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Metric dimension and neighbourhood resolving set for the zero divisor graphs of order at most 10 of a small finite commutative ring

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Abstract

Let R be a commutative ring and $\Gamma(R)$ be its zero-divisor graph. All the vertices of zero divisor graphs are the non-zero divisors of the commutative ring, with two distinct vertices joined by an edge in case their product in the commutative ring is zero. In this paper, we study the metric dimension and neighbourhood resoling set for the zero divisor graphs of order 3,4,5,6,7,8,9,10 of a small finite commutative ring with a unit.

Keywords: Commutative ring, Zero divisor graph, Resolving set, Metric dimension, Neighbourhood set
2020 MSC: Primary: 05C12; Secondary: 05E30

1 Introduction

The concept of zero-divisor graph of a commutative ring was introduced by I. Beck in 1988 [2]. He let all elements of the ring be vertices of the graph and was interested mainly in colorings. In [1], Anderson and Livingston introduced and studied the zero-divisor graph whose vertices are the non-zero divisors. This graph turns out to best exhibit the properties of the set of zero-divisors of a commutative ring. The zero-divisor graph helps us to study the algebraic properties of rings using graph theoretical tools.

The concept metric dimension of connected graphs and its related properties are first introduced by Slater [8] in 1975, independently by Harary and Melter [5] in 1976. A subset of vertices S resolves a graph G if every vertex of G is uniquely determined by its vector of distances to the vertices in S. A resolving set of minimum cardinality for a graph G is called a minimum resolving set. A minimum resolving set is usually called a basis for G and the cardinality of basis is called the metric dimension of G, denoted by $\beta(G)$.

Let $G(V,E)$ be a graph. For any element $v \in V$, the collection $\bar{N}[v]$ of all elements which are adjacent to v and also v itself. A subcollection N is known as $\bar{n} - set$ of G if the total graph G is the finite union of $\bar{N}[v]$ for each $v \in N$ .The least number of elements in $\bar{n}set$ is called the neighbourhood number of G and is denoted by $n(G)$. Sampathkumar [7] is first introduce the concept of neighbourhood number. The least number of elements in $\bar{r} - set$ is known as resolving number of G and is represented by $r(G)$. A neighbourhood resolving number $nr(G)$ is the least number of elements in the $\bar{nr}set$. The concepts of resolving set introduced by Slater [8] and independently by Harary [5].

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2 Preliminaries

Lemma 2.1. [10] For any graph G of order k, $nr(G) \leq k-1$.

Lemma 2.2. [9] For any positive integer n, $nr(P_n) = \begin{cases} \lceil \frac{n}{2} \rceil & \text{for } n \leq 3 \\ \lfloor \frac{n}{2} \rfloor & \text{for } n \geq 4. \end{cases}$

Lemma 2.3. [9] For any positive integer $k \geq 3$, $nr(C_k) = \begin{cases} 3, & \text{for } k = 4 \\ \lceil \frac{k}{2} \rceil, & otherwise \end{cases}$

The following definition will be used in the paper:

Definition 2.4. [4] A Commutative Ring $< R, +, . >$ is a set R together with two binary operations $+$, and ., which we call addition and multiplication, defined on R such that the following axioms are satisfied.
1. $< R, + >$ is an abelian group.
2. Multiplication is associative
3. For $a, b, c \in R$, $a.(b+c) = a.b + a.c$ and $(a+b).c = a.c + b.c$, hold.
4. $a.b = b.a$ for all $a, b \in R$.
5. A unity (or identity) in a ring is a nonzero element. that is an identity under multiplication.

The set of all zero divisors in R is denoted by $Z(R)$ and the set of all non zero zero divisors in R is denoted by $Z^*(R)$.

Definition 2.5. [3] Let $v_1, v_2, v_3 \in V$. If the metric between v_1 , v_2 and v_3 , v_2 are different then we say that vertex v_2 is resolve the vertices v_1 and v_3.

Definition 2.6. [10] Let $P = \{p_1, p_2, ..., p_r\}$ of V and any $w \in V$. The distance of w to the subset P is the r touple of metric between w and p_i, $i = 1, 2, ...r$, represented by $r(w/P) = (x_1, x_2, ..., x_r)$ where $x_i = d(w, p_i)$. If $r(x/P) \neq r(y/P)$, for any $x, y \in V$ then P is known as resolving set for G.

Definition 2.7. [11] Let $R \subseteq V$ and $R = \{k_1, k_2, ..., k_r\}$ $r \geq 1$ and $w \in V$. A binary neighbourhood metric of w to the r touple $(k_1, k_2, ..., k_r)$ is defined by $M_R(w) = (p_1, p_2, ..., p_r)$, where $p_i = \begin{cases} 1 & \text{if } w \in \bar{N}[k_i],\ 1 \leq i \leq r \\ 0 & \text{otherwise.} \end{cases}$

If $M_R(x) \neq M_R(y)$ for each $x, y \in V$, then R is known as $\bar{n}r - set$ or neighbourhood resolving set.

Example 2.8. Consider the ring $R = Z_6 = \{0, 1, 2, 3, 4, 5\}$, $Z^*(R) = \{2, 3, 4\}$ and corresponding zero divisor graph $G = K_{1,2}$ is given below. From the figure $W = \{2\}$ is the resolving set and metric dimension $dim(K_{1,2}) = 1$ and since $N[2] \neq G$ So $S = \{2, 4\}$ is also a resolving set for G and $N[2] \cup N[4] = G$ so the least cardinality of neighbourhood resolving set for G is $nr(K_{1,2}) = 2$.

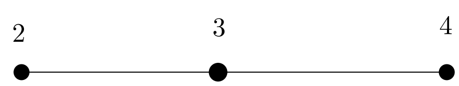

Figure. 1
$K_{1,2}$

3 Metric dimension and Neighbourhood resolving set of zero divisor graphs

In this section, we consider a simple connected zero divisor graphs G with countable number of vertices, examined some graphs for find the metric dimension and neighbourhood resolving set obtained the following results.

Theorem 3.1. If any Zero divisor graph with 3 vertices have the metric dimension ($\beta(G)$) and the least cardinality of neighbourhood resolving set ($nr(G)$) are given in the table 1.

Table 1: Zero divisor graphs with 3 vertices

No. Vertices	Ring	$\|R\|$	Graph G	$\beta(G)$	$nr(G)$
3	Z_6	6	$K_{1,2}$	1	2
	Z_8	8			
	$\dfrac{Z_2[x]}{(x^3)}$	8			
	$\dfrac{Z_4[x]}{(2x, x^2-2)}$	8			
3	$\dfrac{Z_2[x,y]}{(x,y)^2}$	8	K_3	1	0
	$\dfrac{Z_4[x]}{(2,x)^2}$	8			
	$\dfrac{F_4[x]}{(x^2)}$	16			
	$\dfrac{Z_4[x]}{(x^2+x+1)}$	16			

Proof . **Case.1:** The zero divisor graphs $K_{1,2}$ is the special cases of bipartate graph $K_{m,n}$, with $m = 1, n = 2$. Therefore $\beta(K_{1,2}) = 2$, $nr(K_{1,2}) = 2$.
Case.2: The zero divisor graphs K_3 is the complete graph K_n with $n = 3$. Therefore $\beta(K_3) = 2$ and $nr(K_3) = 0$. □

Theorem 3.2. If any Zero divisor graph with 4 vertices have the metric dimension ($\beta(G)$) and the least cardinality of neighbourhood resolving set ($nr(G)$) are given in the table 2.

Table 2: Zero divisor graphs with 4 vertices

No. Vertices	Ring	$\|R\|$	Zero divisor graph	$\beta(G)$	$nr(G)$
4	$Z_2 \times Z_4$	8	$K_{1,3}$	2	3
	$Z_3 \times Z_3$	9	$K_{2,2}$	2	3
	Z_{25}	25	K_4	3	0
	$\dfrac{Z_5[x]}{(x^2)}$	25	K_4	3	0

Proof . **Case.1:** The zero divisor graphs $K_{1,3}$ and $K_{2,2}$ are the special cases of bipartate graph $K_{m,n}$, with $m = 1, n = 3$ and $m = n = 2$.
Therefore $\beta(K_{1,3}) = 2$, $nr(K_{1,3}) = 3$.
Case.2: The zero divisor graphs K_4 is the complete graph K_n with $n = 4$. Therefore $\beta(K_4) = 3$ and $lnr(K_4) = 0$ □

Theorem 3.3. If any Zero divisor graph with 5 vertices have the metric dimension ($\beta(G)$) and the least cardinality of neighbourhood resolving set ($nr(G)$) are given in the table 3

Proof . **Case.1:** The Zero divisor Graph $K_{1,4}$ and $K_{2,3}$ represents the special cases of the bipartate graph $K_{m,n}$ with $m = 1, n = 4$ and $m = 2, n = 3$ respectively. Here $\beta(K_{1,4}) = 3$, $nr(K_{1,4}) = 4$, $\beta(K_{2,3}) = 3$, $nr(K_{2,3}) = 3$
Case.2: The Zero divisor graph for the rings $Z_2 \times Z_4$ and $Z_2 \times \dfrac{Z_2[x]}{x^2}$ is in Figure.2
From this Figure three vertices have the degree one and the remaining two vertices have degree more than one. The set $\{1, 4\}$ is the resolving set for the graph. Therefore $\beta(G) = 2$. Also set $\{2, 3, 4\}$ act as a neighbourhood resolving set for this graph. Therefore $nr(G) = 3$.

Table 3: Zero divisor graphs with 5 vertices

No. Vertices	Ring	$\|R\|$	Zero divisor graph	$\beta(G)$	$nr(G)$
5	$Z_2 \times Z_5$	10	$K_{1,4}$	3	4
	$Z_3 \times F_4$	12	$K_{2,3}$	3	3
	$Z_2 \times Z_4$	8	Fig. 2	2	3
	$Z_2 \times \dfrac{Z_2[x]}{x^2}$	8	Fig. 2	2	3

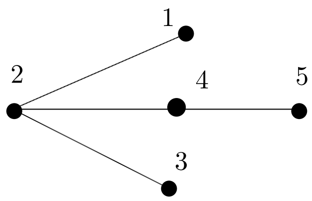

Figure.2

□

Theorem 3.4. If any Zero divisor graph with 6 vertices have the metric dimension ($\beta(G)$) and the least cardinality of neighbourhood resolving set ($nr(G)$) are given in the table 4.

Table 4: Zero divisor graphs with 6 vertices

No. Vertices	Ring	$\|R\|$	Zero divisor graph	$\beta(G)$	$nr(G)$
6	$Z_3 \times Z_5$	15	$K_{2,4}$	4	4
	$F_4 \times F_4$	16	$K_{3,3}$	4	4
	$Z_2 \times Z_2 \times Z_2$	8	Fig. 3	2	4
	$\dfrac{Z_7[x]}{x^2}$	49	K_6	5	0
	Z_{49}	49	K_6	5	0

Proof . Case.1: The zero divisor graphs $K_{2,4}$ and $K_{3,3}$ are the special cases of bipartate graph $K_{m,n}$.
Therefore $\beta(K_{2,4}) = 4$, $nr(K_{2,4}) = 4$ and $\beta(K_{3,3}) = 4$, $nr(K_{3,3}) = 4$.
Case.2: The zero divisor graph for the Ring $Z_2 \times Z_2 \times Z_2$ is in the Figure 3.

From the figure there are 3 vertices with degree 1 and remaining 3 vertices have degree 3. Therefore $\{2, 3\}$ is the resolving set and its metric dimension is 2. Set $\{2, 4, 5, 6\}$ is the least neighbourhood resolving set for the graph shown in the Figure 3.

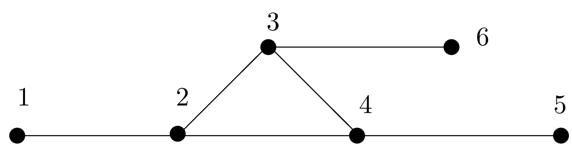

Figure. 3

□

Theorem 3.5. If any Zero divisor graph with 7 vertices have the metric dimension ($\beta(G)$) and the least cardinality of neighbourhood resolving set ($nr(G)$) are given in the table 5

Proof . Case.1: The Zero divisor graph $K_{1,6}$ and $K_{3,4}$ are the special case of $K_{m,n}$ with $m = 1, n = 6$ and $m = 3, n = 4$. Therefore $\beta(K_{1,6}) = 5$, $nr(K_{1,6}) = 6$. $\beta(K_{3,4}) = 5$, $nr(K_{3,4}) = 5$

Table 5: Zero divisor graphs with 7 vertices

No. Vertices	Ring	$\|R\|$	Zero divisor graph	$\beta(G)$	$nr(G)$
7	$Z_2 \times Z_7$	14	$K_{1,6}$	5	6
	$F_4 \times Z_5$	20	$K_{3,4}$	5	5
	$Z_3 \times Z_4$	12	Fig. 4	4	5
	$Z_3 \times \dfrac{Z_2[x]}{x^2}$	12	Fig. 4	4	5
	Z_{16}	16	Fig. 5	4	0
	$Z_2[x]/x^4$	16	Fig. 5	4	0
	$Z_4[x]/x^4+2$	16	Fig. 5	4	0
	$Z_4[x]/x^2+3x$	16	Fig. 5	4	0
	$Z_4[x]/(x^3-2, 2x^2, 2x)$	16	Fig. 5	4	0
	$Z_2[x,y]/(x^3, xy, y^2)$	16	Fig. 6	5	0
	$Z_8[x]/(2x, x^2)$	16	Fig. 6	5	0
	$Z_4[x]/(x^3, 2x^2, 2x)$	16	Fig. 6	5	0
	$Z_4[x]/(x^2+2x)$	16	Fig. 7	3	0
	$Z_8[x]/(2x, x^2+4)$	16	Fig. 7	3	0
	$Z_2[x,y]/(x^2, y^2-xy)$	16	Fig. 7	3	0
	$Z_4[x,y]/(x^2, y^2-xy, xy-2, 2x, 2y)$	16	Fig. 7	3	0
	$Z_4[x,y]/(x^2, y^2, xy-2, 2x, 2y)$	16	Fig. 8	3	0
	$Z_2[x,y]/(x^2, y^2)$	16	Fig. 8	3	0
	$Z_4[x]/(x^2)$	16	Fig. 8	3	0
	$Z_4[x]/(x^3-x^2-2, 2x^2, 2x)$	16	Fig. 9	4	5
	$Z_2[x,y,z]/(x,y,z)^2$	16	K_7	6	0
	$Z_4[x,y]/(x^2, y^2, 2x, 2y)$	16	K_7	6	0
	$F_8[x]/(x^2)$	64	K_7	6	0
	$Z_4[x]/(x^3+x+1)$	64	K_7	6	0

Case.2: The zero divisor graph for the Rings $Z_3 \times Z_4$ and $Z_3 \times \dfrac{Z_2[x]}{x^2}$ are in the Figure. 4. From the Figure we can show the metric dimension and neighbourhood resolving set for the graph. Therefore $\beta(G) = 4$, $nr(G) = 5$.

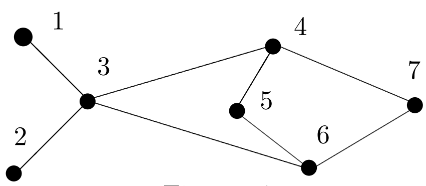

Figure. 4

Case.3: The zero divisor graph for the Rings Z_{16}, $Z_2[x]/x^4$, $Z_4[x]/x^4+2$, $Z_4[x]/x^2+3x$, $Z_4[x]/(x^3-2, 2x^2, 2x)$ are in the Figure. 5. From the Figure we can show the metric dimension and neighbourhood resolving set for the graph. Here $\beta(G) = 4$, $nr(G) = 0$.

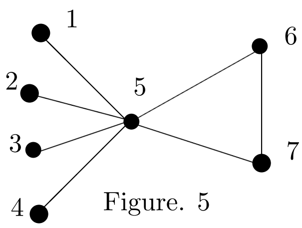

Figure. 5

Case.4: The zero divisor graph for the Rings $Z_2[x,y]/(x^3, xy, y^2)$, $Z_8[x]/(2x, x^2)$, $Z_4[x]/(x^3, 2x^2, 2x)$ are in the Figure.

6. The metric dimension and least neighbourhood resolving set for the graph is $\beta(G) = 5$, $nr(G) = 0$.

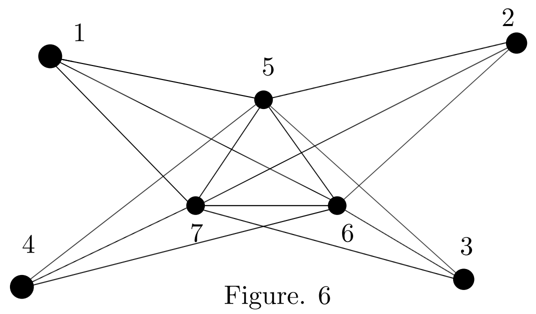

Figure. 6

Case.5: The zero divisor graph for the rings

$$Z_4[x]/(x^2 + 2x), Z_8[x]/(2x, x^2 + 4), Z_2[x, y]/(x^2, y^2 - xy), Z_4[x, y]/(x^2, y^2 - xy, xy - 2, 2x, 2y)$$

are in the Figure. 7. The metric dimension and least cardinality of neighbourhood resolving sets are determined from the figure for the above graphs. Here $\beta(G) = 3$, $nr(G) = 0$.

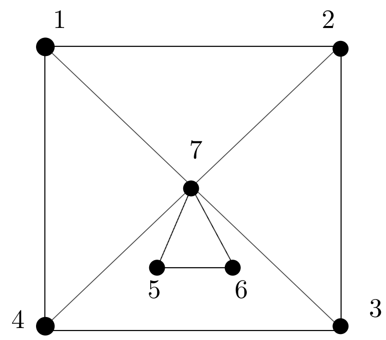

Figure. 7

Case.6: The zero divisor graph for the Rings $Z_4[x, y]/(x^2, y^2, xy - 2, 2x, 2y)$, $Z_2[x, y]/(x^2, y^2)$, $Z_4[x]/(x^2)$ are in the Figure. 8. The metric dimension and least cardinality of neighbourhood resolving set of above graphs are $\beta(G) = 3$, $nr(G) = 0$.

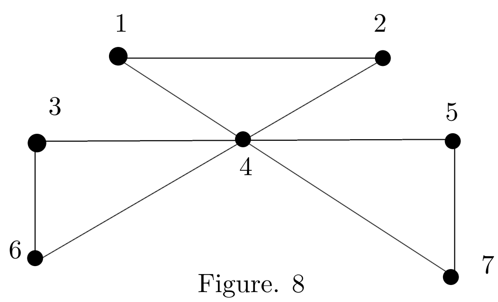

Figure. 8

Case.7: The zero divisor graph for the Ring $Z_4[x]/(x^3 - x^2 - 2, 2x^2, 2x)$ is in the Figure. 9. Also, determined the metric dimension and least cardinality of this graph is $\beta(G) = 4$, $nr(G) = 5$. ,

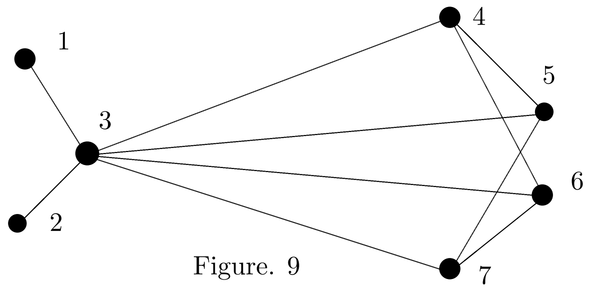

Figure. 9

Case.8: The zero divisor graph of K_7 is the case of K_n with $n = 7$, so the $\beta(K_n) = n - 1$, $nr(K_n) = 0$. Therefore $\beta(K_7) = 6$, $nr(K_7) = 0$. □

Theorem 3.6. If any Zero divisor graph with 8 vertices have the metric dimension ($\beta(G)$) and the least cardinality of neighbourhood resolving set ($nr(G)$) are given in the table 6

Table 6: Zero divisor graphs with 8 vertices

No. Vertices	Ring	$\lvert R\rvert$	Zero divisor graph	$\beta(G)$	$nr(G)$
	$Z_2 \times F_8$	16	$K_{1,7}$	6	7
	$Z_3 \times Z_7$	21	$K_{2,6}$	6	6
	$Z_5 \times Z_5$	25	$K_{4,4}$	6	6
	Z_{27}	27	Fig. 10	6	0
	$Z_9[x]/(3x, x^2 - 3)$	27	Fig. 10	6	0
8	$Z_9[x]/(3x, x^2 - 6)$	27	Fig. 10	6	0
	$Z_3[x]/(x^3)$	27	Fig. 10	6	0
	$Z_3[x, y]/(x, y)^2$	27	K_8	7	0
	$Z_9[x]/(3, x)^2$	27	K_8	7	0
	$F_9[x]/(x^2)$	81	K_8	7	0
	$Z_9[x]/(x^2 + 1)$	81	K_8	7	0

Proof . Case.1: The Zero divisor graph $K_{1,7}$, $K_{2,6}$ and $K_{4,4}$ are the special case of $K_{m,n}$ with $m = 1, n = 7$, $m = 2, n = 6$ and $m = 4, n = 4$. Therefore $\beta(K_{1,7}) = 6$, $nr(K_{1,7}) = 7$. $\beta(K_{2,6}) = 6$, $nr(K_{2,6}) = 6$ $\beta(K_{4,4}) = 6$, $nr(K_{4,4}) = 6$.
Case.2: The zero divisor graph for the Ring Z_{27}, $Z_9[x]/(3x, x^2 - 3)$, $Z_9[x]/(3x, x^2 - 6)$, $Z_3[x]/(x^3)$ From the figure. 10 there are 2 vertices with degree 7 and remaining 6 vertices have degree 2. The resolving set contains 5 vertices of degree 2 and one vertex of degree 7. Therefore least cardinality of resolving set is 6. From the figure there are two vertices have the neighbourhood set contains all vertices of this graph. Therefore neighbourhood resolving set of this graph is zero. Here $\beta(G) = 6$, $nr(G) = 0$.

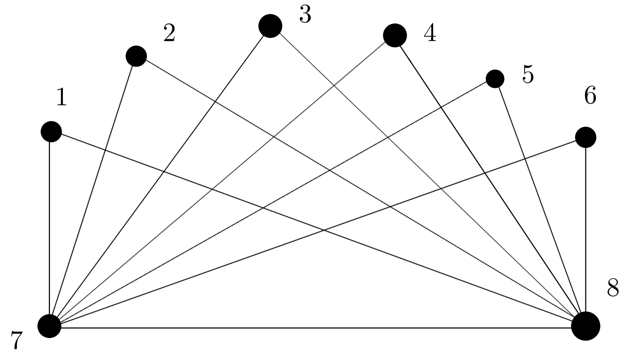

Figure. 10

Case.3: The zero divisor graph of K_8 is the case of K_n with $n = 8$, so the $\beta(K_n) = n - 1$, $nr(K_n) = 0$. Therefore $\beta(K_8) = 7$, $nr(K_8) = 0$. □

Theorem 3.7. If any Zero divisor graph with 9 vertices have the metric dimension ($\beta(G)$) and the least cardinality of neighbourhood resolving set ($nr(G)$) are given in the table 7.

Table 7: Zero divisor graphs with 9 vertices

No. Vertices	Ring	$\|R\|$	Zero divisor graph	$\beta(G)$	$nr(G)$
9	$Z_2 \times F_9$	18	$K_{1,8}$	7	8
	$Z_3 \times F_8$	24	$K_{2,7}$	7	7
	$F_4 \times Z_7$	28	$K_{3,6}$	7	7
	$Z_2 \times Z_2 \times Z_3$	12	Fig.11	3	5
	$Z_4 \times F_4$	16	Fig.12	5	6
	$Z_2[x]/(x^2) \times F_4$	16	Fig.12	5	6

Proof . **Case.1:** The Zero divisor graph $K_{1,8}$, $K_{2,7}$ and $K_{3,6}$ are the special case of $K_{m,n}$ with $m = 1, n = 8$, $m = 2, n = 7$ and $m = 3, n = 6$. Therefore $\beta(K_{1,8}) = 7$, $nr(K_{1,8}) = 8$. $\beta(K_{2,7}) = 7$, $nr(K_{2,7}) = 7$ $\beta(K_{3,6}) = 7$, $nr(K_{3,6}) = 7$.
Case.2: The zero divisor graph for the Ring $Z_2 \times Z_2 \times Z_3$ is in the Figure. 11. From the figure the graph of 9 vertices in which four vertices with degree 1, another four vertices with degree 3 and remaining one vertex of degree 2. Therefore the resolving set contains three vertices. Here the least cardinality of neighbourhood resolving set is 5. Therefore $\beta(G) = 3$, $nr(G) = 5$.

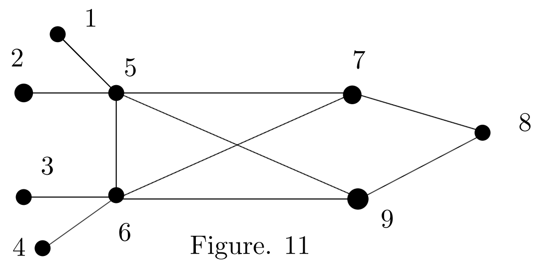

Figure. 11

Case.3: The zero divisor graph for the Ring $Z_4 \times F_4$, $Z_2[x]/(x^2) \times F_4$ is in the Figure. 12. From the figure the graph of 9 vertices in which five vertices with degree 3, three vertices with degree 1 and remaining one vertex of degree 6. The resolving set contains 5 vertices. And the least cardinality of neighbourhood resolving set is 6. Therefore $\beta(G) = 5$, $nr(G) = 6$.

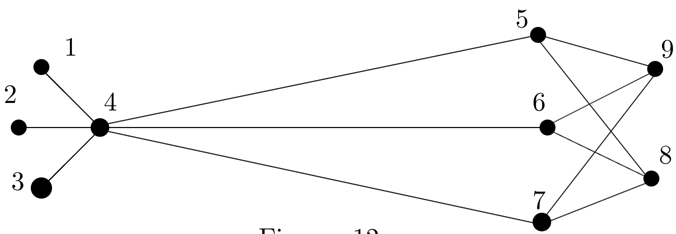

Figure. 12

□

Theorem 3.8. If any Zero divisor graph with 10 vertices have the metric dimension ($\beta(G)$) and the least cardinality of neighbourhood resolving set ($nr(G)$) are given in the table 8

Proof . **Case.1:** The Zero divisor graph $K_{2,8}$, $K_{3,7}$ and $K_{4,6}$ are the special case of $K_{m,n}$. Therefore $\beta(K_{2,8}) = 8$, $nr(K_{2,8}) = 8$. $\beta(K_{3,7}) = 8$, $nr(K_{3,7}) = 8$, $\beta(K_{4,6}) = 8$, $nr(K_{4,6}) = 8$.

Table 8: Zero divisor graphs with 10 vertices

No. Vertices	Ring	$\|R\|$	Zero divisor graph	$\beta(G)$	$nr(G)$
	$Z_3 \times F_9$	27	$K_{2,8}$	8	8
	$F_4 \times F_8$	32	$K_{3,7}$	8	8
10	$Z_5 \times Z_7$	35	$K_{4,6}$	8	8
	Z_{121}	121	K_{10}	9	0
	$Z_{11}[x]/(x^2)$	121	K_{10}	9	0

Case.2: The zero divisor graph K_{10} is a case of K_n complete graph with $n = 10$, so the $\beta(K_n) = n - 1$, $nr(K_n) = 0$ Therefore $\beta(K_{10}) = 9$, $nr(K_{10}) = 0$. □

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