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On a certain subclass of analytic functions involving the modified q-Opoola derivative operator

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Abstract

This paper introduces a new subclass in the open unit disc of analytic functions. It is mainly defined by the modified q-Opoola derivative operator. A coefficient inequality is obtained, and other properties like distortion and closure theorems are derived. Moreover, extreme points of the differential operator are also given. Additionally, Hadamard products (or convolution) of functions respective to the class are also included.

Keywords: Univalent functions, analytic function, q-calculus, Opoola derivative operator, Hadamard products
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1 Introduction

First, we let $\mathbb{U} = \{z : z \in \mathbb{C} : |z| < 1\}$ resemble an open unit disk with respect to the complex plane in which $\mathcal{A}$ refers to the class of functions f given by

$$f(z) = z + \sum_{\iota=2}^{\infty} a_\iota z^\iota. \tag{1.1}$$

This is analytic in $\mathbb{U}$ satisfying the usual normalization conditions given by $f^{'}(0) = 1 + f(0) = 1$. The Hadamard product (also known as convolution) for two analytic functions f as is in equation (1.1) and

$$\mathfrak{h}(z) = z + \sum_{\iota=2}^{\infty} d_\iota z^\iota,$$

is provided by

$$(f * \mathfrak{h})(z) = f(z) * \mathfrak{h}(z) = (h * f)(z) = z + \sum_{\iota=2}^{\infty} a_\iota d_\iota z^\iota.$$

We express $\mathcal{T}$ as the subclass of $\mathcal{A}$ which consists of functions f given by

$$f(z) = z - \sum_{\iota=2}^{\infty} a_\iota z^\iota. \tag{1.2}$$

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This subclass was first established and investigated by Silverman [15] (also see [9], [11]). The q-calculus denotes the possibility of extending the calculus to quantum calculus. Jackson [12, 13] proposed the q-calculus in quantum algebras to generalize the q-series, which has numerous applications in science and engineering. After that, many articles provided and investigated the application of q-calculus (see [3], [4], [6], [7], [10]).

Motivated by the importance of studying the applications of quantum calculus in the physical and mathematical sciences, we introduced the modified derivative operator given by Opoola [5], as follows:

$$\mathsf{D}_q^n(\mu,\vartheta,\mathcal{F},t)f(z)=z+\sum_{\iota=2}^{\infty}\left[\mathcal{F}+\Big([\iota]_q+\vartheta-\mu-\mathcal{F}\Big)t\right]^n a_\iota z^\iota,$$

where $t\geq 0, n\in\mathbb{N}_0$ and $1\leq\mu+\mathcal{F}\leq\vartheta$. Some special operators are also obtained (for example, see [1], [2], [8], [14]).

Definition 1.1. Let $0<\aleph\leq 1$, $\lambda\geq 0$, $n\in\mathbb{N}_0, 1\leq\mu+\mathcal{F}\leq\vartheta$ and $\varphi\in\mathbb{C}\backslash\{0\}$. Then, the function $f\in\mathcal{A}$ is said to be in the class $\mathcal{L}_{q,\mathcal{F},t}^{n,\mu,\vartheta}(\varphi,\aleph)$ if

$$\left|\frac{1}{\varphi}\left(\frac{z\partial_q\mathsf{D}_q^n(\mu,\vartheta,\mathcal{F},t)f(z)}{\mathsf{D}_q^n(\mu,\vartheta,\mathcal{F},t)f(z)}-1\right)\right|<\aleph,\qquad z\in\mathbb{U}.$$

Now, we define the class given by $\mathcal{T}_{q,\mathcal{F},t}^{n,\mu,\vartheta}(\varphi,\aleph)=\mathcal{T}\cap\mathcal{L}_{q,\mathcal{F},t}^{n,\mu,\vartheta}(\varphi,\aleph)$. The aim of this paper is to examine various properties with respect to functions f that belong to the class $\mathcal{T}_{q,\mathcal{F},t}^{n,\mu,\vartheta}(\varphi,\aleph)$.

2 Coefficient Inequalities

Theorem 2.1. Let $f\in\mathcal{T}$. Therefore, $f\in\mathcal{T}_{q,\mathcal{F},t}^{n,\mu,\vartheta}(\varphi,\aleph)$ if and only if

$$\sum_{\iota=2}^{\infty}([\iota]_q+\aleph\,|\varphi|-1)\left[\mathcal{F}+\Big([\iota]_q+\vartheta-\mu-\mathcal{F}\Big)t\right]^n a_\iota\leq\aleph\,|\varphi|. \tag{2.1}$$

Proof . Let $f\in\mathcal{T}_{q,\mathcal{F},t}^{n,\mu,\vartheta}(\varphi,\aleph)$. Then, we have

$$\Re e\left\{\frac{z\partial_q\mathsf{D}_q^n(\mu,\vartheta,\mathcal{F},t)f(z)}{\mathsf{D}_q^n(\mu,\vartheta,\mathcal{F},t)f(z)}-1\right\}>-\aleph|\varphi|.$$

Equivalently,

$$\Re e\left\{\frac{-\sum\limits_{\iota=2}^{\infty}([\iota]_q-1)\left[\mathcal{F}+\Big([\iota]_q+\vartheta-\mu-\mathcal{F}\Big)t\right]^n a_\iota z^\iota}{z-\sum\limits_{\iota=2}^{\infty}\left[\mathcal{F}+\Big([\iota]_q+\vartheta-\mu-\mathcal{F}\Big)t\right]^n a_\iota z^\iota}\right\}>-\aleph|\varphi|. \tag{2.2}$$

Provided that the inequality given above is true with respect to all $z\in\mathbb{U}$, here, we select values of z sitting on the real axis after clearing the denominator in (2.2). By letting $z\to 1^-$ via real values, we gain

$$\begin{aligned}&-\sum_{\iota=2}^{\infty}([\iota]_q-1)\left[\mathcal{F}+\Big([\iota]_q+\vartheta-\mu-\mathcal{F}\Big)t\right]^n a_\iota\\&\qquad\geq-\aleph\,|\varphi|\left(1-\sum_{\iota=2}^{\infty}\left[\mathcal{F}+\Big([\iota]_q+\vartheta-\mu-\mathcal{F}\Big)t\right]^n a_\iota\right).\end{aligned}$$

Thus, we set the desired inequality given by

$$\sum_{\iota=2}^{\infty}([\iota]_q+\aleph\,|\varphi|-1)\left[\mathcal{F}+\left([\iota]_q+\vartheta-\mu-\mathcal{F}\right)t\right]^n a_\iota\leq\aleph\,|\varphi|.$$

On the other hand, we assumed that the inequality given in (2.1) is true and $|z| = 1$, which yields

$$\begin{aligned}\left|\frac{z\partial_q \mathsf{D}_q^n(\mu,\vartheta,F,t)f(z)}{\mathsf{D}_q^n(\mu,\vartheta,F,t)f(z)}-1\right| &= \left|\frac{\sum\limits_{\iota=2}^{\infty}([\iota]_q-1)\left[F+\left([\iota]_q+\vartheta-\mu-F\right)t\right]^n a_\iota z^\iota}{z-\sum\limits_{\iota=2}^{\infty}\left[F+\left([\iota]_q+\vartheta-\mu-F\right)t\right]^n a_\iota z^\iota}\right| \\ &\leq \frac{\aleph|\varphi|\left(1-\sum\limits_{\iota=2}^{\infty}\left[F+\left([\iota]_q+\vartheta-\mu-F\right)t\right]^n a_\iota\right)}{1-\sum\limits_{\iota=2}^{\infty}\left[F+\left([\iota]_q+\vartheta-\mu-F\right)t\right]^n a_\iota} \\ &\leq \aleph|\varphi|.\end{aligned}$$

□

Corollary 2.2. Provided that the function f given by (1.2) is in the class $\mathcal{T}_{q,F,t}^{n,\mu,\vartheta}(\varphi,\aleph)$. Therefore,

$$a_\iota \leq \frac{\aleph|\varphi|}{([\iota]_q+\aleph|\varphi|-1)\left[F+\left([\iota]_q+\vartheta-\mu-F\right)t\right]^n}, \qquad \iota \geq 2,$$

which yields sharp results for the function

$$f(z) = z + \frac{\aleph|\varphi|}{([\iota]_q+\aleph|\varphi|-1)\left[F+\left([\iota]_q+\vartheta-\mu-F\right)t\right]^n} z^\iota, \qquad \iota \geq 2.$$

Growth and distortion properties for functions in the class $\mathcal{T}_{q,F,t}^{n,\mu,\vartheta}(\varphi,\aleph)$ will be given in the following results:

3 Growth and Distortion Theorems

Theorem 3.1. If f an analytic function given by (1.2) is in the class $\mathcal{T}_{q,F,t}^{n,\mu,\vartheta}(\varphi,\aleph)$, and $|z| = \varkappa < 1$. Then, we have

$$1) \quad |f(z)| \geq \varkappa - \frac{\aleph|\varphi|}{(q+\aleph|\varphi|)\left[F+\left(q+1+\vartheta-\mu-F\right)t\right]^n}\varkappa^2.$$

$$2) \quad |f(z)| \leq \varkappa + \frac{\aleph|\varphi|}{(q+\aleph|\varphi|)\left[F+\left(q+1+\vartheta-\mu-F\right)t\right]^n}\varkappa^2.$$

Here, the bounds are sharp, provided that the equality is obtained by the following function

$$f(z) = z + \frac{\aleph|\varphi|}{(q+\aleph|\varphi|)\left[F+\left(q+1+\vartheta-\mu-F\right)t\right]^n} z^2.$$

Proof . With respect to Theorem 2.1, we obtain

$$\sum_{\iota=2}^{\infty}([\iota]_q+\aleph|\varphi|-1)\left[F+\Big([\iota]_q+\vartheta-\mu-F\Big)t\right]^n a_\iota \leq \aleph|\varphi|,$$

and

$$\begin{aligned}&(q+\aleph|\varphi|)\left[F+\Big(1+q+\vartheta-\mu-F\Big)t\right]^n \sum_{\iota=2}^{\infty} a_\iota \\ &\leq \sum_{\iota=2}^{\infty}([\iota]_q+\aleph|\varphi|-1)\left[F+\Big([\iota]_q+\vartheta-\mu-F\Big)t\right]^n a_\iota \\ &\leq \aleph|\varphi|.\end{aligned}$$

Thus, we have

$$\sum_{\iota=2}^{\infty} a_\iota \leq \frac{\aleph\,|\varphi|}{(q+\aleph\,|\varphi|)\left[\digamma(1-t)+\Big(1+q+\vartheta-\mu\Big)t\right]^n}.$$

Therefore, for $f \in \mathcal{T}_{q,\digamma,t}^{n,\mu,\vartheta}(\varphi,\aleph)$ and the fact that $|z| = \varkappa < 1$, we obtain

$$\begin{aligned}|f(z)| &= |z - \sum_{\iota=2}^{\infty} a_\iota z^\iota| \\ &\leq |z| + \sum_{\iota=2}^{\infty} a_\iota |z|^2 \\ &= \varkappa + \varkappa^2 \sum_{\iota=2}^{\infty} a_\iota \\ &\leq \varkappa + \frac{\aleph\,|\varphi|}{(q+\aleph\,|\varphi|)\left[\digamma(1-t)+\Big(1+q+\vartheta-\mu\Big)t\right]^n}\varkappa^2.\end{aligned}$$

Other assertions may be proven as given below

$$\begin{aligned}|f(z)| &= |z - \sum_{\iota=2}^{\infty} a_\iota z^\iota| \\ &\geq |z| - \sum_{\iota=2}^{\infty} a_\iota |z|^2 \\ &= \varkappa - \varkappa^2 \sum_{\iota=2}^{\infty} a_\iota \\ &\geq \varkappa - \frac{\aleph\,|\varphi|}{(q+\aleph\,|\varphi|)\left[\digamma(1-t)+\Big(1+q+\vartheta-\mu\Big)t\right]^n}\varkappa^2.\end{aligned}$$

This completes the proof. □

Upon following the similar method given in Theorem 3.1, we may prove the theorem given below.

Theorem 3.2. If f an analytic function given by (1.2) is in the class $\mathcal{T}_{q,\digamma,t}^{n,\mu,\vartheta}(\varphi,\aleph)$. Therefore, we obtain

$$1)\quad |\partial_q f(z)| \geq 1 - \frac{(1+q)\aleph\,|\varphi|}{(q+\aleph\,|\varphi|)\left[\digamma+\big(q+1+\vartheta-\mu-\digamma\big)t\right]^n}\varkappa.$$

$$2)\quad |\partial_q f(z)| \leq 1 + \frac{(1+q)\aleph\,|\varphi|}{(q+\aleph\,|\varphi|)\left[\digamma+\big(q+1+\vartheta-\mu-\digamma\big)t\right]^n}\varkappa.$$

Here, the bounds are sharp, provided that the equality is obtained by the following function

$$f(z) = z + \frac{\aleph\,|\varphi|}{(q+\aleph\,|\varphi|)\left[\digamma+\big(q+1+\vartheta-\mu-\digamma\big)t\right]^n}z^2.$$

Proof . Let $f \in \mathcal{T}_{q,\digamma,t}^{n,\mu,\vartheta}(\varphi,\aleph)$. Then, we have

$$\begin{aligned}|\partial_q f(z)| &= \left|1 - \sum_{\iota=2}^{\infty}[\iota]_q a_\iota z^{\iota-1}\right| \leq 1 + [2]_q \sum_{\iota=2}^{\infty} a_\iota |z| = 1 + [2]_q \varkappa \sum_{\iota=2}^{\infty} a_\iota \\ &\leq 1 + \frac{(1+q)\aleph\,|\varphi|}{(q+\aleph\,|\varphi|)\left[\digamma(1-t)+\Big(1+q+\vartheta-\mu\Big)t\right]^n}\varkappa.\end{aligned}$$

The remaining assertion may be proven as follows

$$|\partial_q f(z)| = \left|1 - \sum_{\iota=2}^{\infty}[\iota]_q a_\iota z^{k-1}\right| \geq 1 - [2]_q \sum_{\iota=2}^{\infty} a_\iota |z| = 1 - [2]_q \varkappa \sum_{\iota=2}^{\infty} a_\iota$$
$$\geq 1 - \frac{(1+q)\aleph|\varphi|}{(q+\aleph|\varphi|)\left[F(1-t)+\Big(1+q+\vartheta-\mu\Big)t\right]^n}\varkappa.$$

The proof is complete. □

4 Extreme Points

The extreme points with respect to the class $\mathcal{T}_{q,F,t}^{n,\mu,\vartheta}(\varphi,\aleph)$ will now be discussed.

Theorem 4.1. Let $f_1(z) = z$ and

$$f_\iota(z) = z - \frac{\aleph|\varphi|}{([\iota]_q + \aleph|\varphi| - 1)\left[F + \Big([\iota]_q + \vartheta - \mu - F\Big)t\right]^n} z^\iota, \quad (\iota \geq 2).$$

Then, $f \in \mathcal{T}_{q,F,t}^{n,\mu,\vartheta}(\varphi,\aleph)$ if and only if it may be written as

$$f(z) = \sum_{\iota=1}^{\infty} \delta_\iota f_\iota(z),$$

in which $\delta_\iota \geq 0$ and $\sum_{\iota=1}^{\infty} \delta_\iota = 1$.

Proof . Let $f(z) = \sum_{\iota=1}^{\infty} \delta_\iota f_\iota(z)$, where $\delta_\iota \geq 0$ and $\sum_{\iota=1}^{\infty} \delta_\iota = 1$, then

$$\begin{aligned}
f(z) &= \sum_{\iota=1}^{\infty} \delta_\iota f_\iota(z) = \delta_1 f_1(z) + \sum_{\iota=2}^{\infty} \delta_\iota f_\iota(z) \\
&= \delta_1 z + \sum_{\iota=2}^{\infty} \delta_\iota \left\{ z - \frac{\aleph|\varphi|}{([\iota]_q + \aleph|\varphi| - 1)\left[F + \Big([\iota]_q + \vartheta - \mu - F\Big)t\right]^n} z^\iota \right\} \\
&= \delta_1 z + \sum_{\iota=2}^{\infty} \delta_\iota z - \sum_{\iota=2}^{\infty} \delta_\iota \left(\frac{\aleph|\varphi|}{([\iota]_q + \aleph|\varphi| - 1)\left[F + \Big([\iota]_q + \vartheta - \mu - F\Big)t\right]^n} z^\iota \right) \\
&= \sum_{\iota=1}^{\infty} \delta_\iota z - \sum_{\iota=2}^{\infty} \delta_\iota \left(\frac{\aleph|\varphi|}{([\iota]_q + \aleph|\varphi| - 1)\left[F + \Big([\iota]_q + \vartheta - \mu - F\Big)t\right]^n} z^\iota \right) \\
&= z - \sum_{\iota=2}^{\infty} \delta_\iota \left(\frac{\aleph|\varphi|}{([\iota]_q + \aleph|\varphi| - 1)\left[F + \Big([\iota]_q + \vartheta - \mu - F\Big)t\right]^n} z^\iota \right).
\end{aligned}$$

Upon implementing the condition given in (2.1) of Theorem 2.1, we now have

$$\begin{aligned}
&\sum_{\iota=2}^{\infty}([\iota]_q + \aleph|\varphi| - 1)\left[F + \Big([\iota]_q + \vartheta - \mu - F\Big)t\right]^n \left\{ \frac{\aleph|\varphi|}{([\iota]_q + \aleph|\varphi| - 1)\left[F + \Big([\iota]_q + \vartheta - \mu - F\Big)t\right]^n} \right\} \\
&= \sum_{\iota=2}^{\infty} \delta_\iota \aleph|\varphi| = \aleph|\varphi| \left(\sum_{\iota=1}^{\infty} \delta_\iota - \delta_1 \right) = (1-\delta_1)\aleph|\varphi| \leq \aleph|\varphi|.
\end{aligned}$$

Thus, $f \in \mathcal{T}_{q,F,t}^{n,\mu,\vartheta}(\varphi,\aleph)$.

Conversely, suppose that $f \in \mathcal{T}_{q,F,t}^{n,\mu,\vartheta}(\varphi,\aleph)$. We then set the following

$$\delta_\iota = \frac{\aleph\,|\varphi|}{([\iota]_q + \aleph\,|\varphi| - 1)\left[F + \left([\iota]_q + \vartheta - \mu - F\right)t\right]^n} a_\iota \text{ and } \delta_1 = 1 - \sum_{\iota=2}^{\infty} \delta_\iota.$$

Then,

$$\begin{aligned} f_\iota(z) &= z - \sum_{\iota=2}^{\infty} a_\iota z^\iota \\ &= z - \sum_{\iota=2}^{\infty} \delta_\iota \frac{\aleph\,|\varphi|}{([\iota]_q + \aleph\,|\varphi| - 1)\left[F + \left([\iota]_q + \vartheta - \mu - F\right)t\right]^n} z^\iota \\ &= \left(1 - \sum_{\iota=2}^{\infty} \delta_\iota\right) z + \sum_{\iota=2}^{\infty} \delta_\iota f_\iota \\ &= \delta_1 f_1(z) + \sum_{\iota=2}^{\infty} \delta_\iota f_\iota \\ &= \sum_{\iota=1}^{\infty} \delta_\iota f_\iota. \end{aligned}$$

This completes the assertion given in Theorem 4.1. □

Corollary 4.2. The extreme points with respect to class $\mathcal{T}_{q,F,t}^{n,\mu,\vartheta}(\varphi,\aleph)$ are the functions $f_1(z) = z$ and

$$f_\iota(z) = z - \frac{\aleph\,|\varphi|}{([\iota]_q + \aleph\,|\varphi| - 1)\left[F + \left([\iota]_q + \vartheta - \mu - F\right)t\right]^n} z^\iota, \quad k \geq 2.$$

5 Closure Theorems

Theorem 5.1. Let the function $\delta_\tau \geq 0$ for $\tau = 1, 2, \ldots$ and $\sum\limits_{\tau=1}^{\ell} \delta_\tau = 1$. If the function f_τ expressed by

$$f_\tau(z) = z - \sum_{\iota=2}^{\infty} a_{\iota,\tau} z^\iota \left(a_{\iota,\tau} \geq 0, \tau = 1, 2, \ldots, \ell\right), \tag{5.1}$$

are in the class $\mathcal{T}_{q,F,t}^{n,\mu,\vartheta}(\varphi,\aleph)$ for every $\tau = 1, 2, \ldots, \ell$. Therefore, the function $h(z)$ may be expressed by

$$h(z) = z - \sum_{\iota=2}^{\infty} \left(\sum_{\tau=1}^{\ell} \delta_\tau a_{\iota,\tau}\right) z^\iota,$$

which is in the class $\mathcal{T}_{q,F,t}^{n,\mu,\vartheta}(\varphi,\aleph)$.

Proof . Since $f_\tau(z) \in \mathcal{T}_{q,F,t}^{n,\mu,\vartheta}(\varphi,\aleph)$ for every $\tau = 1, 2, \ldots, \ell$. Therefore, we obtain

$$\sum_{\iota=2}^{\infty} ([\iota]_q + \aleph\,|\varphi| - 1)\left[F + \left([\iota]_q + \vartheta - \mu - F\right)t\right]^n a_{\iota,\tau} \leq \aleph\,|\varphi|.$$

Hence, we obtain

$$\begin{aligned}&\sum_{\iota=2}^{\infty}\left\{([\iota]_q+\aleph|\varphi|-1)\left[F+\Big([\iota]_q+\vartheta-\mu-F\Big)t\right]^n\right\}\left(\sum_{\tau=1}^{\ell}\delta_\tau a_{\iota,\tau}\right)\\&=\sum_{\tau=1}^{\ell}\delta_\tau\left(\sum_{\iota=2}^{\infty}([\iota]_q+\aleph|\varphi|-1)\left[F+\Big([\iota]_q+\vartheta-\mu-F\Big)t\right]^n a_{\iota,\tau}\right)\\&\le\aleph|\varphi|\sum_{\tau=1}^{\ell}\delta_\tau\le\aleph|\varphi|.\end{aligned}$$

This shows that $h\in\mathcal{T}_{q,F,t}^{n,\mu,\vartheta}(\varphi,\aleph)$. Therefore, the theorem's proof is completed. □

Corollary 5.2. The class $\mathcal{T}_{q,F,t}^{n,\mu,\vartheta}(\varphi,\aleph)$ is closed with respect to convex linear combination.

Proof . We assume that the functions $f_\iota(\iota=1,2)$ stated in (5.1) are in $\mathcal{T}_{q,F,t}^{n,\mu,\vartheta}(\varphi,\aleph)$. It is adequate enough to prove that the function $\mathfrak{h}$ expressed by

$$\mathfrak{h}(z)=\gamma f_1(z)+(1-\gamma)f_2(z),\qquad 0\le\gamma\le1$$

belongs to the class $\mathcal{T}_{q,F,t}^{n,\mu,\vartheta}(\varphi,\aleph)$. By taking $\ell=2$, $\delta_1=\gamma$ and $\delta_2=1-\gamma$ in Theorem 5.1, we obtain the corollary. □

6 Convolution Properties

Theorem 6.1. Let $\mathfrak{g}(z)$ of the form

$$\mathfrak{g}(z)=z-\sum_{\iota=2}^{\infty}d_\iota z^\iota$$

be analytic in $\mathbb{U}$. Suppose we have $f\in\mathcal{T}_{q,F,t}^{n,\mu,\vartheta}(\varphi,\aleph)$. Therefore, the function $f*\mathfrak{g}$ also sits in the class $\mathcal{T}_{q,F,t}^{n,\mu,\vartheta}(\varphi,\aleph)$. Note that the symbol $''*''$ denotes the Hadamard product (also known as convolution).

Proof . Since $f\in\mathcal{T}_{q,F,t}^{n,\mu,\vartheta}(\varphi,\aleph)$, we have

$$\sum_{\iota=2}^{\infty}([\iota]_q+\aleph|\varphi|-1)\left[F+\Big([\iota]_q+\vartheta-\mu-F\Big)t\right]^n a_\iota\le\aleph|\varphi|.$$

Upon employing the last inequality and the fact that

$$(f*\mathfrak{g})(z)=f(z)*\mathfrak{g}(z)=z-\sum_{\iota=2}^{\infty}a_\iota d_\iota z^\iota,$$

we now have

$$\begin{aligned}&\sum_{\iota=2}^{\infty}([\iota]_q+\aleph|\varphi|-1)\left[F+\Big([\iota]_q+\vartheta-\mu-F\Big)t\right]^n a_\iota d_\iota\le\\&\sum_{\iota=2}^{\infty}([\iota]_q+\aleph|\varphi|-1)\left[F+\Big([\iota]_q+\vartheta-\mu-F\Big)t\right]^n a_\iota\le\aleph|\varphi|.\end{aligned}$$

Hence, with respect to Theorem 2.1, the result follows. □

Concluding Remark

With the aid of q-calculus, we have investigated a new subclass of analytic functions involving the modified q-derivative operator, which we called the q-Opoola derivative operator. For the general subclass of q-starlike univalent functions, we have provided some geometric properties, for example, distortion and growth, coefficient estimates, extreme points, closure theorem, and convolution. The concept outlined in this article can be employed to easily study a large range of analytic and univalent (or multivalent) functions linked to several disciplines, notably those that employ the q-Opoola derivative operator. This may open numerous new lines of inquiry into the Geometric Function Theory of Complex Analysis and appropriate areas.

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