fraction of nonconforming items is taken as GIF number with both linear and nonlinear membership and non-membership functions. It is developed based on the concept α_1-cut, α_2-cut and (α_1, α_2)-cut of GIFN_Bs.

The structure of the present paper is organized as follows: Section 2 presents basic concepts of GIFN_Bs. The GIF probability mass function is represented in Section 3, with (α_1, α_2)-cut set. Also, based on the numerical example, different cut sets of the GIF probability and membership and non-membership function of the GIF probability are provided. Section 4 gives acceptance single sampling (ASS) with the GIF parameter. In Section 5, the GIF operating characteristic is provided. Regarding a numerical example, the operating characteristic surfaces are depicted based on the zero-value of the acceptance number.

2 Preliminaries

In this section, we briefly review several definitions and terminologies related to the GIF numbers used throughout the paper.

Definition 2.1. (Baloui Jamkhaneh and Nadarajah [13]) Consider the non-empty set X and the degree of membership and degree of non-membership of x in A, respectively as $(\mu_A(x), \nu_A(x))$. A GIF set (GIFS_B) A in X, is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, such that $(\mu_A(x), \nu_A(x)) \in L$ and $L = \{(x, y) \in [0,1]^2 \,|x^\delta + y^\delta \leq 1\}$, where $\delta = n$ or $\frac{1}{n}$, $n = 1, 2, \ldots, N$.

Definition 2.2. (Shabani and Baloui Jamkhaneh [22]) The membership and non-membership functions of the especial class of GIFN$_B$A are defined, respectively, as

$$\mu_A(x) = \begin{cases} \left(\dfrac{x-a}{b-a}\right)^{\frac{1}{\delta}}, & a \le x \le b \\ 1, & b \le x \le c \\ \left(\dfrac{d-x}{d-c}\right)^{\frac{1}{\delta}}, & c \le x \le d \\ 0, & o.w \end{cases}, \qquad \nu_A(x) = \begin{cases} \left(\dfrac{b-x}{b-a_1}\right)^{\frac{1}{\delta}}, & a_1 \le x \le b \\ 0, & b \le x \le c \\ \left(\dfrac{x-c}{d_1-c}\right)^{\frac{1}{\delta}}, & c \le x \le d_1 \\ 1, & o.w \end{cases},$$

where $a_1 \le a \le b \le c \le d \le d_1$. The GIFN$_B A$ is denoted as $A = (a_1, a, b, c, d, d_1, \delta)$.

Notation 2.3. (i) The GIFN$_B$ in Definition 2.2 covers the trapezoidal intuitionistic fuzzy and trapezoidal fuzzy numbers as especial cases, such that for $\delta = 1$, it reduces to the trapezoidal intuitionistic fuzzy number, in addition, if $a_1 = a$ and $d = d_1$ reduces to the trapezoidal fuzzy number.

(ii) In Definition 2.2, if the relation $a \le a_1 \le b \le c \le d_1 \le d$ be established, then the GIFN$_B A$ is denoted as $A = (a, a_1, b, c, d_1, d, \delta)$.

Definition 2.4. (Baloui Jamkhaneh [14]) Consider the fixed numbers $(\alpha_1, \alpha_2) \in L$, the (α_1, α_2)-cut set generated by a GIFN$_B A$ is defined by

$$A[\alpha_1, \alpha_2, \delta] = \Big\{ \langle x, \mu_A(x) \ge \alpha_1, \nu_A(x) \le \alpha_2 \rangle : x \in X \Big\}.$$

The α_1-cut set of the GIFN$_B A$ is a crisp subset of $\mathbb{R}$, which is defined as

$$A_\mu[\alpha_1, \delta] = \{\langle x, \mu_A(x) \ge \alpha_1, \rangle \; : x \in X\} = \Big[A_\mu^L[\alpha_1], A_\mu^U[\alpha_1]\Big], \qquad 0 \le \alpha_1 \le 1,$$

$$A_\mu^L[\alpha_1] = a + (b-a)\,\alpha_1^\delta, \qquad A_\mu^U[\alpha_1] = d - (d-c)\,\alpha_1^\delta.$$

Analogously, the α_2-cut set of the GIFN$_B A$ is a crisp subset of $\mathbb{R}$ and is defined as below

$$A_\nu[\alpha_2, \delta] = \{\langle x, \nu_A(x) \le \alpha_2 \rangle : x \in X\} = \Big[A_\nu^L[\alpha_2], A_\nu^U[\alpha_2]\Big], \qquad 0 \le \alpha_2 \le 1,$$

$$A_\nu^L[\alpha_2] = b\left(1 - \alpha_2^\delta\right) + a_1\alpha_2^\delta, \qquad A_\nu^U[\alpha_2] = c\left(1 - \alpha_2^\delta\right) + d_1\alpha_2^\delta.$$

Therefore, the (α_1, α_2)-cut set of the GIFN$_B$ is given as $A[\alpha_1, \alpha_2, \delta] = A_\mu[\alpha_1, \delta] \cap A_\nu[\alpha_2, \delta]$. The GIFN$_B$ based on the α_1-cut and α_2-cut sets are shown as

$$A(\alpha_1, \alpha_2, \delta) = \Big(A_\mu[\alpha_1, \delta], A_\nu[\alpha_2, \delta]\Big).$$

3 Generalized Intuitionistic Fuzzy Probability Mass Function

Consider the finite sample space $S = \{x_1, x_2, \ldots, x_n\}$ and the probability measure defined to nonempty sigma field of subsets of S, $(\mathcal{F}(S))$, denoted as P, such that

$$P\{x_r\} = k_r, \qquad 1 \le i \le n, \qquad \sum_{r=1}^{n} k_r = 1.$$

Regarding to the vagueness of the value k_r, which maybe encountered in some real cases, hence, it is necessary to consider k_r as a non-crisp number. For this purpose, let $\tilde{k}_r$, $r = 1, 2, \ldots, n$ are GIFNs and the probability of the event $X = x_r$ equals $\tilde{k}_r$. In this case, $\tilde{P}\{x_r\} = \tilde{k}_r = (\tilde{k}_{r\mu}[\alpha_i], \tilde{k}_{r\nu}[\alpha_i])$, where $\tilde{k}_{r\mu}[\alpha_i]$, $\tilde{k}_{r\nu}[\alpha_i] \subset [0,1]$ and there are $k_{r\mu} \in \tilde{k}_{r\mu}[1]$ and $k_{r\nu} \in \tilde{k}_{r\nu}[0]$, such that $\sum_{r=1}^{n} k_{r\mu} = 1$ and $\sum_{r=1}^{n} k_{r\nu} = 1$.

Let $A \subset \mathcal{F}(S)$, in this case, the cut set of the GIF probability $\tilde{P}(A)$ is defined as

$$P_j(A)[\alpha_i, \delta] = \Big\{ \sum_{r \in I_A} k_{rj} \Big| (k_{1j}, k_{2j}, \ldots, k_{nj}) \in S_{\alpha_i} \Big\}, \qquad (i,j) = (1,\mu), (2,\nu),$$

where $S_{\alpha_i} = \left\{(k_{1j}, k_{2j}, \ldots, k_{nj}) \middle| k_{rj} \in \tilde{k}_{rj}[\alpha_i], \sum_{r=1}^{n} k_{rj} = 1\right\}$ and $I_A = \{i \in (1,2,\ldots,n) \,|\, x_i \in A\}$. Finally, it is shown as

$$\tilde{P}(A) = P(A)(\alpha_1, \alpha_2, \delta) = \Big(P_\mu(A)[\alpha_1, \delta], P_\nu(A)[\alpha_2, \delta]\Big),$$

and the (α_1, α_2)-cut set of the GIF probability is defined by

$$P(A)[\alpha_1, \alpha_2, \delta] = P_\mu(A)[\alpha_1, \delta] \cap P_\nu(A)[\alpha_2, \delta].$$

Theorem 3.1. For all $A \subseteq S$ and $(\alpha_1, \alpha_2) \in L$, $P_j(A)[\alpha_i, \delta]$ is the α_i-cut of the GIFN$_B$ $\tilde{P}(A)$.

Proof . In the first step, we show that $P_j(A)[\alpha_i, \delta]$ are the α_i-cut of the GIFS$_B$, for $i = 1, 2$. For this purpose, we need to prove $P_\mu(A)[\alpha_1, \delta]$ and $P_\nu(A)[\alpha_2, \delta]$ are the α_1-cut of fuzzy set $P_\mu(A)$ and $(1-\alpha_2)$-cut of fuzzy set $P_{1-\nu}(A)$, respectively. Hence, it suffices to show that for $\alpha_i^{(1)} < \alpha_i^{(2)}$,

$$P_\mu(A)\left[\alpha_1^{(2)}, \delta\right] \subseteq P_\mu(A)\left[\alpha_1^{(1)}, \delta\right] \quad \text{and} \quad P_\nu(A)\left[\alpha_2^{(1)}, \delta\right] \subseteq P_\nu(A)\left[\alpha_2^{(2)}, \delta\right].$$

Since $\tilde{k}_r s$ are GIFN$_B$, i.e. $\tilde{k}_{r\mu}[\alpha_1]$ and $\tilde{k}_{r(1-\nu)}[1-\alpha_2]$ are fuzzy numbers, therefore if $\alpha_i^{(1)} < \alpha_i^{(2)}$ then for $r = 1, 2, \ldots, n$,

$$\tilde{k}_{r\mu}[\alpha_1^{(2)}] \subseteq \tilde{k}_{r\mu}[\alpha_1^{(1)}] \quad \text{and} \quad \tilde{k}_{r\nu}[\alpha_2^{(1)}] \subseteq \tilde{k}_{r\nu}[\alpha_2^{(2)}],$$

then from $k_{r\mu} \in \tilde{k}_{r\mu}[\alpha_1^{(2)}]$ it concluded that $k_{r\mu} \in \tilde{k}_{r\mu}[\alpha_1^{(1)}]$, subsequently $k_{r\nu} \in \tilde{k}_{r\nu}[\alpha_2^{(1)}]$ leads to $k_{r\nu} \in \tilde{k}_{r\nu}[\alpha_2^{(2)}]$. Finally $S_{\alpha_1^{(2)}} \subseteq S_{\alpha_1^{(1)}}$ and $S_{\alpha_2^{(1)}} \subseteq S_{\alpha_2^{(2)}}$, that is

$$P_\mu(A)\left[\alpha_1^{(2)}, \delta\right] \subseteq P_\mu(A)\left[\alpha_1^{(1)}, \delta\right],$$

and

$$P_\nu(A)\left[\alpha_2^{(1)}, \delta\right] \subseteq P_\nu(A)\left[\alpha_2^{(2)}, \delta\right].$$

In second step, let $S = \left\{(k_1, k_2, \ldots, k_n) \middle| k_r \in [0,1], \sum_{r=1}^{n} k_r = 1\right\}$, and define $\mathrm{Dom}[\alpha_i] = \prod_{r=1}^{n} \tilde{k}_{rj}[\alpha_i] \cap S$, and $f_i : \mathrm{Dom}[\alpha_i] \to [0,1]$, so

$$f_i\left(a_1^{(j)}, a_2^{(j)}, \ldots, a_n^{(j)}\right) = \sum_{r \in I_A} a_r^{(j)}.$$

As regard f_i is a continuous function and $\mathrm{Dom}[\alpha_i]$ is connected, bounded and closed, therefore, the image of f_i, defined as $\Gamma(\alpha_i) = f_i(\mathrm{Dom}[\alpha_i])$, is a bounded and closed interval of $[0,1] \subset \mathbb{R}$. For each α_i, using the definition of $P_j(A)[\alpha_i, \delta]$, we have $P_j(A)[\alpha_i, \delta] = \Gamma(\alpha_i)$. Hence $P_\mu(A)[1, \delta] \neq \emptyset$ and $P_\nu(A)[0, \delta] \neq \emptyset$, therefore $\tilde{P}(A)$ is a GIFN$_B$. $\square$

Example 3.2. Consider the sample space $S = \{x_1, x_2, x_3, x_4\}$ and $P(X = x_r) = \tilde{k}_r$, $r = 1, 2, 3, 4$, such that

$$\tilde{k}_1 = \Big(0.4, 0.43, 0.45, 0.45, 0.47, 0.5, 1\Big), \qquad \tilde{k}_2 = \Big(0.1, 0.11, 0.12, 0.12, 0.13, 0.15, 1\Big),$$

$$\tilde{k}_3 = \Big(0.2, 0.23, 0.25, 0.25, 0.28, 0.3, 1\Big), \qquad \tilde{k}_4 = \Big(0.13, 0.15, 0.18, 0.18, 0.19, 0.2, 1\Big),$$

the α_i-cuts of $\tilde{k}_r$, for $i = 1, 2$, are represented as

$$k_{1\mu}[\alpha_1] = \Big[0.43 + 0.02\alpha_1, 0.47 - 0.02\alpha_1\Big], \qquad k_{1\nu}[\alpha_2] = \Big[0.45 - 0.05\alpha_2, 0.45 + 0.05\alpha_2\Big],$$

$$k_{2\mu}[\alpha_1] = \Big[0.11 + 0.01\alpha_1, 0.13 - 0.01\alpha_1\Big], \qquad k_{2\nu}[\alpha_2] = \Big[0.12 - 0.02\alpha_2, 0.12 + 0.03\alpha_2\Big],$$

$$k_{3\mu}[\alpha_1] = \Big[0.23 + 0.02\alpha_1, 0.28 - 0.03\alpha_1\Big], \qquad k_{3\nu}[\alpha_2] = \Big[0.25 - 0.05\alpha_2, 0.25 + 0.05\alpha_2\Big],$$

$$k_{4\mu}[\alpha_1] = \Big[0.15 + 0.03\alpha_1, 0.19 - 0.01\alpha_1\Big], \qquad k_{4\nu}[\alpha_2] = \Big[0.18 - 0.05\alpha_2, 0.18 + 0.02\alpha_2\Big].$$

Consider the event $A = \{x_1, x_3\}$, the (α_1, α_2)-cuts of $\tilde{P}(A)$ are provided by

$$P(A)(0,1,1) = \Big(\Big[0.68, 0.74\Big], \Big[0.65, 0.77\Big]\Big), \qquad P(A)(1,0,1) = \Big(\Big[0.7, 0.7\Big], \Big[0.7, 0.7\Big]\Big),$$

$$P(A)(0.2,0.6,1) = \Big(\Big[0.684, 0.732\Big], \Big[0.67, 0.742\Big]\Big), \qquad P(A)(0.5,0.2,1) = \Big(\Big[0.69, 0.72\Big], \Big[0.69, 0.714\Big]\Big).$$

Let X be the success number in n independent Bernoulli trials with the probability of success $\tilde{p}$, where $\tilde{p}$ is GIFN$_B$. Accordingly, the discrete random variable X has the Binomial GIF probability mass function denoted as $P(x, \tilde{p})$, where its crisp counterpart is given as $P(x)$. Then, the α_1-cut set of membership and α_2-cut set of non-membership functions of the GIF probability mass function are defined as

$$P_j(r)[\alpha_i, \delta] = \{P(x) | S_{\alpha_i}\} = \Big[P_j^L(r)[\alpha_i], P_j^U(r)[\alpha_i]\Big],$$

$$P_j^L(r)[\alpha_i] = \min\{P(x) | S_{\alpha_i}\}, \qquad P_j^U(r)[\alpha_i] = \max\{P(x) | S_{\alpha_i}\}.$$

where

$$S_{\alpha_i} = \Big\{(p_j, q_j)\Big| p_j \in p_j[\alpha_i],\ q_j \in q_j[\alpha_i],\ p_j + q_j = 1,\ (i,j) = (1,\mu), (2,\nu)\Big\},$$

$$P(x) = \binom{n}{x} p^x (1-p)^{n-x}, \qquad x = 0, 1, 2, \ldots, n,$$

Finally, it is shown as

$$\tilde{P}(x) = P(x)(\alpha_1, \alpha_2, \delta) = \Big(P_\mu(x)[\alpha_1, \delta], P_\nu(x)[\alpha_2, \delta]\Big),$$

and the (α_1, α_2)-cut set of the GIF probability mass function is defined by

$$P(x)[\alpha_1, \alpha_2, \delta] = P_\mu(x)[\alpha_1, \delta] \cap P_\nu(x)[\alpha_2, \delta].$$

Example 3.3. Suppose the random variable of the number of successes in n independent trials is modeled by Binomial distribution with the GIF parameter $\tilde{p} = (0.08, 0.09, 0.1, 0.1, 0.11, 0.12, 0.5)$. Then cut sets of the GIF probability of $X = 1$ with $\delta = 0.5$ is represented as follows

$$P_j(1)[\alpha_i, 0.5] = \Big\{15p(1-p)^{14} | S_\alpha\Big\} = \Big[15p_j^U[\alpha_i]\left(1 - p_j^U[\alpha_i]\right)^{14}, 15p_j^L[\alpha_i]\left(1 - p_j^L[\alpha_i]\right)^{14}\Big].$$

The α_i-cuts of the GIF parameter are

$$p_\mu[\alpha_1] = \Big[p_\mu^L[\alpha_1], p_\mu^U[\alpha_1]\Big] = \Big[0.09 + 0.01\sqrt{\alpha_1}, 0.11 - 0.01\sqrt{\alpha_1}\Big],$$

$$p_\nu[\alpha_2] = \Big[p_\nu^L[\alpha_2], p_\nu^U[\alpha_2]\Big] = \Big[0.1 - 0.02\sqrt{\alpha_2}, 0.1 + 0.02\sqrt{\alpha_2}\Big].$$

The different cut sets of the GIF probability and membership and non-membership function of the GIF probability are provided respectively in Table 1 and Figure 1. Based on Table 1 and Figure 1, by increasing α_1 and decreasing α_2, the ambiguity decreases in the GIF probability. As we expected, the vagueness in the GIF probability is decreased by increasing α_1 and decreasing α_2, so that, the minimum length of the interval is attained for $\alpha_1 = 1$ and $\alpha_2 = 0$.

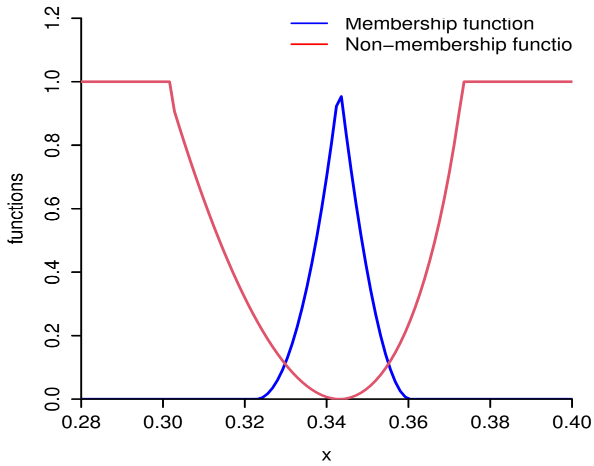

Figure 1: The membership and non-membership functions of the GIF probability.

Table 1: The different cut sets of the GIF probability.

(α_1, α_2)	$P_\mu[\alpha_1]$	$P_\nu[\alpha_2]$	$P[\alpha_1, \alpha_2]$
$(0, 1)$	$[0.3228, 0.3605]$	$[0.3006, 0.3734]$	$[0.3228, 0.3605]$
$(0.05, 0.6)$	$[0.3276, 0.3570]$	$[0.3108, 0.3683]$	$[0.3276, 0.3570]$
$(0.08, 0.5)$	$[0.3288, 0.356]$	$[0.3138, 0.3665]$	$[0.3288, 0.356]$
$(0.1, 0.45)$	$[0.3296, 0.3554]$	$[0.3154, 0.3655]$	$[0.3296, 0.3554]$
$(0.13, 0.4)$	$[0.3304, 0.3547]$	$[0.3171, 0.3644]$	$[0.3304, 0.3547]$
$(0.2, 0.3)$	$[0.3322, 0.3532]$	$[0.3208, 0.3620]$	$[0.3322, 0.3532]$
$(0.4, 0.13)$	$[0.3288, 0.3499]$	$[0.3287, 0.356]$	$[0.3288, 0.3499]$
$(0.45, 0.1)$	$[0.3360, 0.3493]$	$[0.3306, 0.3546]$	$[0.3360, 0.3493]$
$(0.5, 0.08)$	$[0.3374, 0.3486]$	$[0.3319, 0.3534]$	$[0.3374, 0.3486]$
$(0.6, 0.05)$	$[0.3388, 0.3474]$	$[0.3343, 0.3514]$	$[0.3388, 0.3474]$
$(1, 0)$	$[0.3432, 0.3432]$	$[0.3432, 0.3432]$	$[0.3432, 0.3432]$

4 ASS with GIF Parameter

In this section, the SSP of classical attributes characteristics is introduced. An ASP is used to determine how many units can be selected from a lot, or consignment, and how many defective units are allowed in that sample. If the number of defective units is above the predefined number of defective items, the lot is excluded. Suppose we want to check a lot based on the random sample of size n and enumerate the counts of defective items (D). The consignment is accepted if the number of observed defective items (d) is less than or equal to the acceptance number c, otherwise, it is rejected. If the size of the lot is very large, the random variable of the defective items D has a Binomial distribution with parameters n and p, such that p is the rate of the defective units in the lot. If the proportion of the defective items is a GIFN$_B$, then the α_1-cut set of membership and α_2-cut set of non-membership functions of the

GIF acceptance probability is defined as

$$P_j^a[\alpha_i,\delta]=\Big\{P(X\le c)\,|S_{\alpha_i}\Big\}=\Big[P_j^{aL}(c)[\alpha_i],\ P_j^{aU}(c)[\alpha_i]\Big],$$

$$P_j^{aL}(c)[\alpha_i]=\min\Big\{P(X\le c)\,\big|S_{\alpha_i}\Big\},\quad P_j^{aU}(c)[\alpha_i]=\max\Big\{P(X\le c)\,\big|S_{\alpha_i}\Big\}.$$

where

$$\begin{aligned} S_{\alpha_i} &= \Big\{(p_j,q_j)\Big|p_j\in p_j[\alpha_i],\ q_j\in q_j[\alpha_i],\ p_j+q_j=1,\ (i,j)=(1,\mu),(2,\nu)\Big\},\\ P(X\le c) &= \sum_{x=0}^{c}\binom{n}{x}p^x(1-p)^{n-x},\qquad c=0,1,2,\dots,n, \end{aligned}$$

Finally, it is shown as

$$\tilde{p}_a=p_a(\alpha_1,\alpha_2,\delta)=\Big(P_\mu^a[\alpha_1,\delta],P_\nu^a[\alpha_2,\delta]\Big),$$

and the (α_1,α_2)-cut set of the GIF acceptance probability is defined as

$$p_a[\alpha_1,\alpha_2,\delta]=P_\mu^a[\alpha_1,\delta]\cap P_\nu^a[\alpha_2,\delta].$$

Corollary 4.1. If $\delta_1\le\delta_2$ then $p_\mu[\alpha_1,\delta_1]\subset p_\mu[\alpha_1,\delta_2]$ and $p_\nu[\alpha_2,\delta_2]\subset p_\nu[\alpha_2,\delta_1]$, therefore if $P_j^a[\alpha_i,\delta],(i,j)=(1,\mu),(2,\nu)$ be monotone, then $P_\mu^a[\alpha_1,\delta_1]\subset P_\mu^a[\alpha_1,\delta_2]$ and $P_\nu^a[\alpha_2,\delta_2]\subset P_\nu^a[\alpha_2,\delta_1]$, which holds, for instance, for $c=0$ and $c=1$.

Example 4.2. Let the acceptance number in the single sampling plan be zero, and the defective items in the lot are the GIF parameter $\tilde{p}=(0,0.005,0.015,0.017,0.02,0.025,2)$. Then cut sets of the GIF probability of lot acceptance with $n=12$ is provided as follows

$$P_j^a[\alpha_i,2]=\Big\{(1-p)^{12}\,|S_{\alpha_i}\Big\}=\Big[\big(1-p_j^U[\alpha_i]\big)^{12},\big(1-p_j^L[\alpha_i]\big)^{12}\Big],$$

where

$$p_\mu[\alpha_1]=\Big[0.005+0.01\alpha_1^2,0.02-0.003\alpha_1^2\Big],$$

$$p_\nu[\alpha_2]=\Big[0.015-0.015\alpha_2^2,0.017+0.008\alpha_2^2\Big].$$

Hence,

$$P_\mu^a[\alpha_1,2]=\Big[\big(0.98+0.003\alpha_1^2\big)^{12},\big(0.995-0.01\alpha_1^2\big)^{12}\Big],$$

$$P_\nu^a[\alpha_2,2]=\Big[\big(0.983-0.008\alpha_2^2\big)^{12},\big(0.985+0.015\alpha_2^2\big)^{12}\Big].$$

The membership and non-membership functions of the GIF probability of the lot acceptance are given, respectively, as follows

$$\mu_P(x)=\begin{cases}\Big(\dfrac{\sqrt[12]{x}-0.98}{0.003}\Big)^{0.5}, & (0.98)^{12}\le x\le(0.983)^{12}\\ 1, & (0.983)^{12}\le x\le(0.985)^{12}\\ \Big(\dfrac{0.995-\sqrt[12]{x}}{0.01}\Big)^{0.5}, & (0.985)^{12}\le x\le(0.995)^{12}\\ 0, & o.w.\end{cases},$$

$$\nu_P(x)=\begin{cases}\Big(\dfrac{0.983-\sqrt[12]{x}}{0.008}\Big)^{0.5}, & (0.975)^{12}\le x\le(0.983)^{12}\\ 0, & (0.983)^{12}\le x\le(0.985)^{12}\\ \Big(\dfrac{\sqrt[12]{x}-0.985}{0.015}\Big)^{0.5}, & (0.985)^{12}\le x\le 1\\ 1, & o.w.\end{cases}.$$

The membership and non-membership functions of the GIF probability of the lot acceptance with $c = 0$ are given in Figure 2, and different cut sets of the GIF probability of lot acceptance with $c = 0$ are reported in Table 2.

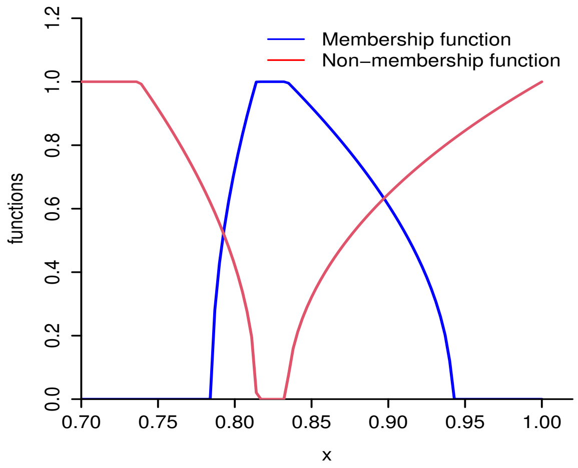

Figure 2: The membership and non-membership functions of the GIF probability of lot acceptance with $c = 0$.

Table 2: The different cut sets of the GIF probability of lot acceptance with $c = 0$.

(α_1, α_2)	$P_\mu^a[\alpha_1, \delta]$	$P_\nu^a[\alpha_2, \delta]$	$P^a[\alpha_1, \alpha_2, \delta]$
$(0, 1)$	$[0.7847, 0.9416]$	$[0.738, 1]$	$[0.7847, 0.9416]$
$(0.2, 0.9)$	$[0.7859, 0.9371]$	$[0.7519, 0.9663]$	$[0.7859, 0.9371]$
$(0.4, 0.7)$	$[0.7893, 0.9236]$	$[0.7759, 0.912]$	$[0.7893, 0.912]$
$(0.5, 0.5)$	$[0.792, 0.9136]$	$[0.7944, 0.873]$	$[0.7944, 0.873]$
$(0.7, 0.4)$	$[0.799, 0.8875]$	$[0.8014, 0.8589]$	$[0.8014, 0.8589]$
$(0.8, 0.3)$	$[0.8034, 0.8715]$	$[0.8069, 0.848]$	$[0.8069, 0.848]$
$(0.9, 0.1)$	$[0.8084, 0.8536]$	$[0.8132, 0.8357]$	$[0.8132, 0.8357]$
$(1, 0)$	$[0.814, 0.8341]$	$[0.814, 0.8341]$	$[0.814, 0.8341]$

The cut sets of the GIF probability of the lot acceptance with $n = 12$ and $c = 1$ are represented as follows

$$\begin{aligned} P_j^a[\alpha_i, 2] &= \left\{ (1 + 11p)(1-p)^{11} | S_{\alpha_i} \right\}, \qquad (i, j) = (1, \mu), (2, \nu), \\ &= \left[\left(1 + 11p_j^U[\alpha_i]\right)\left(1 - p_j^U[\alpha_i]\right)^{11}, \left(1 + 11p_j^L[\alpha_i]\right)\left(1 - p_j^L[\alpha_i]\right)^{11} \right], \end{aligned}$$

where

$$\begin{aligned} p_\mu[\alpha_1] &= \left[p_\mu^L[\alpha_1], p_\mu^U[\alpha_1] \right] = \left[0.005 + 0.01\alpha_1^2, 0.02 - 0.003\alpha_1^2 \right], \\ p_\nu[\alpha_2] &= \left[p_\nu^L[\alpha_2], p_\nu^U[\alpha_2] \right] = \left[0.015 - 0.015\alpha_2^2, 0.017 + 0.008\alpha_2^2 \right]. \end{aligned}$$

The membership and non-membership functions of the GIF probability of the lot acceptance with $c = 1$ are depicted in Figure 3, which has the same shape as Figure 2, with different values of the support x.

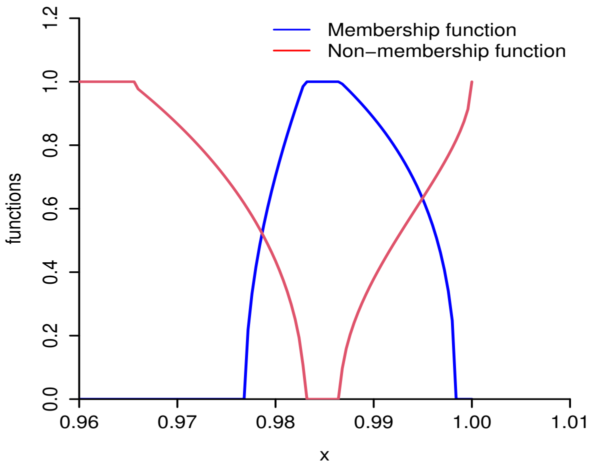

Figure 3: The membership and non-membership functions of the GIF probability of lot acceptance with $c = 1$.

5 GIF Operating Characteristic

One important criterion in the sampling plan is the operating characteristic (OC) curve, which indicates the probabilities of accepting a lot versus the proportion of defective items. Knowing the uncertainty value of the proportion of the defective items and the variation of its position on the x-axis, we have drawn GIF operating characteristic (OC surface) against the y-axis as α_i. If $\tilde{p} = (a_1, a_2, a_3, a_4, a_5, a_6, \delta)$, to achieve this aim, using transformation of the fuzzy proportion of defective items (k), we consider the structure of $\tilde{p}$ as follows

$$\tilde{p}_k = (k, k + b_2, k + b_3, k + b_4, k + b_5, k + b_6, \delta),$$

where $b_i = a_i - a_1, i = 2, 3, \dots, 6$, $0 \leq k \leq 1 - b_6$, and

$$p^L_{k\mu}[\alpha_1] = k + b_2 + (b_3 - b_2)\,\alpha_1^\delta, \qquad p^U_{k\mu}[\alpha_1] = k + b_5 - (b_5 - b_4)\,\alpha_1^\delta,$$
$$p^L_{k\nu}[\alpha_2] = k + b_3 - b_3\alpha_2^\delta, \qquad p^U_{k\nu}[\alpha_2] = k + b_4 + (b_6 - b_4)\,\alpha_2^\delta.$$

Then

$$\tilde{p}^a_k = P^a_{kj}[\alpha_i, \delta] = \{P(X \leq c) | S_{k\alpha_i}\} = \left[P^{aL}_{kj}(c)[\alpha_i], P^{aU}_{kj}(c)[\alpha_i]\right],$$
$$P^{aL}_{kj}(c)[\alpha_i] = \min\left\{\sum_{x=0}^{c}\binom{n}{x}p^x(1-p)^{n-x}|S_{k\alpha_i}\right\}, \qquad P^{aL}_{kj}(c)[\alpha_i] = \min\left\{\sum_{x=0}^{c}\binom{n}{x}p^x(1-p)^{n-x}|S_{k\alpha_i}\right\},$$

where

$$S_{k\alpha_i} = \left\{(p_{kj}, q_{kj})\Big|p_{kj} \in p_{kj}[\alpha_i],\ q_{kj} \in q_{kj}[\alpha_i],\ p_{kj} + q_{kj} = 1,\ (i, j) = (1, \mu), (2, \nu)\right\}.$$

The functions $P^a_{kj}[\alpha_i, \delta]$, $(i, j) = (1, \mu), (2, \nu)$ are two-variates in terms of α_i, $i = 1, 2$ and k. For k_0, $\tilde{p}^a_{k_0}$ is a GIF number. In this method, for every especially α_{10} and α_{20}, shapes of $P^a_{kj}[\alpha_{i0}, \delta], (i, j) = (1, \mu), (2,\ \nu)$ are like bands with upper and lower bounds.

Example 5.1. Let $\tilde{p} = (0.001, 0.005, 0.01, 0.012, 0.017, 0.021, 2)$, then

$$\tilde{p}_k = (k, k + 0.004, k + 0.009, k + 0.011, k + 0.016, k + 0.02, 2), \qquad 0 \leq k \leq 0.98,$$

hence

$$p^L_{k\mu}[\alpha_1] = k + 0.004 + 0.005\alpha_1^2, \quad p^U_{k\mu}[\alpha_1] = k + 0.016 - 0.005\alpha_1^2,$$
$$p^L_{k\nu}[\alpha_2] = k + 0.009 - 0.009\alpha_2^2, \quad p^U_{k\nu}[\alpha_2] = k + 0.011 + 0.009\alpha_2^2.$$

Finally, for $c = 0$, the OC surface is given by

$$P^a_{k\mu}(0)\,[\alpha_1, \delta] = \Big[\left(0.984 - k + 0.005\alpha_1^2\right)^n, \left(0.996 - k - 0.005\alpha_1^2\right)^n\Big],$$

$$P^a_{k\nu}(0)\,[\alpha_2, \delta] = \Big[\left(0.989 - k - 0.009\alpha_2^2\right)^n, \left(0.991 - k + 0.009\alpha_2^2\right)^n\Big].$$

The OC surface for $n = 5$ and 10, with $c = 0$, are plotted in Figure 4 and the OC bands for $n = 5$ and 10, with $(\alpha_1, \alpha_2) = (0, 1), c\ = 0$, are represented in Figure 5. Based on Figure 5, increasing k leads to decreasing of the OC membership and non-membership bands, for $n = 5$ and $n = 10$.

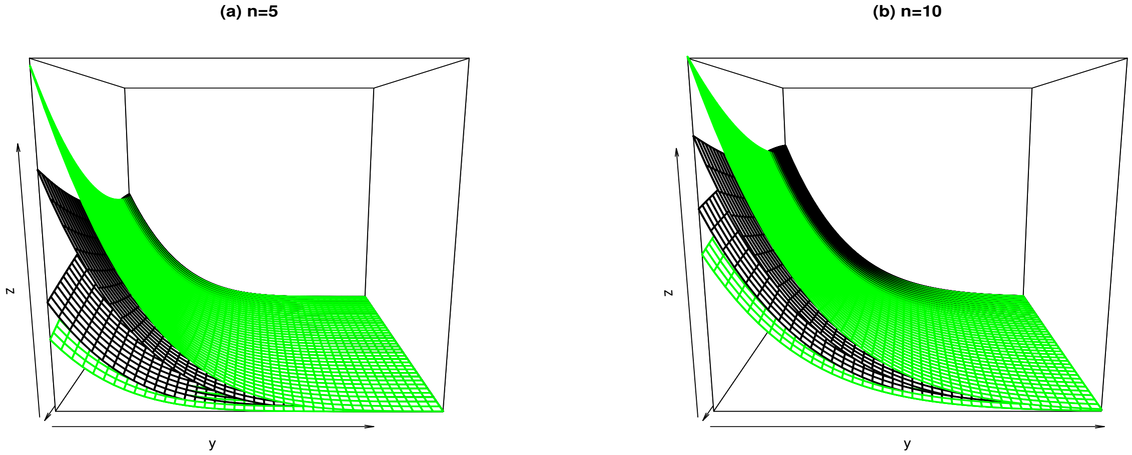

Figure 4: The OC surface for $n = 5$ and 10 with $c = 0$.

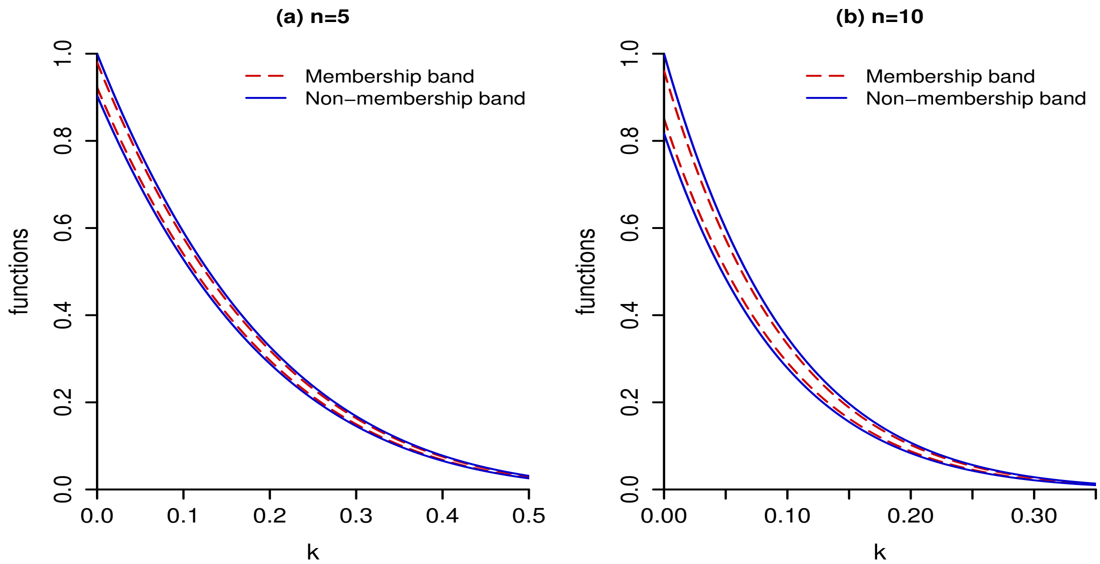

Figure 5: The OC bands for (a) $n = 5$ and (b) $n = 10$ with $(\alpha_1, \alpha_2) = (0, 1), c\ = 0$.

Considering different values of k and $n = 5, 10$, the cut sets of OC surface with $(\alpha_1, \alpha_2) = (0, 1)$ and $c = 0$ are provided in Table 3. As can be seen, by increasing the values of k, the length of the intervals is decreased, which leads to more precise results.

Table 3: The cut sets of OC surface for different values of k with $(\alpha_1, \alpha_2) = (0, 1)$ and $c = 0$.

n	k	$P^a_{k\mu}(0)\,[0,2]$	$P^a_{k\nu}(0)\,[1,2]$	$\tilde{p}^a_k$
	0	$[0.8510, 0.9607]$	$[0.8171, 1]$	$[0.8510, 0.9607]$
	0.01	$[0.7684, 0.8685]$	$[0.7374, 0.9044]$	$[0.7684, 0.8685]$
10	0.03	$[0.6244, 0.7076]$	$[0.5987, 0.7374]$	$[0.6244, 0.7076]$
	0.05	$[0.5052, 0.5740]$	$[0.4840, 0.5987]$	$[0.5052, 0.5740]$
	0.07	$[0.4069, 0.4636]$	$[0.3894, 0.4840]$	$[0.4069, 0.4636]$
	0	$[0.9225, 0.9802]$	$[0.9039, 1]$	$[0.9225, 0.9802]$
	0.01	$[0.8763, 0.9319]$	$[0.8587, 0.9510]$	$[0.8763, 0.9319]$
5	0.03	$[0.7902, 0.8412]$	$[0.7738, 0.8587]$	$[0.7902, 0.8412]$
	0.05	$[0.7108, 0.7576]$	$[0.6957, 0.7738]$	$[0.7108, 0.7576]$
	0.07	$[0.6379, 0.6809]$	$[0.6240, 0.6957]$	$[0.6379, 0.6809]$

For $c = 1, n = 10$, the operating surface is given by

$$P^a_{k\mu}(1)\,[\alpha_1, 2] = \Big[\left(1 + 9\left(k + 0.016 - 0.005\alpha_1^2\right)\right)\left(0.984 - k + 0.005\alpha_1^2\right)^9, \left(1 + 9\left(k + 0.004 + 0.005\alpha_1^2\right)\right) \left(0.996 - k - 0.005\alpha_1^2\right)^9 \Big],$$

$$P^a_{k\mu}(1)\,[\alpha_1, 2] = \Big[\left(1 + 9\left(k + 0.011 + 0.009\alpha_2^2\right)\right)\left(0.989 - k - 0.009\alpha_2^2\right)^9, \left(1 + 9\left(k + 0.009 - 0.009\alpha_2^2\right)\right) \left(0.991 - k + 0.009\alpha_2^2\right)^9 \Big].$$

The GIF operating bands for $n = 10$ with $(\alpha_1, \alpha_2) = (0, 1)$ and $c = 1$ is illustrated in Figure 6, such that by increasing k, the membership and non-membership bands of the GIF operating are decreasing functions.

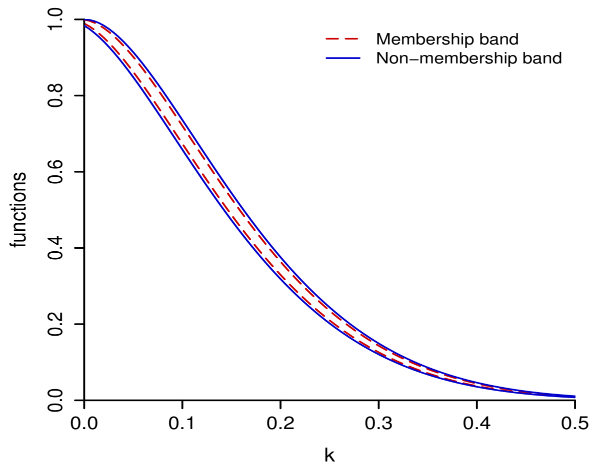

Figure 6: The GIF operating bands for $n = 10$ with $(\alpha_1, \alpha_2) = (0, 1)$, $c = 1$.

Conclusion

In the present paper, the acceptance single sampling plan has been successfully designed using Binomial distribution in the GIF condition. In our approach, based on the effect of parameters vagueness in the value assigned to acceptance probability, the probability is obtained through GIFN$_B$, and it is compatible with the method proposed by Baloui Jamkhaneh et al. [9] for the fuzzy environment. The GIF operating characteristic surfaces are designed for different combinations of sample sizes and parameters. Also, the cut sets of the GIF operating characteristic are two-variate functions in terms of α_i and k. For k_0, they are the GIF numbers, and for every especially α_{10} and α_{20}, they are like a band with upper and lower bounds. The proposed sampling plan can be extended to neutrosophic statistics distributions in future research.

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