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Behavioral approach in multi-period portfolio optimization using genetic algorithm

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Abstract

This paper discusses a multi-period portfolio optimization problem by considering a conditional value-at-risk (CVaR) constraint Based on prospect theory, which considers the loss-averse utility, the transaction cost and the lower bound and upper bound investment in each asset. A genetic algorithm is proposed to solve the portfolio model. The results based on the average optimal ultimate wealth and Sharp ratio criteria showed that loss-averse investors tend to concentrate most of their wealth and perform better than rational investors. The impact of CVaR on investment performance was identified. When the market falls, investors with higher risk aversion avoid extreme losses and obtain more gains.

Keywords: Loss aversion, Multi-period optimization, Conditional value at risk, Genetic algorithm optimization
2020 MSC: 34Nxx, 60J70, 68W50

1 Introduction

Determining the portfolio according to classical theories can lead to a scenario in which the investors changes the composition of their portfolio in response to short market fluctuations and observation of losses. By repeating these changes in asset allocation, negative long-term consequences are inevitable and ultimately the result of this process is lower than optimal consequences. Because of the desires and goals of the investors are not taken into account. Therefore, it is necessary to identify behavioral biases such as loss aversion, mental accounting and asymmetric risk-seeking before designing and implementing the asset portfolio selection model to avoid these problems. Choosing the right investment portfolio, in a way that achieves the investor's goals and in line with their interests, leads to the reduction of immediate behaviors without intellectual support and creates psychological security for investors, even in falling markets. In single-period portfolio selection models, it is assumed that the investor's preferences are constant and the portfolio composition does not change until the end of investment horizon. However, due to the impact of each period from previous periods and the existence of transaction costs, the multi-period optimization model is used. For institutional and individual investors who are concerned about the impact of business risk and psychological issues on investment choice behaviors. In classical financial theories, investors have always assumed risk aversion who want maximize the concave utility function. The problem of selecting classic portfolio is based on the theory of expected utility, which arises from the risk aversion and rationality of investors [17].

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Risk aversion as one of the most important aspects of investor behavior has always been considered in research. However, in the field of behavioral finance, since the issue of portfolio selection is considered with regard to psychological issues such as loss aversion, regret aversion, ambiguity aversion, etc., the risk aversion aspect of investors has not been considered. Numerous studies show that investors behave irrationally and contrary to the expected utility theory. [11] study from perspective of behavioral economics and propose a prospect theory. This theory is based on several assumptions: 1. People make investment decisions based on changes in wealth rather than total wealth. 2. Investors are not always risk averse and show risk-seeking behavior in losses. 3. Objective weights are replaced by mental weights [14].

[11] stated that the utility function is S-shaped and not its traditional concave form. This theory describes asymmetric attitudes toward gains and losses, which show loss aversion behavior of investors and that avoiding loss is stronger than the desire for gain.

The issues of asset selection and portfolio are financial theories that have always been discussed. In recent years, loss aversion is receiving more interests. Several researchers introduce loss aversion in portfolio selection models and study optimal investment decision of investors. [10] Presented a single-period portfolio selection model with S-shaped utility functions by considering loss aversion. [19], study the Market Equilibrium Behavior of prospect investors with Reference Dependence and Loss Aversion Characteristics. They argued that preferences based on prospect theory, can lead investors to limited trading behavior in a market interaction process. [13], examined the effect of myopic loss aversion on investment decisions in financial markets. They argued that higher myopic loss-aversion is associated with investing in fewer stocks. In most of these researches, the issue of choosing a single-period portfolio with constant loss aversion has been investigated.

Another topic of interest in recent years in the field of portfolio selection is multi-period portfolio optimization. One of the factors that have led to more attention to this issue is that in the real world, investors are constantly evaluating and reallocating their assets due to changing market conditions and changing their preferences, which is ignored in single-period models. Several studies have shown that economic conditions change over time, and investors in a changing economic environment tend to adjust their asset allocation and make multi-period investment decisions [1, 4]. Due to the cost of transactions and uncertainty of risky assets, single-period models cannot well describe the intermediate investment of investors. The degree of loss aversion of investors also depends on the results of their previous investments. This means that loss aversion, as a function of investment performance, changes over time [1]. Accordingly, [20] conducted an empirical analysis and concluded that investors' past returns or losses have asymmetric effects on their loss-aversion and investment behavior. [7], studied the issue of a linear asset allocation of loss-aversion investor and concluded that by dynamically updating the loss-aversion parameters, the performance of loss-aversion portfolios is significantly improved. [16], proposed an optimization model based on prospect theory, which was solved using a particle swarm algorithm. [9], investigated portfolio optimization based on prospect theory using genetic algorithms. [4] used particle swarm algorithm to solve the model based on prospect theory and concluded that the particle swarm algorithm is highly efficient in solving the model. Most research on loss-aversion investor portfolio optimization examines the maximizing expected utility, and different investor attitudes toward risk are less considered. As mentioned, loss-averse investors are risk-averse in gains and risk-seek in the losses. Although [8], empirically shows that loss-aversion investors in large and excessive losses do not become more risk-seeking, but risk aversion is always present in gains. Therefore, there is a relationship between utility and risk, and investors who seek to maximize the utility of loss aversion, avoid excessive losses and exhibit risk aversion behaviors.

As a result, it is essential to keep investors' risk within a safe range. Investors with higher risk aversion avoid excessive losses and make more profit. Since investors are risk averse in large losses. There are two motivations for this research. First, modeling the loss-aversion coefficient and the reference point that change over time as a function of the previous wealth of investors, according to the work [7]. Then, we create a multi-period portfolio model to examine their effects. In this model we maximize loss aversion utility and keep investment risk in a limited threshold to avoid severe losses. Second, a Genetic algorithm is used to solve the model.

The remaining part of this paper is organized as follows: In section2, we create a multiperiod loss aversion portfolio model by considering CVaR constraints, the transaction cost and the lower bound and upper bound investment in each asset. section3, design and creation of genetic algorithms which is used to solve the model. Section4, a numerical experiment of Tehran Stock Exchange is presented. Section5, the results are discussed. In Section 6. We gave the concluding remarks.

Table 1: Summery of the parameters

Parameter	Meaning	value	refrence
kti	unit transaction cost when the investor buys or sells asset i in the tth period	0.8%	Assumed
$W0$	Initial wealth	1	Assumed
r_t^f	initial reference point in the tth period,	0.017	Assumed
λ_0	Initial loss aversion period	2.25	Kahneman and Tversky,1979
α	return curvature	0.88	Kahneman and Tversky,1979
β	loss curvature	0.88	Kahneman and Tversky,1979
τ	confidence level of CVaR	0.95	Krokhmal, J. Palmquist, and S. Uryasev,2002
ω	risk thresholds	$\{0.3, 0.5, 0.7\}$	Assumed
u_i	Upper bound	0.8	Assumed
l_i	Lower bound	0.01	Assumed

2 Mukti-period portfolio model based on prospect theory

2.1 Utility function

According to prospect theory by [11], the S-shaped utility function for prospect theory (PT) is defined as follows:

$$\varphi(y) = (y-u)^{\alpha}\varepsilon_{y\geq u} - \lambda(u-y)^{\beta}(1-\varepsilon_{y\geq u}) \tag{2.1}$$

where

$$\varepsilon_{y\geq u} = \begin{cases} 1 & y \geq u \\ 0 & y < u \end{cases} \tag{2.2}$$

Y is a Portfolio return rate and $u \in R$ are the reference point that determines the gains and losses. u is equal to the risk-free interest rate, which is usually the bank deposit rate. With multi-period investment terms, it is assumed that the investor enters the financial market with the initial wealth w_0 and allocates this wealth to n risky and risk-free asset. The total investment horizon includes T periods and the portfolio is adjusted in each period. w_t is wealth at the beginning of period t, and x_{it} is the weight of assets $i \in \{1, .., n\}$ in the $t \in \{1, .., T\}$ period after adjusting the portfolio. $\Delta x_{i,t}$ is the weight of asset i in the t period that $\Delta x_{i,t} > 0(\Delta x_{i,t} < 0)$ represents the buy (sell) operations. $k_{i,t}$ is the transaction cost at the time of buying or selling the i share in period t. Therefore, the expected net return rate of the portfolio is calculated by removing transaction costs using the following formula:

$$y_t = \sum_{i=1}^{n} r_{it}x_{it} - \sum_{i=1}^{n} k_{it}\Delta x_{it} \tag{2.3}$$

Assuming that λ_t and u_t are the loss aversion coefficient and the reference point in the t period, respectively, so the PT function in the t period is written as follows:

$$\varphi(y_t) = (y_t-u_t)^{\alpha}\varepsilon_{y_t\geq u_t} - \lambda_t(u_t-y_t)^{\beta}(1-\varepsilon_{y_t\geq u_t}) \tag{2.4}$$

[6], state that the reference point and loss aversion coefficient of investors change with changes in past losses and gains. $\lambda_t \geq \lambda_0 \geq 0$ where λ_0 is the initial loss aversion coefficient of investors. $0 \leq u_t \leq r_t^f$ is the risk-free rate of return at time t. On the multi-period investment horizon, λ_0 is the lowest loss aversion factor and r_t^f is the smallest reference point. Therefore, the dynamic function of the loss aversion coefficient and the reference point are defined as follows:

$$\lambda_t == \begin{cases} \lambda_0 & y_t \geq y_{t-1} \\ \lambda_0 + (\dfrac{y_{t-1}}{y_t} - 1) & y_t < y_{t-1} \end{cases} \tag{2.5}$$

$$u_t == \begin{cases} r_t^f(\frac{y_{t-1}}{y_t}) & y_t \geq y_{t-1} \\ r_t^f & y_t < y_{t-1} \end{cases} \tag{2.6}$$

where y_t is the portfolio return rate in period t. Given the above two equations, we use w_t as an alternative to y_t . As a result, the dynamic function of loss aversion coefficient and the modified reference point are used as follows, and wealth w_t is used as a substitute for y_t [15].

$$\lambda_t^{'} == \begin{cases} \lambda_0 & w_t \geq w_{t-1} \\ \lambda_0 + (\frac{w_{t-1}}{w_t} - 1) & w_t < w_{t-1} \end{cases} \tag{2.7}$$

$$u_t^{'} == \begin{cases} r_t^f(\frac{w_{t-1}}{w_t}) & w_t \geq w_{t-1} \\ r_t^f & w_t < w_{t-1} \end{cases} \tag{2.8}$$

As a result, the multi-period PT utility function is written as follows:

$$\sum_{t=1}^{T} \varphi(y_t) = \sum_{t=1}^{T}((y_t - u_t^{'})^{\alpha}\varepsilon_{y_t \geq u_t^{'}} - \lambda_t(u_t^{'} - y_t)^{\beta}(1 - \varepsilon_{y_t \geq u_t^{'}})) \tag{2.9}$$

2.2 Multiperiod CVaR

[18], proposed a measure called conditional value at risk for measuring risk. Accordingly, conditional value at risk is defined as the average of risks that exceeding the risk value. Conditional value at risk is a coherent measure of risk. And is defined as follows:

$$CVaR = \frac{1}{1-\alpha}\int_{f(x,y)\geq\alpha} f(x,y)p(y)dy \tag{2.10}$$

X is the portfolio weight vector, y is the portfolio return rate, and f (x, y) is the portfolio loss function, $p(y)$ is the density function y, and α is the confidence level. Therefore, the multi-period Conditional value at risk function is written as follows, The symbols in this function are: $X = (x_1, \ldots, x_t, ..., x_T)$ is weight sequence vector at time t , η_t, value $VaR_\alpha(t)$ at confidence level α and $\eta = (\eta_1, \ldots, \eta_t, \ldots, \eta_T)$. Therefore, the equation is written as follows:

$$CVaR(T) = \phi(X,\eta) = \sum_{t=1}^{T}\frac{1}{1-\alpha}\int_{f_t(x_t,y_t)} f_t(x_t,y_t)p(y)dy \tag{2.11}$$

where $f_t(x_t, y_t)$ is the portfolio loss function at time t. Based on the Conditional value at risk auxiliary function, the above equation is written as follows [2]:

$$CVaR(T) = \sum_{t=1}^{T}(\eta_t + \frac{1}{(1-\alpha)q})\sum_{j=1}^{q}(f_t(x_t, y_t^j) - \eta_t)^+) \tag{2.12}$$

where the loss function at time t, $f_t(x_t, y_t^j) = -x_t^{'}r_t^j$ and r_t^j as jth return rate vector that occurs at time t. Its elements are the return rate of each asset r_{it} and $j \in \{1, 2, \ldots, q\}$. Let $h_{jt} = (-x_t^{'}r_t^j - \eta_t)^+$ therefore, the above relation is written as follows:

$$CVaR(T) = \sum_{t=1}^{T}(\eta_t + \frac{1}{(1-\alpha)q})\sum_{j=1}^{q}(h_{jt}) \tag{2.13}$$

$$Where\ h_{jt} \geq -x_t^{'}r_t^j - \eta_t \tag{2.14}$$

$$and\ h_{jt} \geq 0 \tag{2.15}$$

CVaR constraint defined as

$$CVaR_\alpha(T) \leq \omega \tag{2.16}$$

By creating q types of possible returns $r_t^j, (j = 1, 2, \ldots, q)$ (stock behavior is simulated in the t period. Q scenario is for the future of the portfolio each year. In order to generate this q scenario for three risk stocks, we consider the probability density function of the simultaneous rate of return of these three risky assets equal to the normal multivariate distribution with mean μ and covariance $\sum$. In this way, the future behavior of three risky assets is simulated for the next three periods each year. By holding each scenario constant, the model is solved. Where (2.16) is the constraint for the sum of $CVaR$ in T periods and ω is a risk threshold indicating the risk constraint limit. As decreases, the degree of risk-averse increases; (2.14) and (2.15) are the constraints on h_{jt}.

2.3 Self-financing

Without losing of the generality, the assumption of the whole investment process is self-financing and the investor is not allowed to add or withdraw during the investment period. Similar to [16], in general, the transaction cost of risk-free assets is equal, so the transaction cost is zero. $\Delta x_{i,t}$ is adjusting the value (relative to total funds) of investing in risky assets in the tth period. $k_{i,t}$ is the cost of risky asset transactions in the tth period. This is considered equal to 0.8%. Based on the above topics, the self-financing constraint is written as follows:

$$\sum_{i=0}^{n} \Delta x_{i,t} + \sum_{i=1}^{n} k_{i,t} |\Delta x_{i,t}| = 0, \quad t = 1, ..., T \tag{2.17}$$

2.4 Short selling

According to [5], the short selling constraint in the model means that the adjusted value of the asset is a non-negative value. Therefore, the construction of short selling is as follows:

$$M\{x_{it} < 0\} = 0 \quad i = 0, \ldots, n, t = 1, ..., T \tag{2.18}$$

2.5 Upper bound and lower bound

In the real world, investors may set up or low investment rates for each asset. which is known as the portfolio relative diversity constraints [3]. Where u_i and l_i denote the upper nd lower bounds on the position of asset i in the portfolio model, respectively.

$$l_i \leq \frac{(x_{it})}{\sum_{i=0}^{n}(x_{it})} \leq u_i \quad i = 0, \ldots, n, t = 1, ..., T \tag{2.19}$$

As a result, the model is written as follows:

We assume that the investor's goal is to maximize loss-aversing utility and minimize excessive loss over the T period. He also considers self-financing constraint, short sell constraint, and up and low bound constraint. As a result, the model can be written as the following multi-period model:

$$\begin{cases} max\ U_T = \sum_{t=1}^{T} u(y_t) \\ s.t. CVaR_\alpha(T) \leq \omega \\ h_{jt} \geq -x_t^{'} r_t^j - \eta_t \\ h_{jt} \geq 0 \\ x_{it} = \dfrac{1 + r_{(t-1)i}}{1 + \sum_{i=1}^{n} r_{(t-1)i} x_{(t-1)i}} x_{(t-1)i} + \Delta x_{i,t} \\ w_{(t+1)} = w_{(t)}(1 + y_t) \\ \sum_{i=1}^{n} x_{it} = 1 \\ \sum_{i=0}^{n} \Delta x_{i,t} + \sum_{i=1}^{n} k_{i,t} |\Delta x_{i,t}| = 0, \quad t = 1, ..., T \\ M\{x_{it} < 0\} = 0 \\ l_i \leq \dfrac{(x_{it})}{\sum_{i=0}^{n}(x_{it})} \leq u_i \end{cases} \tag{2.20}$$

2.6 Multi period mean-CVaR model

The benchmark model is Markowitz mean-variance model that meets the CVaR criterion. Therefore, we use this model to review and compare the results of the prospect model. Objective function is to maximize returns. The only difference between this model and the above model is that the goal is defined as follows:

$$max\ V_T = \sum_{t=1}^{T} y_t \tag{2.21}$$

3 Genetic algorithm

Genetic algorithm is one of the most famous intelligent optimization algorithms that has provided acceptable results. Richenberg first presented in 1960 in a study of evolutionary strategy. Many researchers later studied his theory until John Holland proposed the Genetic Algorithm [**?**]. Because the optimization model is non-convex, it cannot be solved efficiently using liner, nonlinear or other or other mathematical programming methods. For this purpose, intelligent optimization algorithm derived from natural phenomena or social behaviors are used to solve complex and multidimensional problems, which usually provide optimal solutions in less time. According to the research background, Genetic Algorithm (GA) is used to solve the model. It originates from the two theories of genetics and evolution. Other algorithms may work as well but are out of the scope of this article due to the length restrictions.

3.1 Chromosome

Each chromosome contains the necessary information to examine and solve the problem. Chromosomes are made up of smaller units called genes. Genetic algorithm is a random (probabilistic) algorithm. The initial response (primary chromosomes) plays an important role in the final optimal response. In this study, the answer to the variable x in part 2.2. is presented by a chromosome denoted as $X = [X_{11}, \dots, X_{1n}; \dots; X_{t1}, \dots, X_{tn}; X_{T1}, \dots, X_{Tn}]$ That genes are $X_{t1}, X_{t2}, \dots, X_{tn}$ In the range $[0, 1] t \in \{1, 2, \dots, T\}$.

3.2 Population

The initial population is created as follows: 1) The number $T \times n$ produces a random value of y_{ti} in the range $[0, 1]$ and the matrix with T rows and n columns is formulated as follows: $[y_{ti}]_{T\times n} = [y_{11}, ..., y_{1i}, \dots, y_{1n}; \dots; y_{t1}, ..., y_{ti}, \dots, y_{tn}; y_{T1}, \dots$

2) $X_{ti} = Y_{ti} / \sum_{i=1}^{n} y_{ti}$.

3) Repeat steps 1 and 2 for M times to generate the initial population.

3.3 Crossover operation

A binary genetic algorithm based on the roulette wheel is used to determine the next generation chromosome selection system. In the Crossover operator, I generate a random number of chromosomes, and if the random number kth is less than the rate of the Crossover, chromosome i is selected as a parent. We convert the two parent chromosomes X_1 and X_2 into two children Z_1 and Z_2 as follows:

$$Z_1 = \beta.X_1 + (1 - \beta).X_2 \tag{3.1}$$
$$Z_2 = \beta.X_2 + (1 - \beta).X_1 \tag{3.2}$$

By repeating the above operations N times, we can get a population of new offspring population. In this article, we set up $P_c = 0.8$.

3.4 Mutation operation

The purpose of mutation in GAs is to introduce diversity into the sampled population. Mutation operators are used in an attempt to avoid local minimums by preventing the population of chromosomes from becoming too similar to each other, thereby reducing or even stopping global optimal convergence. A parameter P_m is predefined as a mutation probability of GA first. In this article we use uniforms Mutation operator to perform mutation operations. A chromosome is randomly selected, $X_s = [X_{1i}, \dots, X_{1n}; \dots; X_t1, \dots, X_{ti}; X_{T1}, \dots, X_{Ti}; \dots; X_{Tn}]$. The probability of mutation is used to mutate a gene X_{ti} from the chromosome X_s to gene X^*_{ti} in the range $[0, 1]$. Then, the other genes except X^*_{ti} that belonging to the tth row are changed to $X^{**}_{tj} = X^*_{tj}(1 - X^{**}_{tj})/(1 - X^*_{ti})$. we set up $Pm = 0.1$.

4 Computational investigations

In this section, we examine the performance of an optimal asset portfolio that mentioned above. The optimal portfolio weight x_t^* , optimal ultimate wealth and sharp ratio [7] of model (2-20) are calculated. To compare the PT model with classical portfolio approaches [17] We also show the results of the benchmark optimization models. As we know, the M-CVaR model is a modified form the mean-variance model presented in [17], which is considered CVaR constraint as a PT model (2-20). Therefore, we compare these two models to reveal the impact of CVaR constraint on the PT model. Also the Tehran Stock Exchange (TSE) index, which shows the market trends used as an evaluation criterion for investment performance. So the benchmark models include a multi-period model constraint by M-CVaR model (2-20) that utility function is replaced with (2-21) and TSE index.

4.1 Data

We have three risky assets and one non-risk asset in our portfolio. Risky assets are metals, Petroleum products; Automobile sector index of TSE index, non-risky asset is monthly return rate of Central Bank bonds in the period under review. The period of 2009 to 2020 is considered for the purpose of extracting historical data, which the years 2009 to 2015 as the in sample period and 2016 to 2020 as the out of-sample period. Each investment horizon T includes 4 months. Investors reallocate wealth at the end of each period. We use $r_{i,t} = \frac{P_{it}}{P_{it-1}} - 1$ to drive the return rate of assets i in the tth period, that P_{it} is the closing price of asset i at the end of the tth month. The transaction cost is considered 0.8%. The initial reference point is the return rate of bank deposits for three months equal to 10% and the initial wealth is 1.

4.2 Model solution steps

1. Using support vector regression method with genetic algorithm (GA-SVR), the expected return rates of all assets over the out of sample horizon are estimated. The return rates of all assets from April 2009 to March 2016 are used to simulate the monthly return rates from April 2016 to April 2020.
2. The average optimal portfolio weight under the multi-period PT and M-CVaR models with CVaR constraint using risk threshold ω using MATLAB software, GA algorithm and expected return rates in the first step, is calculated. Loss aversion coefficient $\lambda = 2.25$ and $\alpha = \beta = 0.88$ [11], and the confidence level in CVaR, 95% [12] and the risk threshold level, which indicates the risk constraint as a percentage of initial wealth (0.3, 0.5, 0.7).
3. We calculate the average optimal ultimate wealth and Sharp ratio of the multi period PT model and the M-CVaR model. Based on the expected returns estimated in the first step, and the optimal portfolio weights estimated in the second step, the average of the optimal ultimate wealth and the Sharp ratio of the PT model, the M-CVaR model and the TSE index are calculated and compared them.
4. for check the strength of the model, the initial loss coefficient and the initial reference point are changed and the stability test of the model is performed.

5 Discussion of result

In order to transfer the constraints to the objective function, the penalty factor method has been used. A set of scenarios is designed.

5.1 Optimal portfolio weights

Table 2 shows the average weight of the optimal portfolio in both the PT M-CVaR models. As the results show, by increasing the risk threshold (ω), the weight of non- risk asset decreases. At similar values for ω, the results show that in PT model, asset allocation is more concentrated. And rational investors are so conservative that they tend to diversify the allocation of their assets to expand the risk.

5.2 Optimal ultimate wealth and sharp ratio

The above table shows the average of the optimal ultimate wealth and the Sharp ratio for both models and the index. In 2018 and 2019, the ultimate wealth of the index had the highest value, and this result shows that the market was rising. With similar values of ω, the ultimate wealth for the PT model is less than the M-CVaR model and TSE index, while it has a higher Sharp ratio than the other two models. In 2017, the stock market experienced a period

Table 2: Optimal portfolio weights for the two models: PT and M-CVaR

	assets	PT			M-CVaR		
		ω=0.3	ω=0.5	ω=0.7	ω=0.3	ω=0.5	ω=0.7
2016	Metals	0.049	0.062	0.096	0.175	0.154	0.224
	Petroleum products	0.064	0.129	0.147	0.103	0.136	0.157
	Automobile	0.245	0.230	0.253	0.139	0.221	0.269
	Non-Risk	0.642	0.579	0.504	0.583	0.489	0.350
2017	Metals	0. 02	0.096	0.125	0.111	0.270	0.152
	Petroleum products	0.103	0.131	0.168	0.101	0.131	0.217
	Automobile	0.292	0.313	0.341	0.292	0.252	0.310
	Non-Risk	0.585	0.460	0.366	0.496	0.347	0.321
2018	Metals	0.082	0.093	0.057	0.242	0.185	0. 249
	Petroleum products	0.145	0.314	0.288	0.259	0.331	0.315
	Automobile	0.334	0.208	0.354	0.128	0.205	0.192
	Non-Risk	0.439	0.385	0.301	0.371	0.279	0.244
2019	Metals	0.024	0.069	0.162	0.276	0.340	0.301
	Petroleum products	0.253	0.141	0.112	0.119	0.168	0.310
	Automobile	0.419	0.498	0.511	0.321	0.294	0.273
	Non-Risk	0.304	0.292	0.215	0.284	0.198	0.116

Table 3: Optimal ultimate wealth and sharp ratio ($u_0 = 0.1, \lambda_0 = 2.25$)

	assets	PT			M-CVaR			TSE index
		ω=0.3	ω=0.5	ω=0.7	ω=0.3	ω=0.5	ω=0.7	
2016	Ultimate Wealth	1.070	1.004	0.995	1.022	0.996	0.905	0.985
	Sharp Ratio	1.412	0.232	0.147	0.114	0.005	-0.103	-0.567
2017	Ultimate Wealth	1.229	1.578	1.843	1.150	1.369	1.212	1.169
	Sharp Ratio	1.664	1.787	2.091	0.981	1.001	1.622	2.549
2018	Ultimate Wealth	1.343	1.700	2.009	1.042	1.664	1.986	1.485
	Sharp Ratio	2.581	3.786	4.534	2.454	3.080	3.709	2.072
2019	Ultimate Wealth	1.441	1.909	2.327	1.268	1.775	2.102	1.997
	Sharp Ratio	6.168	5.892	4.850	5.926	5.135	4.587	5.746

of rise and falls, and as a result, the ultimate wealth index is a little more than one. With similar ω, in this year the ultimate wealth and sharp ratio of the PT model is higher than the classical model and TSE index. In 2016, the ultimate wealth of the TSE index is less than one, which shows that the stock market has been falling. With similar ω, the ultimate wealth and Sharp ratio PT model is higher than the other two models.

6 Conclusion and future work

With attention to the issues of psychology and risk in selecting an investment, the two issues of risk management and financial-behavioral in selecting portfolio should be considered. One of the important topics that have been paid less attention is the study of the effect of different psychological features in behavioral finance, including loss aversion, ambiguity aversion and etc. on investment select compared to classical models of rational investment select. Numerous studies show that the investment behavior of investors is irrational and follows the expected utility theory. [11] Examined investor behavior from the perspective of behavioral economics and propose a prospect theory (PT). According to this theory, the S-shaped utility function replaces the classical concave utility function. The core of PT theory is that investors are loss averse and also this theory explains asymmetric attitudes to gains and losses. According to this theory, investors in the gains are risk-averse and have a concave utility function, and in the losses are risk-seek and are a convex utility function. But [8] empirically shows that loss-averse investors always risk averse in the gains, but does not always risk seek in the face of extreme losses. As a result, investors seeking to maximize the loss aversion utility avoid extreme losses and display risk-averse behaviors. Therefore, there is a relationship between utility and risk, and it is essential that risk be kept within a safe range and extreme losses are avoided.

In this research, maximizing the expected utility based on prospect theory and at the same time attitudes toward risk is considered. [17] In his research shows that loss-averse investors in the face of large and extreme losses be risk averse again. It is therefore necessary to limit the risk to prevent extreme losses. According to the mentioned topics, the multi-period PT model has been presented and tested with the aim of maximizing the multi-period utility and limiting the investment risk, to prevent extreme losses.

Conditional value at risk was added as a constraint in the model and a Genetic algorithm was used to solve the model. Using the data of Tehran Stock Exchange, the performance of the PT model and the mean-CVaR model have been studied. The results show that loss-averse investors tend to allocate their wealth to fewer assets than rational investors. Rational investors are conservative and tend to diversify assets to share risk. As the degree of risk aversion increases, the weight of risk-free assets increases. In 2016 and 2017, due to the negative return on risky assets, often in lower amounts ω or higher degree of risk aversion, loss-averse investors to increase wealth and prevent losses, invest more of their wealth in risk-free assets. This has led to more optimal ultimate wealth and a sharp ratio of loss-averse investors than rational investors. As ω decreases in 2018 and 2019, when the market has been rising, optimal Ultimate wealth and sharp ratio are also decrease.

In 2016, when the market is falling, with the increase of ω, the optimal ultimate wealth and the Sharp ratio decrease. This shows the effect of conditional value at risk on the performance of loss-averse investor, especially in a falling market. Loss-averse investors with conditional value at tend to allocate more of their wealth to lower-risk assets to avoid extreme losses. loss-averse investors with a higher degree of risk aversion can avoid large losses and have an optimal ultimate wealth and a higher Sharp ratio. In a situation where the market is rising, the optimal ultimate wealth of loss-averse investors is less.

Despite the complexities of the real world, it is not possible to conduct a comprehensive study of the real situation. So the models are somewhat abstract. In other words, modeling is based on assumptions that simplify the situation compared to the real world. One of the advantages of this work is achieving important and significant relationships and results, and one of the disadvantages is distancing from reality. One of these assumptions in the present study is to consider the normal distribution for the return on risky assets. It is suggested that in future research, the model be reviewed using the expected shortfall constraint and the systematic expected shortfall as a substitute for conditional value at risk. The investment horizon in this research is one year and the investment periods are four months. This research can also be examined in shorter or longer investment horizons.

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