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Bicontinuous biseparating operators on Orlicz spaces in the context of hypergroups

Seyyed Mohammad Tabatabaie*, Mahdi Latifpour

Department of Mathematics, University of Qom, Qom, Iran

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Abstract

In this paper, first, we study bicontinuous biseparating left multipliers on Orlicz algebras in the context of a compact hypergroup and give some formula for them. Also, we assume that Φ is a Δ_2-regular Young function with $\Phi \in \Delta'$ (globally) which is not equivalent to $|x|^2$, and prove that if there is an isometry algebra isomorphism between convolution Orlicz algebras $L^\Phi(G_1)$ and $L^\Phi(G_2)$, then the underlying locally compact groups G_1 and G_2 are isomorphic.

Keywords: locally compact hypergroup, locally compact group, Orlicz algebra, convolution, biseparating operator, left multiplier
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1 Introduction

In [10, 23, 6, 16, 21] the authors prove that for each $1 \leq p < \infty$ ($p \neq 2$) and locally compact groups G_1 and G_2 (compact if $p \neq 1$), if there is a bipositive or isometric isomorphism from $L^p(G_1)$ onto $L^p(G_2)$, then G_1 and G_2 are isomorphic; see also [24, 25, 26]. In [27] and [8], some similar facts for weighted group algebras are given. Y. Kuznetsova and S. Zadeh in [13] prove that if there is a bicontinuous biseparating isomorphism between two weighted Lebesgue algebras, then their underlying locally compact groups are isomorphic. Recently, in [22] we consider Orlicz algebras in the context of locally compact groups and prove that for each Young function Φ with $\Phi \in \Delta_2 \cap \Delta'$, if there exists a bicontinuous biseparating algebra isomorphism between convolution Orlicz algebras $L^\Phi(G_1)$ and $L^\Phi(G_2)$, then G_1 and G_2 are isomorphic. S. Degenfeld-Schonburg and R. Lasser in [3, Theorem 1] proved that if K is a commutative hypergroup and T is a bounded linear operator on $L^1(K)$, then T is a left multiplier if and only if there exists a unique measure $\mu \in \mathcal{M}(K)$ such that $T(f) = \mu * f$ for all $f \in L^1(K)$. Locally compact hypergroups are extensions of locally compact groups which were introduced in [9, 4, 20]. See [2] as a monograph on hypergroups. In this paper, first we study bicontinuous biseparating operators on Orlicz spaces in the context of hypergroups, and show that if K is a compact hypergroup and M is a bicontinuous biseparating left multiplier on the convolution Orlicz algebra $L^\Phi(K)$, then there exist a number $c \in \mathbb{C}$ and a positive measure $\mu \in \mathcal{M}(K)$ such that $Mf = c\, \mu * f$ for all $f \in L^\Phi(K)$, where $\Phi \in \Delta'$ is a Young function such that $x \leq \Psi(x)$ for all $x \geq 0$ in which Ψ is the complementary of Φ. Also, we prove that if $L^\Phi(G_i)$ is a convolution Orlicz algebra for $i = 1, 2$, and there is an isometry algebra isomorphism between $L^\Phi(G_1)$ and $L^\Phi(G_2)$, then G_1 and G_2 are isomorphic, where G_1 and G_2 are two locally compact groups, Φ is a Δ_2-regular Young function with $\Phi \in \Delta'$ (globally) and Φ is not equivalent to $|x|^2$. For convenience of readers, in

*Corresponding author
Email addresses: sm.tabatabaie@qom.ac.ir (Seyyed Mohammad Tabatabaie), m.latifpour@stu.qom.ac.ir (Mahdi Latifpour)

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Section 2 we recall some preliminaries regarding locally compact hypergroups and Orlicz spaces. See [1, 5, 11, 12, 19] as some recent works on this topic.

2 Preliminaries

2.1 Locally Compact Hypergroups

Let K be a locally compact Hausdorff space. We denote the space of all bounded Radon measures on K by $\mathcal{M}(K)$, and the set of non-negative elements of $\mathcal{M}(K)$ is denoted by $\mathcal{M}^+(K)$. The support of each measure $\mu \in \mathcal{M}(K)$ and the Dirac measure at the point $x \in K$ are denoted by $\text{supp}\mu$ and δ_x, respectively.

Definition 2.1. Let K be a locally compact Hausdorff space with the following property:

1. there is a mapping $* : \mathcal{M}(K) \times \mathcal{M}(K) \to \mathcal{M}(K)$ (called *convolution*) such that $(\mathcal{M}(K), *, +)$ is a complex Banach algebra;
2. for each $\mu, \nu \in \mathcal{M}^+(K)$, $\mu * \nu$ is a non-negative measure in $\mathcal{M}(K)$ and the mapping $(\mu, \nu) \mapsto \mu * \nu$ from $\mathcal{M}^+(K) \times \mathcal{M}^+(K)$ into $\mathcal{M}^+(K)$ is continuous, where $\mathcal{M}^+(K)$ is equipped with the cone topology;
3. for all $x, y \in K$, $\delta_x * \delta_y$ is a compact supported probability measure;
4. the mapping $(x, y) \mapsto \text{supp}(\delta_x * \delta_y)$ from $K \times K$ into $\mathcal{K}(K)$ equipped with the Michael topology, is continuous;
5. there is an element e such that for each $x \in K$, $\delta_e * \delta_x = \delta_x = \delta_x * \delta_e$;
6. there is a homeomorphism $x \mapsto x^-$ from K onto K (called *involution*) such that for each $x, y \in K$ we have $(x^-)^- = x$ and $(\delta_x * \delta_y)^- = \delta_{y^-} * \delta_{x^-}$;
7. for each $x, y \in K$, $e \in \text{supp}(\delta_x * \delta_y)$ if and only if $x = y^-$.

Then, $K \equiv (K, *, .^-, e)$ is called a *(locally compact) hypergroup.*

Any locally compact group G is a hypergroup with the inverse mapping as involution and the usual convolution

$$\mu * \nu(f) := \int_G \int_G f(xy)\, d\mu(x) d\nu(y), \quad (f \in C_0(G))$$

for all $\mu, \nu \in \mathcal{M}(G)$. So, hypergroups are extension of locally compact groups. In the book [2] one can see several classes of hypergroups.

A Radon nonzero nonnegative measure λ on a hypergroup K is called a (left) Haar measure if for each $x \in K$, $\delta_x * \lambda = \lambda$. By [9, 5.3], there is a continuous positive function Δ on K (called *modular function*) such that for each $x \in K$, $\lambda * \delta_{x^-} = \Delta(x)\lambda$. Throughout, K is a locally compact hypergroup with a given left Haar measure λ.

For each measurable function $f : K \to \mathbb{C}$ and each $a, x, y \in K$ we put

$$f_x(y) = f^y(x) = f(x * y) := \int_K f(t)\, d(\delta_x * \delta_y)(t) \quad \text{and} \quad r_a f(x) := f(x * a^-).$$

Let $f, g : K \to \mathbb{C}$ be measurable functions and $\mu \in \mathcal{M}(K)$. For each $x \in K$ we define

$$(f * g)(x) := \int_K f(x * y) g(y^-)\, d\lambda(y) \quad \text{and} \quad (\mu * f)(x) := \int_K f(y^- * x)\, d\mu(y)$$

while these integrals exist.

2.2 Orlicz Spaces

In this part, we give definition of an Orlicz space in the context of a hypergroup K. For more details see monographs [18, 17].

A convex function $\Phi : [0, \infty) \to [0, \infty]$ is called a *Young function* if $\Phi(0) = \lim_{x\to 0} \Phi(x) = 0$ and $\lim_{x\to\infty} \Phi(x) = \infty$. A continuous Young function Φ is called an *N-function* if $\lim_{x\to 0} \frac{\Phi(x)}{x} = 0$, $\lim_{x\to\infty} \frac{\Phi(x)}{x} = \infty$, and $\Phi(x) = 0$ implies that $x = 0$. We say that an N-function Φ satisfies a globally Δ'-condition and write $\Phi \in \Delta'$ (globally) if there is a constant $c > 0$ such that for each $x, y \geq 0$, $\Phi(xy) \leq c\, \Phi(x)\, \Phi(y)$; see [18, Definition 7, Ch. II]. If Φ is a Young function, the function Ψ defined by

$$\Psi(x) := \sup\{xy - \Phi(y) : y \geq 0\}, \quad (x \geq 0)$$

is called the *complementary* of Φ. In this paper, always Φ is a strictly increasing Young function, and Ψ is its complementary.

The set of all Borel measurable functions $f : K \to \mathbb{C}$ for which there exists an $\alpha > 0$ such that

$$\int_K \Phi(\alpha|f(x)|)\, d\lambda(x) < \infty,$$

is denoted by $L^\Phi(K)$. We denote the set of all measurable functions $g : K \to \mathbb{C}$ such that $\int_K \Psi(|g(x)|)\, d\lambda(x) \leq 1$ by B_Ψ, and for each $f \in L^\Phi(K)$ we put

$$\|f\|_\Phi := \sup \left\{ \int_K |f(x)g(x)|\, d\lambda(x) : \, g \in B_\Psi \right\}.$$

Then, $(L^\Phi(K), \|\cdot\|_\Phi)$ is called an *Orlicz space*. For each $p \geq 1$ we define $\Phi_p(x) := x^p$ for all $x \geq 0$. Then, $L^{\Phi_p}(K) = L^p(K, \lambda)$. So, Orlicz spaces are extension of Lebesgue spaces.

3 Main Results

In this section we will formulate bicontinuous biseparating left multipliers on $L^\Phi(K)$, where K is a compact hypergroup. We also prove that for a family of Young functions, if $L^\Phi(G_1)$ and $L^\Phi(G_2)$ are isometrically isomorphic, then two locally compact groups G_1 and G_2 are isomorphic too. Throughout we assume that Φ is a Δ_2-regular Young function.

Definition 3.1. Let $\mathcal{A}$ be a Banach algebra. A linear operator $M : \mathcal{A} \to \mathcal{A}$ is called a *left multiplier* if for each $a, b \in \mathcal{A}$, $M(ab) = M(a)b$.

Remark 3.2. Note that for each $a \in K$ and measurable functions f, g on K we have $r_a(f * g) = f * r_a g$ because for every $x \in K$,

$$\begin{aligned}
(f * r_a g)(x) &= \int_K f(x * y) r_a g(y^-)\, d\lambda(y) \\
&= \int_K f_x(y) g(y^- * a^-)\, d\lambda(y) \\
&= \int_K f_x(a^- * y) g(y^-)\, d\lambda(y) \\
&= \int_K g(y^-) f^y(x * a^-)\, d\lambda(y) \\
&= \int_K \int_K f(t * y) g(y^-)\, d\lambda(y) d(\delta_x * \delta_{a^-})(t) \\
&= \int_K (f * g)(t)\, d(\delta_x * \delta_{a^-})(t) \\
&= (f * g)(x * a^-) \\
&= r_a(f * g)(x),
\end{aligned}$$

thanks to [9, Lemma 3.1G].

Proposition 3.3. Let K be a compact hypergroup, and M be a bounded left multiplier on the convolution Orlicz algebra $L^\Phi(K)$. Then, for each $a \in K$, $r_a M = M r_a$.

Proof . Let M be a left multiplier on $L^\Phi(K)$, $a \in K$ and $f \in C(K)$. By [15, Theorem 4.1], for each $\varepsilon > 0$, there is some $g \in L^{\Phi}(K)$ such that $\|f * g - f\|_{\Phi} \leq \varepsilon$. For each $v \in B_{\Psi}$ we have $r_{a^-} v \in B_{\Psi}$ because

$$\begin{aligned}\int_K \Psi(|r_{a^-} v(x)|)\, d\lambda(x) &\leq \int_K \Psi(|v|(x * a))\, d\lambda(x) \\ &\leq \int_K \Psi(|v|)(x * a)\, d\lambda(x) \\ &= \int_K \Psi(|v|) * \delta_{a^-}\ d\lambda \\ &= \int_K \Psi(|v|)\, d(\lambda * \delta_a) \\ &= \int_K \Psi(|v|)\, d\lambda \leq 1,\end{aligned}$$

thanks to Jensen's inequality [18, Proposition 5 page 62],[2, Section 1.2.15] and [2, Theorem 1.3.25]. This implies that for every $h \in L^{\Phi}(K)$,

$$\begin{aligned}\|r_a h\|_{\Phi} &= \sup\left\{\int_K |h(x * a^-)v(x)|\, d\lambda(x) :\ v \in B_{\Psi}\right\} \\ &\leq \sup\left\{\int_K |h|(x * a^-)\, |v|(x)\, d\lambda(x) :\ v \in B_{\Psi}\right\} \\ &= \sup\left\{\int_K |h(x)|\ |v|(x * a)\, d\lambda(x) :\ v \in B_{\Psi}\right\} \\ &\leq \|h\|_{\Phi},\end{aligned}$$

by applying a version of [9, Theorem 5.1D] for right Haar measures (note that since K is compact, it is unimodular, and so λ would be also a right Haar measure). Hence, the mapping

$$r_a : L^{\Phi}(K) \to L^{\Phi}(K), \qquad f \mapsto r_a f$$

is a bounded linear operator with $\|r_a\| \leq 1$. In addition, note that by Remark 3.2, we have

$$r_a(M(f * g)) = r_a(M(f) * g) = M(f) * r_a g = M(f * r_a g) = M(r_a(f * g)).$$

Therefore,

$$\begin{aligned}\|r_a M(f) - M(r_a f)\|_{\Phi} &\leq \|r_a\big(M(f - f * g)\big)\|_{\Phi} + \|M\big(r_a(f * g - f)\big)\|_{\Phi} \\ &\leq 2\|r_a\|\, \|M\|\ \|f * g - f\|_{\Phi} \\ &\leq 2\|M\|\, \varepsilon.\end{aligned}$$

Since $\varepsilon > 0$ is arbitrary, we have $r_a M(f) = M(r_a f)$. But because of the assumption $\Phi \in \Delta_2$, the set $C(K)$ is dense in $L^{\Phi}(K)$. Hence, by continuity the proof is complete. □

Definition 3.4. Let $(\Omega_1, \Sigma_1, \mu_1)$ and $(\Omega_2, \Sigma_2, \mu_2)$ be two measure spaces. A set map P from Σ_1 into Σ_2 defined modulo null sets is called a *regular set isomorphism* if

(i) $P(\Omega_1 \setminus A) = P(\Omega_1) \setminus P(A)$ for all $A \in \Sigma_1$.

(ii) $P(\cup_{n=1}^{\infty} A_n) = \cup_{n=1}^{\infty} P(A_n)$ for disjoint $A_n \in \Sigma_1$

(iii) $\mu_2(P(A)) = 0$ if and only if $\mu_1(A) = 0$

In sequel, the Borel σ-algebra on a hypergroup K is denoted by $\mathfrak{B}_K$. The previous fact is applied to prove the next result which is a hypergroup version of [14, Theorem 1] and [22, Theorem 3.8]. Although the proofs are similar, we write some details in this proof for convenience of readers.

Theorem 3.5. Let K be a compact hypergroup. Let M be a bicontinuous biseparating left multiplier on the convolution Orlicz algebra $L^{\Phi}(K)$. Then, there exist a constant $k \in \mathbb{C}$ and a regular set isomorphism T on $\mathfrak{B}_K$ such that

$$M(\chi_A) = k\, \chi_{T(A)} \quad \text{a.e.} \qquad (A \in \mathfrak{B}). \tag{3.1}$$

Proof . Assume that $M : L^\Phi(K) \to L^\Phi(K)$ is a bicontinuous biseparating left multiplier. We define $T : \mathfrak{B}_K \to \mathfrak{B}_K$ by

$$T(A) := \{x \in K : M(\chi_A)(x) \neq 0\}, \quad (A \in \mathfrak{B}_K).$$

up to null sets. By some calculations as in the proof of [22, Theorem 3.8] one can see that

$$M(\chi_A) = h \cdot \chi_{T(A)} \quad \text{a.e.} \tag{3.2}$$

for all $A \in \mathfrak{B}_K$, where h is the image of the constant function one by the operator M. This implies that for each $x \in K$,

$$r_x h = r_x M(\mathbf{1}) = M(r_x \mathbf{1}) = M(\mathbf{1}) = M(\chi_K) = h\chi_{T(K)} = h\,\chi_K = h.$$

So, $\lambda(E_x) = 0$ where $E_x := \{y \in K : h(y * x^-) \neq h(y)\}$. Hence, $h = k$ a.e. for a constant number $k \in \mathbb{C}$, and by (3.2),

$$M(\chi_A) = k\,\chi_{T(A)} \quad \text{a.e.} \qquad (A \in \mathfrak{B}_K). \tag{3.3}$$

□

R.E. Edward in [6, Proposition 1] states that if G is a compact group, $1 \leq p < \infty$ and T is a positive left multiplier on $L^p(G)$, then there exists a positive Radon measure μ on G such that $Tf = \mu * f$ for all $f \in L^p(G)$. The next fact can be concluded by a similar argument as in the proof of [6, Proposition 1].

Theorem 3.6. Let K be a compact hypergroup and T be a positive left multiplier on $L^p(K)$ $(1 \leq p < \infty)$. Then, there exists a positive Radon measure $\mu \in \mathcal{M}(K)$ such that $Tf = \mu * f$ for all $f \in L^p(K)$.

Theorems 3.5 and 3.6 help us to give a similar formula for any bicontinuous biseparating left multiplier on $L^\Phi(K)$.

Theorem 3.7. Let K be a compact hypergroup and $\Phi \in \Delta'$ (globally). Let M be a bicontinuous biseparating left multiplier on the convolution Orlicz algebra $L^\Phi(K)$. Then, there exist a number $c \in \mathbb{C}$ and a positive measure $\mu \in \mathcal{M}(K)$ such that $Mf = c\,\mu * f$ for all $f \in L^\Phi(K)$.

Proof . By Theorem 3.5, there exist $k \in \mathbb{C}$ and regular set isomorphism T such that

$$\Phi^{-1}\left(\frac{1}{\lambda(A)}\right) \leq \frac{2\|M\|}{|k|}\,\Phi^{-1}\left(\frac{1}{\lambda(T(A))}\right),$$

Then, we can find a linear bounded operator $F : L^1(K) \to L^1(K)$ such that for every simple function s on K, $F(s) = \frac{1}{k}\,M(s)$. So, for each $A \in \mathfrak{B}_K$, $F(\chi_A) = \chi_{T(A)} = \frac{1}{k}\,M(\chi_A)$. Because of properties of M, the operator F is left multiplier and bipositive on $L^1(K)$. Finally, thanks to Theorem 3.6 the proof is complete. □

Remark 3.8. For every $\mu \in \mathcal{M}(K)$ we denote $A_\mu(f) := \mu * f$ for all $f \in L^\Phi(K)$. For each $a \in K$, set $A_a := A_{\delta_a}$. In general, the relation $A_a(fg) = A_a f\, A_a g$ is not valid for each $a \in K$. But, this equality holds whenever a belongs the center of the hypergroup K defined by $Z(K) := \{x \in K : \delta_x * \delta_{x^-} = \delta_e = \delta_{x^-} * \delta_x\}$. In fact, thanks to [9, Lemma 10.4B] for each $a \in Z(K)$ and $x \in K$, $\delta_a * \delta_x$ is a Dirac measure. So,

$$\begin{aligned} A_a(fg)(x) &= \int_K f(t)g(t)\,d(\delta_{a^-} * \delta_x)(t) \\ &= \int_K f(t)\,d(\delta_{a^-} * \delta_x)(t) \int_K g(t)\,d(\delta_{a^-} * \delta_x)(t) \\ &= A_a f(x)\,A_a g(x). \end{aligned}$$

This shows that A_a is separating.

Corollary 3.9. Let K_1, K_2 be locally compact hypergroups and $\Phi \in \Delta'$ (globally). Assume that T is a bicontinuous biseparating algebra isomorphism from $L^\Phi(K_1)$ onto $L^\Phi(K_2)$. Then, for each $a \in Z(K_1)$ there exists a measure $h(a) \in \mathcal{M}(K_2)$ such that $T\,A_a = A_{h(a)}T$.

Proof . Let $a \in Z(K_1)$. Then, the mapping $T\,A_a\,T^{-1} : L^\Phi(K_2) \to L^\Phi(K_2)$ is a bicontinuous biseparating left multiplier. Indeed, for each $f, g \in L^\Phi(K_2)$,

$$\begin{aligned}(T\,A_a\,T^{-1})(f * g) &= (TA_a)(T^{-1}(f) * T^{-1}(g)) \\ &= T((A_a(T^{-1}(f)) * T^{-1}(g))) \\ &= T(A_a(T^{-1}(f))) * g.\end{aligned}$$

This implies that $T\,A_a\,T^{-1}$ is a left multiplier. Since a is a center element, $T\,A_a\,T^{-1}$ is invertible. Also, since T, T^{-1} and A_a are separating, $T\,A_a\,T^{-1}$ is biseparating. Then, by Theorem 3.7, there exists a measure $h(a) \in \mathcal{M}(K_2)$ such that $T\,A_a\,T^{-1} = A_{h(a)}$. This completes the proof. □

Theorem 3.10. Let K_i be a hypergroup and $L^\Phi(K_i)$ be a convolution Orlicz algebra for $i = 1, 2$. Assume that $T : L^\Phi(K_1) \to L^\Phi(K_2)$ is an isometry algebra isomorphism and Φ is not equivalent to $|x|^2$. Then, T is bicontinuous and biseparating.

Proof . Since T is a bijective isometry, T and T^{-1} are continuous. Thanks to [7, Theorem 5.3.5], there exist a Borel measurable function h and a regular set isomorphism S from $\mathcal{B}_1$ onto $\mathcal{B}_2$ such that

$$Tf(t) = h(t)S_1f(t) \quad \text{for all } f \in E_1, \tag{3.4}$$

where $\mathcal{B}_i$ $(i = 1, 2)$ is the Borel σ-algebra on K_i, and S_1 is the operator induced by S. By [7, Remark 3.2.4], we have $S_1(fg) = (S_1f)(S_1g)$ for all Borel measurable functions f, g on K_1. This shows that T is biseparating. □

Here, we recall the result [22, Corollary 3.11].

Theorem 3.11. Let G_i be a locally compact group for $i = 1, 2$, Φ be a Δ_2-regular Young function with $\Phi \in \Delta'$ (globally), and $L^\Phi(G_i)$ be a convolution Orlicz algebra for $i = 1, 2$. If there is a bicontinuous biseparating algebra isomorphism between $L^\Phi(G_1)$ and $L^\Phi(G_2)$, then G_1 and G_2 are isomorphic.

Now, thanks to Theorems 3.10 and 3.11 one can directly conclude the next fact.

Corollary 3.12. Let G_1 and G_2 be two locally compact groups, Φ be a Δ_2-regular Young function with $\Phi \in \Delta'$ (globally) and Φ is not equivalent to $|x|^2$. If $L^\Phi(G_i)$ is a convolution Orlicz algebra for $i = 1, 2$, and there is an isometry algebra isomorphism between $L^\Phi(G_1)$ and $L^\Phi(G_2)$, then G_1 and G_2 are isomorphic.

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