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A semi-oriented radial measure for Malmquist Productivity Index: A case study of regional electricity companies in Iran

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Abstract

One of the important applications of data envelopment analysis is to determine the progress and regress of the units under evaluation at two different times, which has been addressed in many papers. Also, one of the distinctions of data coverage analysis technique with other methods is the introduction of achievable and flexible benchmarks. In the present paper, we intend to study the progress and regress of Iranian regional electricity companies during two consecutive years of 2015 and 2016. Since some of the evaluated indicators are semi-positive and semi-negative indicators, in this study we will develop Emrooznejad et al. [7] to determine the productivity index of Malmquist for semi-positive and semi-negative indicators. Finally, for further explanation, we have used the proposed models to determine the progress and regression of 16 regional electricity companies in Iran with 3 semi-positive and semi-negative indices in the presence of the limitation on the benchmark, an undesirable index and 11 completely positive indices in the nature of input with constant scale returns as a black box.

Keywords: Progress and Regression, Semi-oriented radial measure, Malmquist Productivity Index, Regional Electricity Companies, Data Envelopment Analysis
2020 MSC: 90C08

1 Introduction

Every business, whether manufacturing or service, is always looking to improve its current situation. In fact, it tries to put its firm or organization in the best working conditions that have an efficient performance. The fact that an enterprise is at its optimal point in terms of performance or how far it is from that point and what is its growth trend from year to year, requires measurement methods and tools, which are called performance measurement of progress and regress. Therefore, by identifying the weaknesses and strengths of the past and proper planning according to the facilities, equipment, and available human and financial resources works to improve the weaknesses and maintain the strengths of the organization in the future.

In research and studies to measure relative efficiency, the Data Envelopment Analysis (DEA) model, which has grown significantly in the last four decades, has become one of the most widely used methods in research in operations

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(OR) and management science (MS) and it has been widely used in public and private organizations [13].With this method, the researcher has evaluated the efficiency of 16 regional electricity companies in 2016 [12]. Regional electricity companies are state-owned companies and subsidiaries of Tavanir Company as an infrastructure and parent industry that after receiving energy produced from different power plants in Iran (thermal, hydro and renewable) are responsible for voltage conversion, transmission, transit and supply of reliable and stable electricity to all Home, industrial, agricultural, public and commercial subscribers through electricity distribution companies. These companies are the regional electricity companies of Azerbaijan, Isfahan, Bakhtar, Tehran, Khorasan, Khuzestan, Zanjan, Semnan, Sistan and Baluchestan, Gharb, Fars, Kerman, Gilan, Mazandaran, Hormozgan and Yazd, which are considered as black boxes in this paper.

For more than two decades, the performance measurement of Iran's electricity industry has been of special interest to decision-making units and to achieve the goals with the growth of science and technology and sometimes in parallel, many different models and approaches such as key performance indicators (KPI), balanced scorecard (BSC), organizational excellence model (EFQM), taste and so on have been used. In the mentioned methods, if in practice the standard imposed to any criteria is beyond the potential of the organization due to environmental conditions, the obtained results will not be consistent with the facts. Therefore, in order to avoid ambiguity in managers' decisions, data envelopment analysis (DEA) technique is presented for Iran's electricity industry, which consists of a set of mathematical relations and models based on non-parametric linear programming and has grown significantly in the last four decades and large organizations in developing countries use it. The biggest advantage of this method is the relative comparison of unit performance, which was founded by Farrell [9] and later used by Charnes et al.[4] to develop the term of data envelopment analysis, which aims to construct a nonparametric envelopment limit function on data points. In such a way that all inefficiently observed units are placed below and efficiently observed units placed on the production limit function. For each efficiency decision unit, we obtain the ratio of all products UY_j to all inputs VX_j where U and V are the weight vectors of products and inputs, respectively. We use linear programming to select the optimal weights, given the constraint that all the performances of the decision units are less than or equal to one. Khoveyni et al. [18] investigated congestion in the presence of negative data using non-radial models. Lin, Yang, Huang [20] introduced a two-step approach for dealing with negative data based on the SBM model. Soltanifar et al. [23] built a non-radial linear model in the presence of negative data and used cross-efficiency to rank decision-making units.

2 Literature review

Fare et al. [8] developed the Malmquist index in the DEA framework. This model can decompose the intertemporal efficiency change into catch-up and innovation (Frontier-shift) effects.Caves et al. [2] build the Productivity Index. Berg et al.[1] study growth and productivity in the Norwegian banking industry. Introducing the Malmquist productivity index, they stated that total productivity can be found in the growth of borders and the change in the distance of each bank from the borders. They saw a decline in medium-sized banks. Malmquist productivity index is based on distance function, which provides desirable properties by Farrell radial measurement. Grifell et al. [10] stated that a measure of excessive radial efficiency increases the actual efficiency and Malmquist negative productivity index is affected by that. So this has led them to come up with a new definition of "one-sided" efficiency and to come up with a new non-practical, unusual measure called non-radial productivity, which they call the quasi-Malmquist productivity index. Orea [21] evaluated Spanish savings banks by parametrically analyzing a generalized Malmquist productivity index that considers economies of scale, and his research results show the progress of branches, which can mainly be related to their technical progress and positive effect. The Malmquist Productivity Index is a performance appraisal over time. Assuming it is non-parametric, it can be measured as retrieval and innovation, both of which are performed by data envelopment analysis.

Tone [24] has proposed three different methods in this regard to measure the related issues of efficiency and Malmquist index. The geometric mean of the Malmquist Productivity Index and its components can provide different measures of productivity change. Pastor et al. [22] presented the Malmquist Global Productivity Index, which is circular and represents a single measure of productivity change. Camanho et al. [3] stated that the Malmquist productivity index distinguishes internal inefficiencies of decision-making units from those related to the characteristics of their group (or program). They practically worked on bank branches and obtained a list to compare the efficiency distribution within the group, an index to compare border productivity, and an evaluation of the efficiency of internal management. Essid et al. [5] examined the productivity of Tunisian high schools during the period 2001-2002 and 2003-2004 using the Malmquist productivity index. This analysis allowed them to identify the source of productivity changes to find quasi-fixed factors. Their study showed that there was no significant change in productivity during the study period. They showed that inefficiency can be related almost to technical regression and to a lesser extent to technical inefficiency. The common approach at the macroeconomic level is to measure energy and environmental performance, which can be done using data envelopment analysis models. In this regard, Wang et al.[25] have studied the energy of the whole region and the environmental efficiency of China. They have improved the Malmquist model to measure energy and environmental efficiency in 29 administrative districts of China during the period 2000-2008. Mavi et al. [19] emphasized the least adverse environmental impacts on economic development, which led to the development of the Malmquist model based on shared weight. Their findings show that Ireland and the United States are continuously achieved energy improvement and environmental productivity. Therefore, this index is done by using data envelopment analysis models and based on measuring the distance function or technical efficiency of the coordinates of the observations of unit o at time t and $t+1$ relative to the production boundaries with output technology at a constant scale at times t and $t+1$. In Section 3, we discuss it in detail. Adverse indicators have also been used in this paper, and various methods have been proposed in the DEA to deal with adverse indicators.

Zhou et al.[26] have proposed a new measure to evaluate the performance of 38 Chinese industries with adverse outputs. The results of their research show that Chinese industry has not performed well in most sectors, including energy extraction. They have provided a model for improving China's industry based on the results of performance and productivity assessments. Halkos et al. [11] stated that so far four different scenarios have been presented in dealing with unfavorable indicators. The first scenario involves those papers that ignore the undesirable index from the production function. The second scenario is those papers that consider the adverse index as normal inputs. In the third scenario, the undesirable index is considered as natural output. The fourth scenario makes the necessary changes to the undesirable index to consider them. In their paper, they examine the advantages and disadvantages of existing methods. Their paper is a useful summary of papers with an unfavorable index.

The version of the Malmquist productivity index introduced by Fare et al. [8] is based on assumptions such as the non-negativeness of the indices, on the other hand, sometimes the indices have special characteristics and it is obvious that the benchmark should also follow this characteristic. Mostly indicators can only be changed by a percentage of their value. Indicators can only be changed by a percentage of their value. In this paper, a method based on data envelopment analysis for Malmquist productivity index will be used to measure the efficiency, progress and regression of 16 regional electricity companies in Iran due to the presence of negative data. Therefore, with the presence of semi-positive and semi-negative indices, we also use the semi-oriented radial measurement (SORM) technique. There are suggested methods for dealing with negative inputs. We use the SORM method when we encounter negative data in data envelopment analysis. One of the features of SORM is the management of semi-positive and negative indices that calculate each input and output variable basically as a sum. That is, one of the two variables is negative and the other is positive. So that it shows the sum in the initial value. The continuation of the sections of the paper is as follows. In Section 3, we will review the literature, which includes a summary of the Malmquist method and radial measurements with negative data. In Section 4, which includes research innovation, we have developed the Malmquist Productivity Index for semi-positive and semi-negative data with undesirable indicators and control constraints. Section 4 is a practical example for regional electricity companies in Iran, and in Section 5, we analyze the results of the research.

3 Background

3.1 Malmquist Productivity Index

Suppose n units under evaluation $DMU_j(j=1,\ldots,n)$ are assumed at time t and $t+1$. The $j-th$ unit in year $t+1$ and with input consumption $X_j^{t+1}=(x_{1j}^{t+1},\ldots,x_{sj}^{t+1})$has produced output $Y_j^{t+1}=(y_{1j}^{t+1},\ldots,y_{sj}^{t+1})$.

In order to investigate the progress or regression of the activity performed by the DMU_o unit $o\in\{1,\ldots,n\}$at time t + 1 compared to time t, we have used the Malmquist method, whose multiplication input-oriented model (CCR) is as follows:

Step (1): Find the optimal answer to each of the following models.

$$\begin{aligned} min \quad & D_o^l(x_o^k, y_o^k) = Max \sum_{r=1}^{n} u_r y_{ro}^k \\ s.t. \quad & \\ & \sum_{r=1}^{s} u_r y_{rj}^l - \sum_{i=1}^{m} v_i x_{ij}^l \leq 1, \qquad j = 1, \dots, n, \\ & \sum_{i=1}^{m} v_i x_{io}^k = 1 \\ & u_r \geq 0, r = 1, \dots, s \\ & v_i \geq 0, i = 1, \dots, m \end{aligned} \tag{3.1}$$

where $k, l \in \{t, t+1\}$. Therefore, calculate the four models and the values of $D_o^t(x_o^t, y_o^t)$, $D_o^t(x_o^{t+1}, y_o^{t+1})$, $D_o^{t+1}(x_o^t, y_o^t)$ and $D_o^{t+1}(x_o^{t+1}, y_o^{t+1})$.

Step (2): Insert:

$$\begin{aligned} MPI_o =& [\frac{D_o^t(x_o^{t+1}, y_o^{t+1})}{D_o^t(x_o^t, y_o^t)} . \frac{D_o^{t+1}(x_o^{t+1}, y_o^{t+1})}{D_o^{t+1}(x_o^t, y_o^t)}]^{1/2} = \frac{D_o^{t+1}(x_o^{t+1}, y_o^{t+1})}{D_o^t(x_o^t, y_o^t)} [\frac{D_o^t(x_o^t, y_o^t)}{D_o^{t+1}(x_o^t, y_o^t)} . \frac{D_o^t(x_o^{t+1}, y_o^{t+1})}{D_o^{t+1}(x_o^{t+1}, y_o^{t+1})}]^{1/2} \\ =& \text{Performance changes} \times \text{Technical changes}. \end{aligned} \tag{3.2}$$

Call MPI_o the criterion of Malmquist's progress and regress.

Step (3): The result of the criterion of progress and regression

i1) If $MPI_o > 1$ then DMU_o has progressed at time $t + 1$ relative to time t.

i2) If $MPI_o = 1$ then DMU_o at time $t + 1$ has not changed from time t.

i3) If $MPI_o < 1$ then DMU_o has regressed at time $t + 1$ relative to time t.

3.2 Radial measurement with negative data

Semi-negative and semi-positive data in data envelopment analysis was developed by Emrouznejad et al. [7].They considered the case that the input and output indices have semi-positive and semi-negative values. Emrouznejad et al. [6] investigated the necessary and sufficient conditions for boundedness of the input and output orientations of the variable returns to scale SORM DEA model.Kazemi Metin et al. [16] showed that the existence of negative outputs as inputs is inconsistent with the assumptions of the production process. Furthermore, they showed that the standard SORM may not be able to distinguish between efficient and inefficient units under certain conditions.Also, Kazemi Metin et al. [16] showed that in the SORM model, it is possible to classify two DMUs that are non-dominated as efficient and inefficient, while this does not happen in the modified SORM. Kaffash et al. [15] improved the Modified SORM model from computational and target settings perspective and allows for the dual formulation of linear programming which is useful in some contexts like estimating shadow prices or imposing weight restrictions. Among other researches in this field, [14] and [17] can be mentioned.

In this method, inputs and outputs are divided into two categories: completely positive, semi-positive and semi-negative.Suppose $I = 1, 2, \dots, m$ and $R = 1, 2, \dots, s$ are sets of input and output indices and suppose $I^{'}$ and $I^{''}$ are totally positive, semi-positive and semi-negative input indices, respectively and obviously $I^{'} \cap I^{''} = \emptyset$ and $I^{'} \cup I^{''} = I$ as well as $R^{'}$ and $R^{''}$ represent a set of output indices that are completely positive, semi-positive and semi-negative, respectively. Also $R^{'} \cap R^{''} = \emptyset$ and $R^{'} \cup R^{''} = I$. Emrouznejad et al. [7] performed the following to calculate the efficiency in the nature of input in the presence of semi-positive and semi-negative indices with returns on a variable scale.

Step (1): For $i \in I^{''}$ and $r \in R^{''}$:

$$\begin{aligned} x_{ij} &= x_{ij}^1 - x_{ij}^2; \quad x_i^1, x_j^2 \geq 0, \\ y_{rj} &= y_{rj}^1 - y_{rj}^2; \quad y_i^1, y_j^2 \geq 0. \end{aligned} \tag{3.3}$$

Therefore,

$$x_{ij}^1 = \begin{cases} x_{ij} & , x_{ij} \geqslant 0 \\ 0 & , x_{ij} < 0 \end{cases} \tag{3.4}$$

$$x_{ij}^2 = \begin{cases} 0 & , x_{ij} \geqslant 0 \\ -x_{ij} & , x_{ij} < 0 \end{cases} \tag{3.5}$$

$$y_{rj}^1 = \begin{cases} y_{rj} & , y_{rj} \geqslant 0 \\ 0 & , y_{rj} < 0 \end{cases} \tag{3.6}$$

$$y_{rj}^2 = \begin{cases} 0 & , y_{rj} \geqslant 0 \\ -y_{rj} & , y_{rj} < 0 \end{cases} \tag{3.7}$$

Step (2): Solve the following model.

$$\begin{aligned} h^* = min \quad & h \\ s.t. \quad & \\ & \sum_j \lambda_j x_{ij} \leq h x_{io} \quad & i \in I^{'} \\ & \sum_j \lambda_j x_{ij}^1 \leq h x_{io}^1 \quad & i \in I^{''} \\ & \sum_j \lambda_j x_{ij}^2 \geq h x_{io}^2 \quad & i \in I^{''} \\ & \sum_j \lambda_j y_{rj} \geq y_{ro} \quad & r \in R^{'} \\ & \sum_j \lambda_j y_{rj}^1 \geq y_{ro}^1 \quad & r \in R^{''} \\ & \sum_j \lambda_j y_{rj}^2 \leq y_{ro}^2 \quad & r \in R^{''} \\ & \sum_j \lambda_j = 1 & \\ & \lambda_j \geq 0 & j = 1, \ldots, n. \end{aligned} \tag{3.8}$$

The optimal answer of the above model, h^*, indicates the efficiency for DMU_o in the presence of semi-positive and semi-negative indices.

4 SORM-Malmquist index

Note that some indicators may not apply to DEA defaults. Since in data envelopment analysis, it is the pattern or benchmark that determines the performance and the pattern should be appropriate to reality and applicable, so the actual conditions of the indicators should be seen in the model. For example, if x_{1p} represents DMU_P human resources, it may be controllable to some extent depending on the circumstances of the company. For example, if α percent is controllable, the following constraint should be added to the constraint of the problem so that the corresponding pattern is real and applicable.

$$(1-\alpha)x_{1p} \leq \sum_j \lambda_j x_{1j} \tag{4.1}$$

Similarly, if the output index is qualitative, for example suppose y_{1p} is qualitative. Suppose to convert it to a quantitative index, values in the range $[1, 5]$ are assigned to it. Therefore, the constraint $1 \leq \sum_j \lambda_j y_{1j} \leq 5$ must be added to the constraint of the problem. In general, suppose the input indicators to the two categories M and $\bar{M}$ are the category that applies to the DEA default and the category that does not apply to the DEA default and suppose the controllable percentage of the $i \in \bar{M}$ index is α, so the following constraint will be added to the model.

$$\sum_j \lambda_j x_{ij} \geq (1-\alpha) x_{io} \quad ; i \in \bar{M} \tag{4.2}$$

In the presence of semi-positive and semi-negative indicators, the following constraints will be added to the model:

$$\sum_j \lambda_j x_{ij} \geq (1-\alpha) x_{io}; i \in \bar{M}, i \in I^{'} \tag{4.3}$$

$$\sum_j \lambda_j x_{ij}^1 \geq (1-\alpha) x_{io}^1; i \in \bar{M}, i \in I^{''} \tag{4.4}$$

$$\sum_j \lambda_j x_{ij}^2 \leq (1+\alpha)x_{io}^2; i \in \bar{M}, i \in I^{''} \tag{4.5}$$

where x_{ij}^1 and x_{ij}^2 and $i \in I^{''}$ are the same as in the previous section. Now, if the output indices are divided into two categories M and $\bar{M}$, ie the category that apply by default and the category that do not apply by default, and assume that the controllable percentage of the index $r \in \bar{H}$ is equal to β_rThen the following constraint will be added to the model.

$$\sum_j \lambda_j y_{rj} \geq (1+\beta_r)y_{ro}; r \in \bar{H} \tag{4.6}$$

In the presence of semi-positive and semi-negative indicators, the following constraints will be added to the model.

$$\sum_j \lambda_j y_{rj} \leq (1+\beta_r)y_{ro}; r \in \bar{H}, r \in R^{'} \tag{4.7}$$

$$\sum_j \lambda_j y_{rj}^1 \leq (1+\beta_r)y_{ro}^1; r \in \bar{H}, r \in R^{''} \tag{4.8}$$

$$\sum_j \lambda_j y_{rj}^2 \geq (1-\beta_r)y_{ro}^2; r \in \bar{H}, r \in R^{''} \tag{4.9}$$

Note that if the output y_{rj} contains two categories of desirable and undesirable, namely y_{rj}^g and y_{rj}^b, then the similar constraint in the model for evaluating DMU_p will be as follows.

$$\begin{aligned} &\sum_j \delta_j \lambda_j y_{rj}^g \geq y_{ro}^g \quad ; r = 1, ..., s \\ &\sum_j \delta_j \lambda_j y_{rj}^b \geq y_{ro}^b \quad ; r = 1, ..., s \end{aligned} \tag{4.10}$$

The coefficient δ_j indicates that y_{rj}^g and y_{rj}^b are interdependent. Therefore, considering the change control, the following constraints will be added to the model.

$$\begin{aligned} &\sum_j \delta_j \lambda_j y_{rj}^g \leq (1+\beta_r)y_{ro}^g \qquad ; r \in \bar{M} \\ &(1-\beta_r^{'})y_{ro}^b \leq \sum_j \delta_j \lambda_j y_{rj}^b \leq (1-\beta_r^{''})y_{ro}^b; r \in \bar{M} \end{aligned} \tag{4.11}$$

Now, if semi-positive and semi-negative indices are also available, constraints (4.10) and (4.11) will change to constraints (4.12) and (4.13).

$$\begin{aligned} &\sum_j \delta_j \lambda_j y_{rj}^g \geq y_{ro}^g \quad ; r \in \bar{R} \\ &\sum_j \delta_j \lambda_j y_{rj}^{1,g} \geq y_{ro}^{1,g} \quad ; r \in \bar{R}'' \\ &\sum_j \delta_j \lambda_j y_{rj}^{2,g} \leq y_{ro}^{2,g} \quad ; r \in \bar{R}'' \\ &\sum_j \delta_j \lambda_j y_{rj}^b = y_{ro}^b \quad ; r \in \bar{R}' \\ &\sum_j \delta_j \lambda_j y_{rj}^{1,b} = y_{ro}^{1,b} \quad ; r \in \bar{R}'' \\ &\sum_j \delta_j \lambda_j y_{rj}^{2,b} = y_{ro}^{2,b} \quad ; r \in \bar{R}'' \end{aligned} \tag{4.12}$$

and

$$\begin{aligned}
&\sum_j \delta_j\lambda_j y_{rj}^{g} \le (1+\beta_r)y_{ro}^{g} && ;r\in R^{'};r\in\bar{H}\\
&\sum_j \delta_j\lambda_j y_{rj}^{1,g} \le (1+\beta_r)y_{ro}^{1,g} && ;r\in R^{''};r\in\bar{H}\\
&\sum_j \delta_j\lambda_j y_{rj}^{2,g} \ge (1+\beta_r)y_{ro}^{2,g} && ;r\in R^{''};r\in\bar{H}\\
&(1-\beta_r^{'})y_{rp}^{b} \le \sum_j \delta_j\lambda_j y_{rj}^{b} \le (1-\beta_r^{''})y_{rp}^{b} && ;r\in R^{''};r\in\bar{H}\\
&(1-\beta_r^{'})y_{rp}^{1,b} \le \sum_j \delta_j\lambda_j y_{rj}^{1,b} \le (1-\beta_r^{''})y_{rp}^{b} && ;r\in R^{''};r\in\bar{H}\\
&(1-\beta_r^{''})y_{rp}^{b} \le \sum_j \delta_j\lambda_j y_{rj}^{2,b} \le (1-\beta_r^{'})y_{rp}^{2,b} && ;r\in R^{''};r\in\bar{H}\\
&\sum_j \lambda_j = 1.
\end{aligned} \tag{4.13}$$

Note that we intend to propose a model that includes semi-positive and semi-negative indices and control constraints, and we intend to use it to calculate the Malmquist productivity index. The objective function of this model is of the minimization type and includes constraints (4.12) and (4.13).

Suppose $I=\{1,2,\ldots,m\}$ and $R=\{1,2,\ldots,s\}$ are the set of input and output index indices and $I^{'}$and $I^{"}$ are the set of completely positive and semi-positive and semi-negative at time t and $t{+}1$ respectively. Also, $R^{'}$and $R^{"}$represent completely positive, semi-positive and semi-negative output indices at t and $t+1$, respectively. Obviously we have, $I^{'}\cap I^{"}=\emptyset$, $I^{'}\cup I^{"}=I$, $R^{'}\cap R^{"}=\emptyset$ and $R^{'}\cup R^{"}=R$. To linearization this model, we use the following variable change for each unit.

$$\delta_j\lambda_j=\tau_j,\gamma_j=(1-\delta_j)\lambda_j,\tau_j+\gamma_j=\lambda_j. \tag{4.14}$$

The proposed model is in the form of a linear model (4.18). To achieve the stated goal, do the following.

Step (1): Solve the following models.

$$\begin{aligned}
&\theta_N^{*1}(x_o^k,y_o^k)=Min \quad \theta\\
&s.t.\\
&\sum_{j=1}^{n}(\tau_j+\gamma_j)x_{ij}^{l} \le (-\tau_o-\gamma_o+\theta)x_{io}^{k} && i\in I^{'}\\
&\sum_{j=1}^{n}(\tau_j+\gamma_j)x_{ij}^{1,l} \le (-\tau_o-\gamma_o+\theta)x_{io}^{1,k} && i\in I^{''}\\
&\sum_{j=1}^{n}(\tau_j+\gamma_j)x_{ij}^{2,l} \ge (-\tau_o-\gamma_o+\theta)x_{io}^{2,k} && i\in I^{''}\\
&\sum_{j=1}^{n}(\tau_j+\gamma_j)x_{ij}^{l} \ge (1-\alpha_i)x_{io}^{k} && i\in I^{'},i\in\bar{M}\\
&\sum_{j=1}^{n}(\tau_j+\gamma_j)x_{ij}^{1,l} \ge (1-\alpha_i)x_{io}^{1,k} && i\in I^{''},i\in\bar{M}\\
&\sum_{j=1}^{n}(\tau_j+\gamma_j)x_{ij}^{2,l} \le (1-\alpha_i)x_{io}^{2,k} && i\in I^{''},i\in\bar{M}\\
&\sum_{j=1}^{n}\tau_j y_{rj}^{g,l} \ge y_{ro}^{g,k} && ;r\in R^{'}\\
&\sum_{j=1}^{n}\tau_j y_{rj}^{1,g,l} \ge y_{ro}^{1,g,k} && ;r\in R^{''}\\
&\sum_{j=1}^{n}\tau_j y_{rj}^{2,g,l} \ge y_{ro}^{2,g,k} && ;r\in R^{''}\\
&\sum_{j=1}^{n}\tau_j y_{rj}^{b,l} = y_{ro}^{b,k} && ;r\in R^{'}
\end{aligned}$$

$$
\begin{aligned}
&\sum_{j=1}^{n}\tau_j y_{rj}^{1,b,l} = y_{ro}^{1,b,k} && ;r\in R^{''}\\
&\sum_{j=1}^{n}\tau_j y_{rj}^{2,b,l} = y_{ro}^{2,b,k} && ;r\in R^{''}\\
&\sum_{j=1}^{n}\tau_j y_{rj}^{g,l} \leq (1+\beta_r) y_{ro}^{g,k} && ;r\in R^{'}, r\in \bar{H}\\
&\sum_{j=1}^{n}\tau_j y_{rj}^{1,g,l} \leq (1+\beta_r) y_{ro}^{1,g,k} && ;r\in R^{''}, r\in \bar{H}\\
&\sum_{j=1}^{n}\tau_j y_{rj}^{2,g,l} \geq (1-\beta_r) y_{ro}^{2,g,k} && ;r\in R^{''}, r\in \bar{H}\\
&\sum_{j=1}^{n}\tau_j y_{rj}^{b,l} \geq (1-\beta_r) y_{ro}^{b,k} && ;r\in R^{'}, r\in \bar{H}\\
&\sum_{j=1}^{n}\tau_j y_{rj}^{1,b,l} \geq (1-\beta_r) y_{ro}^{1,b,k} && ;r\in R^{''}, r\in \bar{H}\\
&\sum_{j=1}^{n}\tau_j y_{rj}^{2,b,l} \leq (1+\beta_r) y_{ro}^{2,b,k} && ;r\in R^{''}, r\in \bar{H}\\
&\sum_{j=1}^{n}(\tau_j+\gamma_j) = 1\\
&\lambda_j \geq 0 && j=1,\ldots,n.\\
&\tau_j,\gamma_j \geq 0.
\end{aligned}
\tag{4.15}
$$

Therefore,

$$
\begin{aligned}
&x_j^h = x_j^{1,h} - x_j^{2,h}\\
&x_{ij}^{1,h} = \begin{cases} x_{ij}^h & ,x_{ij}^h \geq 0\\ 0 & ,x_{ij}^h < 0\end{cases}\\
&x_{ij}^{2,h} = \begin{cases} 0 & ,x_{ij}^h \geq 0\\ -x_{ij}^h & ,x_{ij}^h < 0\end{cases}
\end{aligned}
\tag{4.16}
$$

for $t+1$ and $h=t$. Also

$$
\begin{aligned}
&y_j^h = y_j^{1,h} - y_j^{2,h}\\
&y_{rj}^{1,h} = \begin{cases} y_{rj}^h & ,y_{rj}^h \geq 0\\ 0 & ,y_{rj}^h < 0\end{cases}\\
&y_{rj}^{2,h} = \begin{cases} 0 & ,y_{rj}^h \geq 0\\ -y_{rj}^h & ,y_{rj}^h < 0\end{cases}
\end{aligned}
\tag{4.17}
$$

Note that $k,l\in\{t,t+1\}$. In this case, it was possible 4 distinct models with the values of the objective function $\theta_N^{*,t}(x_o^t,y_o^t)$, $\theta_N^{*,t+1}(x_o^t,y_o^t)$, $\theta_N^{*,t}(x_o^{t+1},y_o^{t+1})$ and $\theta_N^{*,t+1}(x_o^{t+1},y_o^{t+1})$.

Step (2): Productivity criteria in the presence of semi-positive and semi-negative indicators

$$
\begin{aligned}
&MPI_O^N = [\frac{\theta_N^t(x_o^{t+1},y_o^{t+1})}{\theta_N^t(x_o^t,y_o^t)}.\frac{\theta_N^{t+1}(x_o^t,y_o^t)}{\theta_N^{t+1}(x_o^{t+1},y_o^{t+1})}]^{\frac{1}{2}}\\
&= \frac{\theta_N^{t+1}(x_o^{t+1},y_o^{t+1})}{\theta_N^t(x_o^t,y_o^t)}.[\frac{\theta_N^t(x_o^t,y_o^t)}{\theta_N^{t+1}(x_o^t,y_o^t)}.\frac{\theta_N^t(x_o^{t+1},y_o^{t+1})}{\theta_N^{t+1}(x_o^{t+1},y_o^{t+1})}]^{\frac{1}{2}}\\
&= "Performance""changes" \times "Technical""changes"
\end{aligned}
\tag{4.18}
$$

MPI_O^N is the criterion for determining the progression and regression of Malmquist in the presence of semi-positive and semi-negative indicators.

Step (3): The result of the MPI_O^N criterion

i1) If $MPI_O^N > 1$ then DMU_o has progressed at time $t+1$ relative to time t.

i2) If $MPI_O^N = 1$ then DMU_o has not changed at time $t+1$ relative to time t.

i3) If $MPI_O^N < 1$ then DMU_o has regressed at time $t+1$ relative to time t.

5 Case study of calculating Malmquist productivity index for 16 regional electricity companies in Iran

In this section, we intend to study the progress and regress of 16 regional electricity companies in Iran in 2015 and 2016.

Input indicators:

Human resources (x_{1j}), company history (x_{2j}), financial resources (revenue and resources) (x_{3j}), capacity of transmission and over-distribution lines (x_{4j}) and capacity of transmission and over-distribution substations (x_{5j}).

Output indicators:

Gross loss (y_{1j}), net other non-operating income (expenses) (y_{2j}), net profit (loss) (y_{3j}), The activity of deploy plan to providing client honoring manager and qualitative organizational excellence (qualitative) (y_{4j}), cost absorption and costs of the program (y_{5j}), power generation (y_{6j}), purchase of energy from Igmc and the private sector (y_{7j}), sale of energy to Igmc and subscribers (y_{8j}), percentage of energy losses in the transmission and over-distribution network (y_{9j}) and redistributed energy (y_{10j}).

Between input and output indices in 2015 and 2016, the output indices y2j, y3j and y7j are semi-positive and semi-negative, the y_{9j} index is undesirable and the rest of the indices are completely positive. According to the experts of Iran's regional power companies, change control constraints for each of the indicators are presented in Table (reft1).

Table 1: Change control constraints for each of the indicators

added Constraints to the model	Controllable percentage	Variable title	Variable	Row
$\sum_{j=1}^{n} \lambda_j x_{1j} \geq 0.9x_{1o}$	10	human resources	x_{1j}	1
$\sum_{j=1}^{n} \lambda_j x_{3j} \geq 0.95x_{1o}$	5	Financial resources	x_{3j}	2
$\sum_{j=1}^{n} \lambda_j y_{1j} \leq 1.07y_{1o}$	7	Gross profit	y_{1j}	3
$\sum_{j=1}^{n} \lambda_j y_{2j} \leq 1.1y_{2o}$	10	Net of other income	y_{2j}	4
$\sum_{j=1}^{n} \lambda_j y_{3j} \leq 1.05y_{3o}$	5	Net profit (loss)	y_{3j}	5
$1 \leq \sum_{j=1}^{n} \lambda_j y_{4j} \leq 9$	(Numbers between 1 and 9)	The activity of deploy plan to providing client honoring manager and qualitative organizational excellence	y_{4j}	6
$\sum_{j=1}^{n} \lambda_j y_{5j} \leq 1.05y_{5o}$	5	cost absorption form the program	y_{5j}	7
$\sum_{j=1}^{n} \lambda_j y_{6j} \leq 1.1y_{6o}$	10	Electric power generation	y_{6j}	8
$\sum_{j=1}^{n} \lambda_j y_{7j} \leq 1.03y_{7o}$	3	energy purchasing	y_{7j}	9
$\sum_{j=1}^{n} \lambda_j y_{8j} \leq 1.04y_{8o}$	4	Energy sales	y_{8j}	10
$\sum_{j=1}^{n} \lambda_j y_{9j} \leq 1.04y_{9o}$	4	Percentage of energy losses	y_{9j}	11
$\sum_{j=1}^{n} \lambda_j y_{10j} \leq 1.1y_{10o}$	10	Undistributed energy	y_{10j}	12

Note that for the y_{2j}, y_{3j} and y_{7j} indices which are semi-positive and semi-negative, the constraints mentioned will be converted in accordance with section 3. Statistical summary of the data in the table (2) is displayed.

By entering the data into Gams software and executing model (4.18) for the four modes of table (3), the result is

The first column on the right shows the productivity index of Malmquist in 2016 compared to 2015, which has been calculated using Equation (4.18). The results show that DMU_{15} is the result of progress, $DMU_1 2$ is the result of regression and other 14 companies, including DMU_1, DMU_2, DMU_3, DMU_4, DMU_5, DMU_6, DMU_7, DMU_8, DMU_9, DMU_{10}, DMU_{11}, DMU_{13}, DMU_{14} and DMU_{16}, have not made any progress and regress. Considering these results, the amount of index data, performance evaluation and ranking of companies by DEA method, researcher management knowledge of the status of units, economic conditions and division of units into four groups are very large (4 and 6), large (2 and 11), medium (14, 15, 3, 1, 10, 5 and 12), small units (7, 9, 8, 16 and 13) Based on the peak of electricity consumption, it can be said that unit 15 due to having more appropriate financial resources and agility (Medium) has improved, 14 units out of 4 groups have practically not progressed and regressed due to financial resource constraints, and unit 12 of the intermediate group has declined due to technical network and resource constraints. Therefore, it can be clearly said that the economic (financial) bottlenecks caused by the oppressive sanctions have been one of the important factors in the lack of effective development of companies.

Table 2: Statistical summary of the data in 2015

Indicators	Min	Max	Average	Variance
Human resources	182	2427	1053.3125	331146.8398
Financial resources	719969	7039936	1760585	2.17273E+12
Gross profit	-1106579	1816429	540785.75	3.7434E+11
Net of other income	-974422	54456	-132515.25	85023117258
Net profit (loss)	-8812967	-123916	-2248158.813	6.81063E+12
The activity of deploy plan to providing client honoring manager and qualitative organizational excellence	5	8.3	6.675	0.926875
Cost absorption form the program	8.5	73	35.03125	175.7021484
Electric power generation	246	14145	5491.75	17350939.81
Eenergy purchasing	179	7012	2156	4077809.625
Energy sales	180	17756	7537.25	31500530.19
Percentage of energy losses	1.89	4.1	2.4325	0.32890625
Undistributed energy	2	610	64.3625	20815.82984

Table 3: Malmquist productivity benchmark values of companies

DMU_j	E_{11}	E_{12}	E_{21}	E_{22}	MPI_j^n
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	1	1
4	1	1	1	1	1
5	1	1	1	1	1
6	1	1	1	1	1
7	1	1	1	1	1
8	1	1	1	1	1
9	1	1	1	1	1
10	1	1	1	1	1
11	1	1	1	1	1
12	1	1	1	0.995	0.998
13	1	1	1	1	1
14	1	1	1	1	1
15	0.992	1	1	1	1.004
16	1	1	1	1	1

6 Conclusion

In this paper, in order to determine the progress and regression of the units under evaluation with semi-positive and semi-negative as well as unfavorable indices, the models of Emrouznejad et al. [7]have been developed to determine the productivity index of Malmquist in proportion to semi-positive and semi-negative indices. Finally, using the proposed models and applying input and output data, the units include 3 semi-positive and semi-negative indices, 1 unfavorable index, and 11 completely positive indices in 2015 and 2016 was calculated with their own characteristics and application for semi-positive and semi-negative and unfavorable Malmquist productivity indices. Progress and regression of 16 regional power companies that were considered as block boxes were determined. That 1 unit in 2016 compared to 2015 showed a progress and 1 unit a decline, and the remaining 14 units did not show progress and regression, or in other words remained unchanged. Also, considering the analysis of the results, it can be said that one of the units has actually progressed due to having more appropriate financial resources and relative agility, 14 units have not progressed and regressed due to financial constraints, and 1 unit has declined due to technical constraints and financial resources. Therefore, it is observed that the financial index (financial resources) is a very effective factor in the development of companies. According to the managerial and technical knowledge of the researcher of the situation of companies, the results are close to reality, ie companies have not had effective development in difficult economic conditions caused by oppressive sanctions.

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