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Solution of time fractional Swift-Hohenberg equation by Aboodh transform homotopy perturbation method

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Abstract

In this study, the Aboodh transform homotopy perturbation method has been used to solve fractional differential equations. The Swift–Hohenberg equation accurately explains the creation and evolution of patterns in a wide range of systems. The Swift-Hohenberg (S-H) model is linked to fluid dynamics, temperature, and thermal convection. The problem's convergence analysis is displayed. All of the Surface representation graphs are also represented using MAPLE. ATHPM solution has been compared with the exact solution and LADM to check the accuracy of the proposed method.

Keywords: Swift Hohenberg (S-H) equation, fractional differential equation, Aboodh transform, Homotopy perturbation method
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1 Introduction

In recent years, there have been numerous advancements in the field of solving nonlinear differential equations. Many nonlinear differential equations have no analytic solution. In this paper, we solve the Swift Hohenberg (S-H) model using the Aboodh transform homotopy perturbation method (ATHPM). The S-H equation was first introduced and derived from the equations for thermal convection by J. Swift and P. Hohenberg. [22]. The S–H equation was first offered as a phenomenological comparison to thermodynamic systems by analyzing phase transitions in Rayleigh-Bénard convection behavior. Still, it immediately became clear that it was a highly approximate model for nonlinear pattern production. The importance of the S–H equation stems from its ability to produce results nearly identical to those of the Navier–Stokes equations, which can be difficult to solve numerically. It's worth mentioning that it's been used in a wide range of subjects, including biology, economics, optics, sociology, and fluid dynamics, to name a few. In its most basic form, the S-H equation is [12, 15].

$$\frac{\partial^\alpha \omega}{\partial \xi^\alpha} = b\omega - (1 + \frac{\partial^2}{\partial \chi^2})\omega - N(\omega), \xi > 0 \tag{1.1}$$

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where α is the order of the fractional derivative, ω is a scalar function, b is constant, and N(ω) is a nonlinear element. In various disciplines of thermal physics, the S-H equation is crucial [4, 5, 8, 11, 16]. Many applications of the S-H equation can be found in engineering and research, including physics, biology, laser study fluid, and hydrodynamics [7, 18, 21]. In fluid layers restricted between horizontal well-conducting boundaries, the S-H equation plays a significant role in pattern creation theory[13].

In this study, we combine two powerful techniques, the Aboodh transform and the homotopy perturbation method (HPM), to generate approximate analytic solutions of nonlinear differential equations, with excellent agreement with existing methods such as q-HATM [24], Iterative method [12], and ETDM [15]. Many researchers have utilised various methodologies and strategies to examine the SH equation, including Homotopy analysis [13], perturbation method [18], and residual power series method [19]. ATHPM has been shown to produce correct approximation solutions to a variety of ordinary, partial, and fractional differential equations, whether linear or non-linear [9].

2 Aboodh transform

We consider functions in the set A given by

$$A = \{f(\xi) : \exists M, k_1, k_2 > 0, |f(\xi)| < Me^{-v\xi}\}. \tag{2.1}$$

For a given function in the set A, the constant M must be constant, k_1, k_2 may be finite or infinite. The Aboodh transform of function $f(\xi)$ is defined by

$$A[f(\xi)](v) = k(v) = \frac{1}{v}\int_0^\infty f(\xi)e^{-v\xi}d\xi, \xi \geq 0, k_1 \leq v \leq k_2. \tag{2.2}$$

The variable v in this transform is used to factor the variable ξ in the argument of the function f. The Aboodh transform is defined for functions of exponential order.

$f(\xi)$	$A[f(\xi)]$
1	$\frac{1}{v^2}$
ξ	$\frac{1}{v^3}$
ξ^n	$\frac{n!}{v^{n+2}}$
$e^{a\xi}$	$\frac{1}{v^2-av}$
$\sin a\xi$	$\frac{a}{v(v^2+a^2)}$
$\cos a\xi$	$\frac{1}{v^2+a^2}$
$f^{(n)}(\xi)$	$v^{(n)}k(v) - \prod_{k=0}^{n-1} \frac{f^{(k)}(0)}{v^{2-n+k}}$

Table 1: Aboodh transform of some functions

3 Aboodh transform Homotopy perturbation method in fractional order S-H equation

The time fractional model of Swift Hohenberg (S-H) is

$$\frac{\partial^\alpha \omega}{\partial \xi^\alpha} = b\omega - (1 + \frac{\partial^2}{\partial \chi^2})\omega - N(\omega), \tag{3.1}$$

where $\omega(0, \chi)$ is any function of χ. Apply Aboodh transform on equation (3.1),

$$A[\frac{\partial^\alpha \omega}{\partial \xi^\alpha}] = A[\omega u - (1 + \frac{\partial^2}{\partial \chi^2})\omega - N(\omega)].$$

By Aboodh transform for fractional order,

$$v^\alpha\{k(v) - \frac{1}{v^2}\omega(\chi, 0)\} = A[\omega u - (1 + \frac{\partial^2}{\partial \chi^2})\omega - N(\omega)]$$

which can be written as

$$k(v) = \frac{1}{v^2}\omega(\chi, 0) + \frac{1}{v^\alpha} A[\omega u - (1 + \frac{\partial^2}{\partial \chi^2})\omega - N(\omega)].$$

Now, by taking inverse Aboodh transform

$$\omega(\chi, \xi) = \omega(\chi, 0) + A^{-1}[\frac{1}{v^\alpha} A(\omega u - (1 + \frac{\partial^2}{\partial \chi^2})\omega - N(\omega)].$$

Assuming the solution is of the form

$$\omega = \omega_0 + pu_1 + p^2 u_2 + \cdots .$$

To consider the nonlinear operator, we apply HPM

$$\sum_{n=0}^{\infty} p^n \omega_n(\chi, \xi) = \omega(\chi, 0) + pA^{-1}[\frac{1}{v^\alpha} A(R \sum_{n=0}^{\infty} p^n H_n(\chi, \xi) - N \sum_{n=0}^{\infty} p^n H_n(\chi, \xi))].$$

Comparing coefficients of powers of p both sides, we get

$$p^0 : \omega_0(\chi, \xi) = \omega(\chi, 0)$$

$$p^1 : \omega_1(\chi, \xi) = -A^{-1}[\frac{1}{v^\alpha} A(R\omega_0(\chi, \xi) - Nu_0(\chi, \xi))]$$

$$p^2 : \omega_2(\chi, \xi) = -A^{-1}[\frac{1}{v^\alpha} A(R\omega_1(\chi, \xi) - Nu_1(\chi, \xi))]$$

$$p^3 : \omega_3(\chi, \xi) = -A^{-1}[\frac{1}{v^\alpha} A(R\omega_2(\chi, \xi) - Nu_2(\chi, \xi))]$$

$$p^4 : u_4(\chi, \xi) = -A^{-1}[\frac{1}{v^\alpha} A(R\omega_3(\chi, \xi) - Nu_3(\chi, \xi))]$$

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Approximate solution can be obtained as,

$$\omega(\chi, \xi) = \omega_0 + \omega_1 + \omega_2 + \omega_3 + ...$$

4 Analysis of Convergence

Theorem 4.1. Suppose that X and Y be Banach space and $N : X \to Y$ is a contraction nonlinear mapping, that is [10]

$$\forall \omega, \bar{\omega}; \|N(\omega) - N(\bar{\omega})\| \leq \gamma \|\omega - \bar{\omega}\|, 0 < \gamma < 1$$

which according to Banach's fixed point theorem, having the fixed point ω, that is $N(\omega) = \omega$ The sequence generated by the homotopy perturbation method will be regarded as

$$\omega_n = N(\omega_{n-1}), \omega_{n-1} = \sum_{i=0}^{n-1} \omega_i, n = 1, 2, 3, ...$$

and suppose that $\omega_0 \in B_r(\omega)$ where $B_r(\omega) = \{\omega^* \in X / \|\omega^* - \omega\| < r\}$ then we have the following statements
(1) $\|\omega_n - \omega\| \leq \gamma^n |\omega_0 - \omega\|$,
(2)$\omega_n \in B_r(\omega)$,
(3) $\lim \omega_n = \omega$.

Definition 4.2. For all $i \in N \cup \{0\}; \gamma_i$ can be obtain as [10]

$$\gamma_i = \begin{cases} \frac{\omega_{i+1}}{\omega_i} & \text{if } \|\omega_i\| \neq 0 \\ 0 & \text{if } |\omega_i\| = 0 \end{cases}$$

Corollary 4.3. $\sum_{i=0}^{\infty} \omega_i$ converges to exact solution ω, whem $0 \leq \gamma_i < 1, i = 1, 2, 3, ...$ [10].

5 Some illustrations

Example 5.1.

$$\frac{\partial^{\alpha}\omega(\chi,\xi)}{\partial\xi^{\alpha}}+(1-b)\omega(\chi,\xi)+2\frac{\partial^{2}\omega(\chi,\xi)}{\partial\chi^{2}}+\frac{\partial^{3}\omega(\chi,\xi)}{\partial\chi^{3}}=0$$

with initial condition $\omega(\chi,0)=e^{\chi}$

$$\frac{\partial^{\alpha}\omega(\chi,\xi)}{\partial\xi^{\alpha}}=-(1-b)\omega(\chi,\xi)-2\frac{\partial^{2}\omega(\chi,\xi)}{\partial\chi^{2}}-\frac{\partial^{3}\omega(\chi,\xi)}{\partial\chi^{3}}.$$

Apply Aboodh transform

$$A[\frac{\partial^{\alpha}\omega}{\partial\xi^{\alpha}}]=-A[(1-b)\omega(\chi,\xi)+2\frac{\partial^{2}\omega(\chi,\xi)}{\partial\chi^{2}}+\frac{\partial^{3}\omega(\chi,\xi)}{\partial\chi^{3}}]$$

$$v^{\alpha}\{k(\chi,\xi)-\frac{1}{v^{2}}\omega(\chi,0)\}=-A[(1-b)\omega(\chi,\xi)+2\frac{\partial^{2}\omega(\chi,\xi)}{\partial\chi^{2}}+\frac{\partial^{3}\omega(\chi,\xi)}{\partial\chi^{3}}]$$

$$k(\chi,\xi)=\frac{1}{v^{2}}\omega(\chi,0)-\frac{1}{v^{\alpha}}A[(1-b)\omega(\chi,\xi)+2\frac{\partial^{2}\omega(\chi,\xi)}{\partial\chi^{2}}+\frac{\partial^{3}\omega(\chi,\xi)}{\partial\chi^{3}}].$$

Apply inverse Aboodh transform

$$\omega(\chi,\xi)=\omega(\chi,0)-A^{-1}[\frac{1}{v^{\alpha}}A((1-b)\omega(\chi,\xi)+2\frac{\partial^{2}\omega(\chi,\xi)}{\partial\chi^{2}}+\frac{\partial^{3}\omega(\chi,\xi)}{\partial\chi^{3}})].$$

By HPM

$$\omega_0=\omega(\chi,0)=e^{\chi}$$

$$p^{1}:\omega_1=-A^{-1}[\frac{1}{v^{\alpha}}A((1-b)\omega_0+2\frac{\partial^{2}\omega_0}{\partial\chi^{2}}+\frac{\partial^{3}\omega_0}{\partial\chi^{3}})]=\frac{(b-4)e^{\chi}\xi^{\alpha}}{\alpha!}$$

$$p^{2}:\omega_2=-A^{-1}[\frac{1}{v^{\alpha}}A((1-b)\omega_1+2\frac{\partial^{2}\omega_1}{\partial\chi^{2}}+\frac{\partial^{3}\omega_1}{\partial\chi^{3}})]=\frac{(b-4)^{2}e^{\chi}\xi^{2\alpha}}{(2\alpha)!}$$

$$p^{3}:\omega_3=-A^{-1}[\frac{1}{v^{\alpha}}A((1-b)\omega_2+2\frac{\partial^{2}\omega_2}{\partial\chi^{2}}+\frac{\partial^{3}\omega_2}{\partial\chi^{3}})]=\frac{(b-4)^{3}e^{\chi}\xi^{3\alpha}}{(3\alpha)!}$$

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$$\omega_n=\frac{(b-4)^{n}e^{\chi}\xi^{n\alpha}}{(n\alpha)!}.$$

Approximate solution can be obtained as,

$$\omega(\chi,\xi)=\omega_0+\omega_1+\omega_2+\omega_3+...+\omega_n+\cdots.$$

$$\omega(\chi,\xi)=e^{\chi}+\frac{(b-4)e^{\chi}\xi^{\alpha}}{\alpha!}+\frac{(b-4)^{2}e^{\chi}\xi^{2\alpha}}{(2\alpha)!}+\frac{(b-4)^{3}e^{\chi}\xi^{3\alpha}}{(3\alpha)!}+...\frac{(b-4)^{n}e^{\chi}\xi^{n\alpha}}{(n\alpha)!}+... \tag{5.1}$$

which is approximate solution of Example 5.1 obtained by ATHPM. Applying convergence analysis (for b=5,$\alpha=1$), we have

$$\gamma_0=\frac{\|\omega_1\|}{\|\omega_0\|}=0.050<1$$

$$\gamma_1=\frac{\|\omega_2\|}{\|\omega_1\|}=0.0250<1$$

$$\gamma_2=\frac{\|\omega_3\|}{\|\omega_2\|}=0.01667<1.$$

Hence, we can say that $\sum_{i=0}^{\infty}\omega_i$ is convergent. Therefore the approximate solution $\omega(\chi,\xi)$ is convergent.

The case $\alpha=1$ is convert fractional order S-H equation into classical S-H equation and it has the solution

$$\omega(\chi,\xi)=e^{\chi+(b-4)\xi}$$

which is similar solution to the Elzaki Transform Decomposition Method (ETDM) [15] and Iterative Method [12].

χ	$\xi=0.02$		$\xi=0.04$		$\xi=0.06$		$\xi=0.08$		$\xi=0.1$	
	ATHPM	EXACT	ATHPM	EXACT	ATHPM	EXACT	ATHPM	EXACT	ATHPM	EXACT
0	0.9231163465	0.9231163464	0.8521437890	0.8521437890	0.7866278608	0.7866278611	0.7261490345	0.7261490371	0.6703200305	0.6703200460
0.1	1.020201340	1.020201340	0.9417645335	0.9417645336	0.8693582349	0.8693582354	0.8025187950	0.8025187980	0.7408182034	0.7408182207
0.2	1.127496852	1.127496852	1.040810773	1.040810774	0.9607894387	0.9607894392	0.8869204333	0.8869204367	0.8187307338	0.8187307531
0.3	1.246076730	1.246076731	1.150273798	1.150273799	1.061836546	1.061836547	0.9801986703	0.9801986733	0.9048373972	0.9048374180
0.4	1.377127765	1.377127764	1.271249150	1.271249150	1.173510870	1.173510871	1.083287065	1.083287068	0.9999999770	1
0.5	1.521961556	1.521961556	1.404947590	1.404947591	1.296930087	1.296930087	1.197217359	1.197217363	1.105170892	1.105170918
0.6	1.682027650	1.682027650	1.552707218	1.552707219	1.433329413	1.433329415	1.323129808	1.323129812	1.221402730	1.221402758
0.7	1.858928042	1.858928042	1.716006862	1.716006862	1.584073984	1.584073985	1.462284584	1.462284589	1.349858776	1.349858808
0.8	2.054433210	2.054433211	1.896480879	1.896480879	1.750672500	1.750672500	1.616074395	1.616074402	1.491824663	1.491824698
0.9	2.270499838	2.270499838	2.095935515	2.095935514	1.934792334	1.934792334	1.786038424	1.786038431	1.648721232	1.648721271
1	2.509290388	2.509290390	2.316366976	2.316366977	2.138276217	2.138276220	1.973877726	1.973877732	1.822118756	1.822118800

Table 2: Comparision of ATHPM Solution with Exact Solution at $b=0$

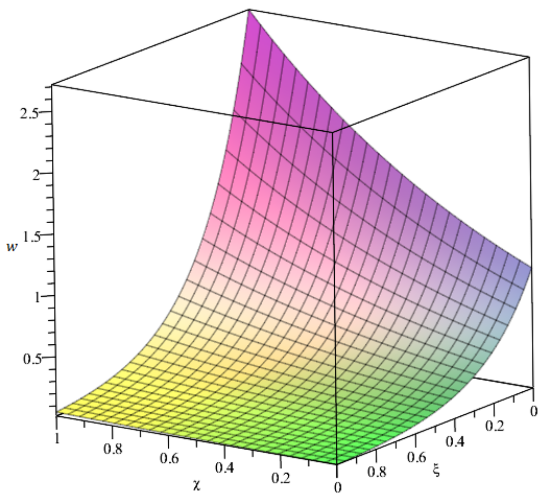

Figure 1: ATHPM solution at $b=0$ & $\alpha=1$

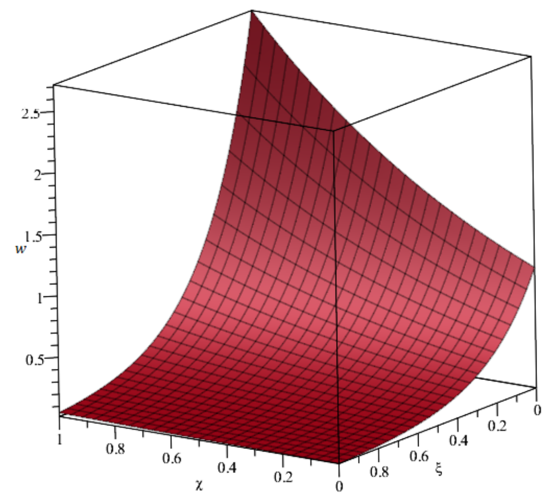

Figure 2: Exact solution $\omega(\chi,\xi)=e^{\chi+(b-4)\xi}$ at $b=0$

Figures 1 and 2 are generated by maple software. Here figure-1 represents the 3-D surface of an approximate solution obtained by the ATHPM method at $b=0$ and $\alpha=1$ of example-5.1. figure-2 represents an exact solution at $b=0$ and $\alpha=1$ of example-5.1. Table 2 represents comparision between the ATHPM solution and the exact solution. From table-3, it can be measured that the ATHPM solution is very accurate because very small absolute error can be negligible and it can be measured from figures 1 and 2 also.

Example 5.2.

$$\frac{\partial^\alpha \omega(\chi,\xi)}{\partial \xi^\alpha}+(1-b)\omega(\chi,\xi)+2\frac{\partial^2\omega(\chi,\xi)}{\partial\chi^2}+\frac{\partial^4\omega(\chi,\xi)}{\partial\chi^4}+\omega^3(\chi,\xi)=0$$

χ	ξ=0.02			ξ=0.06			ξ=0.1		
	ATHPM	Exact Solution	Absolute error	ATHPM	Exact Solution	Absolute error	ATHPM	Exact Solution	Absolute error
0	0.9231163465	0.9231163464	1×10^{-10}	0.7866278608	0.7866278611	8.8×10^{-9}	0.6703200305	0.6703200460	3.096×10^{-7}
0.1	1.020201340	1.020201340	1×10^{-11}	0.8693582349	0.8693582354	9.6×10^{-9}	0.7408182034	0.7408182207	3.42×10^{-7}
0.2	1.127496852	1.127496852	1×10^{-11}	0.9607894387	0.9607894392	1.06×10^{-8}	0.8187307338	0.8187307531	3.778×10^{-7}
0.3	1.246076730	1.246076731	1×10^{-9}	1.061836546	1.061836547	1.1×10^{-8}	0.9048373972	0.9048374180	4.18×10^{-7}
0.4	1.377127765	1.377127764	1×10^{-9}	1.173510870	1.173510871	1.3×10^{-8}	0.9999999770	1.0000000000	4.62×10^{-7}
0.5	1.521961556	1.521961556	1×10^{-11}	1.296930087	1.296930087	1.5×10^{-8}	1.105170892	1.105170918	5.1×10^{-7}
0.6	1.682027650	1.682027650	1×10^{-11}	1.433329413	1.433329415	1.5×10^{-8}	1.221402730	1.221402758	5.64×10^{-7}
0.7	1.858928042	1.858928042	1×10^{-11}	1.584073984	1.584073985	1.7×10^{-8}	1.349858776	1.349858808	6.23×10^{-7}
0.8	2.054433210	2.054433211	1×10^{-9}	1.750672500	1.750672500	2×10^{-8}	1.491824663	1.491824698	6.88×10^{-7}
0.9	2.270499838	2.270499838	1×10^{-11}	1.934792334	1.934792334	2.2×10^{-8}	1.648721232	1.648721271	7.61×10^{-7}
1	2.509290388	2.509290390	2×10^{-9}	2.138276217	2.138276220	2.4×10^{-8}	1.822118756	1.822118800	8.41×10^{-7}

Table 3: Absolute error of Example 5.1 at $b=0$

with initial condition $\omega(\chi,0)=\frac{1}{10}\sin(\frac{\pi\chi}{k})$.

$$\frac{\partial^\alpha\omega(\chi,\xi)}{\partial\xi^\alpha}=-(1-b)\omega(\chi,\xi)-2\frac{\partial^2\omega(\chi,\xi)}{\partial\chi^2}-\frac{\partial^4\omega(\chi,\xi)}{\partial\chi^4}-\omega^3(\chi,\xi).$$

Apply Aboodh transform

$$A[\frac{\partial^\alpha\omega}{\partial\xi^\alpha}]=A[-(1-b)\omega(\chi,\xi)-2\frac{\partial^2\omega(\chi,\xi)}{\partial\chi^2}-\frac{\partial^4\omega(\chi,\xi)}{\partial\chi^4}-\omega^3(\chi,\xi)]$$

$$v^\alpha\{k(\chi,\xi)-\frac{1}{v^2}\omega(\chi,0)\}=A[-(1-b)\omega(\chi,\xi)-2\frac{\partial^2\omega(\chi,\xi)}{\partial\chi^2}-\frac{\partial^4\omega(\chi,\xi)}{\partial\chi^4}-\omega^3(\chi,\xi)]$$

$$k(\chi,\xi)=\frac{1}{v^2}\omega(\chi,0)+\frac{1}{v^\alpha}A[-(1-b)\omega(\chi,\xi)-2\frac{\partial^2\omega(\chi,\xi)}{\partial\chi^2}-\frac{\partial^4\omega(\chi,\xi)}{\partial\chi^4}-\omega^3(\chi,\xi)].$$

Apply inverse Aboodh transform

$$\omega(\chi,\xi)=\omega(\chi,0)+A^{-1}[\frac{1}{v^\alpha}A(-(1-b)\omega(\chi,\xi)-2\frac{\partial^2\omega(\chi,\xi)}{\partial\chi^2}-\frac{\partial^4\omega(\chi,\xi)}{\partial\chi^4})-\omega^3(\chi,\xi)].$$

By HPM

$$\omega_0=\omega(\chi,0)=\frac{1}{10}\sin(\frac{\pi\chi}{k})$$

$$p^1:\omega_1=A^{-1}[\frac{1}{v^\alpha}A(-(1-b)\omega_0-2\frac{\partial^2\omega_0}{\partial\chi^2}-\frac{\partial^4\omega_0}{\partial\chi^4}-\omega_0^3)]=\sin(\frac{\pi\chi}{k})[\frac{2\pi^2}{10k^2}-\frac{(1-b)}{10}-\frac{\pi^4}{10k^4}-\frac{sin^2(\frac{\pi\chi}{k})}{1000}]\frac{\xi^\alpha}{\alpha!}$$

$$p^2:\omega_2=A^{-1}[\frac{1}{v^\alpha}A(-(1-b)\omega_1-2\frac{\partial^2\omega_1}{\partial\chi^2}-\frac{\partial^4\omega_1}{\partial\chi^4}-3\omega_0^2\omega_1)].$$

Therefore,

$$\begin{aligned}\omega_2=&\sin(\frac{\pi\chi}{k})[\frac{(3)(11)(2473)}{800000}-\frac{2\pi^2}{5k^2}+\frac{291\pi^4}{500k^4}-\frac{2\pi^6}{5k^6}+\frac{\pi^8}{10k^8}-\frac{101b}{500}+\frac{2\pi^2b}{5k^8}-\frac{\pi^4b}{2k^8}-b^2\\&-\frac{1}{200000w^4}(-2400\pi^2k^2+8400\pi^4+k^4(403-400b))\cos(\frac{2\pi\chi}{k})+3k^8\cos(\frac{4\pi\chi}{k})]\frac{\xi^{2\alpha}}{(2\alpha)!}\end{aligned}$$

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Applying convergence analysis (for b=1,$\alpha=1$,k=10), we have

$$\gamma_0=\frac{\|\omega_1\|}{\|\omega_0\|}=0.01226<1$$

$$\gamma_1=\frac{\|\omega_2\|}{\|\omega_1\|}=0.007<1.$$

Hence, we can say that $\sum_{i=0}^{\infty}\omega_i$ is convergent. Therefore the approximate solution $\omega(\chi,\xi)$ is convergent. Approximate solution can be obtained as,

$$\omega(\chi,\xi)=\omega_0+\omega_1+\omega_2+\omega_3+\cdots+\omega_n+\cdots.$$

Therefore

$$\begin{aligned}\omega(\chi,\xi)=&\sin(\frac{\pi\chi}{k})[\frac{1}{10}+(\frac{2\pi^2}{10k^2}-\frac{(1-b)}{10}-\frac{\pi^4}{10k^4}-\frac{sin^2(\frac{\pi\chi}{k})}{1000})\frac{\xi^\alpha}{\alpha!}\\&+(\frac{(3)(11)(2473)}{800000}-\frac{2\pi^2}{5k^2}+\frac{291\pi^4}{500k^4}-\frac{2\pi^6}{5k^6}+\frac{\pi^8}{10k^8}-\frac{101b}{500}+\frac{2\pi^2b}{5k^8}-\frac{\pi^4b}{2k^8}-b^2\\&-\frac{1}{200000k^4}(-2400\pi^2k^2+8400\pi^4+k^4(403-400b))\cos(\frac{2\pi\chi}{k})+3k^8\cos(\frac{4\pi\chi}{k}))\frac{\xi^{2\alpha}}{(2\alpha)!}]\end{aligned}$$

which is similar solution to the Homotopy analysis transform method [24].

Here Figure 3 represents the 3-D surface of a solution obtained by the ATHPM method, and Figure 4 represents an exact solution of Example 5.2. Table 4 represents comparison between the ATHPM solution and the LADM solution.

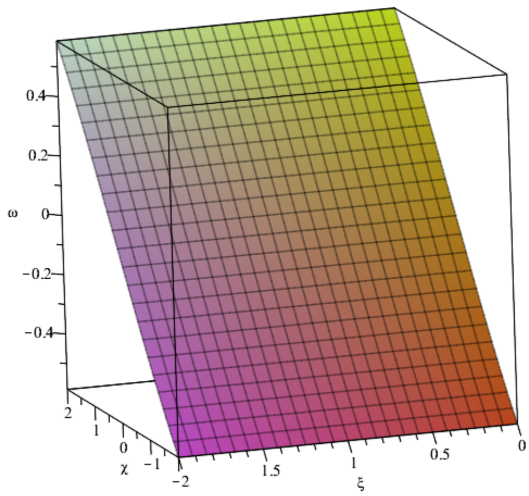

Figure 3: surface represents ATHPM solution $\omega(\chi, \xi)$ for $b = 1, k = 10$ and $\alpha = 1$

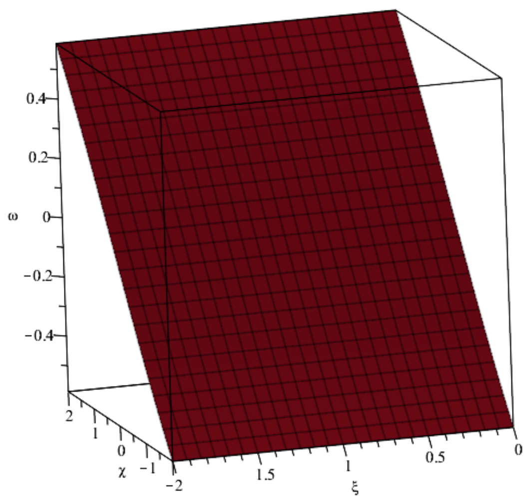

Figure 4: surface represents exact solution $\omega(\chi, \xi)$ for $b = 1, k = 10$ and $\alpha = 1$

	ξ	$\alpha = 0.85$		$\alpha = 0.95$		$\alpha = 1$	
		ATHPM	LADM	ATHPM	LADM	ATHPM	LADM
χ=1	0	0.309017	0.309017	0.309017	0.309017	0.309017	0.309017
	0.125	0.308681	0.308679	0.030878	0.030875	0.0308792	0.0308781
	0.25	0.309419	0.309409	0.0308532	0.0308502	0.0308514	0.0308546
	0.375	0.309153	0.309158	0.0308254	0.0308261	0.0308356	0.0308311
	0.5	0.307788	0.307792	0.0308017	0.0308024	0.0308082	0.0308077
	0.625	0.309456	0.309469	0.0307361	0.0307790	0.0307876	0.0307843
	0.75	0.309237	0.309252	0.0307525	0.0307561	0.0307813	0.0307810
	0.875	0.307067	0.307040	0.0307303	0.0307333	0.0307365	0.0307377

Table 4: Comparison of ATHPM $\omega(\chi, \xi)$ at $k = 10, b = 0.9$ with LADM for different values of α

6 Conclusion

The Aboodh transformation homotopy perturbation method (ATHPM) was successfully used to discover a solution to the fractional model of the Swift Hohenberg problem. The Aboodh transform exceeds all other techniques studied in the literature. It is limited to overcoming nonlinear problems; hence for the nonlinear terms in the shown situations, the homotopy perturbation method (HPM) is used. To conclude, the solution to the S-H equation allows us to investigate a variety of nonlinear situations. The strength of ATHPM is its simplicity and capacity to provide a high-precision solution to nonlinear systems. Every problem includes a convergence analysis, which concludes that all series solutions are convergent, and convergent solutions are similar to other analytical methods such as ETDM, Iterative method and q-HATM. ATHPM solution has been compared with LADM and exact solution and concluded that ATHPM solution has high accuracy and future researchers can be used to solve further nonlinear problems which occur in real world problems.

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