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Analyzing the existence of exponential trapezoidal fuzzy numbers and investigating a method for ranking exponential trapezoidal fuzzy quantities

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Abstract

In 2000, exponential trapezoidal fuzzy numbers were defined for the first time by Chen and Lie [3] without investigating the definition and properties of fuzzy numbers for them. Therefore, in this paper first, some necessary definitions and the difference between fuzzy numbers and fuzzy quantities are presented. Then, the correctness of the definition of exponential trapezoidal fuzzy numbers is investigated. It is concluded that exponential trapezoidal fuzzy numbers do not satisfy general conditions of fuzzy numbers such as bounded amplitude and continuity. In fact, these are fuzzy quantities not fuzzy numbers. As a result, a ranking method is proposed for such quantities to be examined using common tools and test platforms that exist for ranking fuzzy quantities. Moreover, the strength and weaknesses of the method are reported.

Keywords: Uncertainty, Fuzzy set, Exponential trapezoidal fuzzy numbers, Ranking fuzzy numbers
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1 Introduction

The data exist in real world environment are uncertain and they are difficult to be determined exactly. Therefore, a mathematical model of a problem does not generally have accurate output to fulfill sufficient efficiency. As a result, in optimization problems, an appropriate tool is required by which the uncertainty of data is overcome. Fuzzy set theory is one of the most important research approach that can deal with problems relating to ambiguous and uncertainty and has been widely used in various fields of study such as [1, 4, 8, 13, 26, 32, 33] fuzzy set concept was proposed by Zadeh in 1965. A fuzzy set which its reference set is real numbers was called fuzzy number by Dubois and Prade [6]. Fuzzy numbers and fuzzy quantities are two important topics in many applications of fuzzy sets theory such as fuzzy control, fuzzy decision making, fuzzy optimization, [17, 18, 30, 34], fuzzy probability and statistics and even in management prediction in COVID-19 pandemic [28].

Using fuzzy numbers enables one to model linguistic variables. According to various applications of fuzzy numbers, different fuzzy numbers such as LR fuzzy number, bell shape fuzzy number, bipolar triangular fuzzy number and etc. are introduced [10, 31]. In 2000, exponential trapezoidal fuzzy numbers were defined for the first time by Chen and

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Lie [3] without investigating the definition and properties of fuzzy numbers for them. Therefore, in the present paper, the correctness of the aforementioned definition is discussed. Also, comparing a set of options and choosing the best one is such important that leads to a new interesting branch of science called operations research. Specially, in cases where the importance of various options is defined in terms of fuzzy numbers, a method is required to compare and rank fuzzy numbers in order to determine the priority of the options similar to cases of ordinary numbers.

The subject of ranking or ordering fuzzy numbers was first investigated by Jain in 1976 [16]. Afterwards, many researchers dealt with this problem and more than hundred indexes were proposed for ranking and compering fuzzy numbers that have their pros and cons [19, 31, 35, 39]. In 2011, a ranking function was proposed by Ezzati et al. through combining core, α-cut and spreads of unnormal fuzzy numbers [9]. In the same year, a method was proposed by Rezvani for pair compression of two unnormal trapezoidal fuzzy numbers using their area, perimeter and center of gravity [23].

In 2013, Yau and Wu [38] claimed that methods based on degree of deviation are not able to satisfy the symmetry property in all cases. Moreover, such methods ignore the opinion of decision-maker. To overcome such problems, they proposed a ranking index that involves right and left spreads and center of gravity of unnormal fuzzy numbers by considering the decision-maker opinion. The aforementioned method was also used to solve multi-criteria fuzzy decision-making problem. A new ranking method is proposed by Nasseri et al. [20] which is based on coordinates of the center of incircle of triangles. The coordinates of the incircle center is used in their proposed ranking method because each triangle's incircle is defined a unique quantity. Moreover, the consistency between the ranking of fuzzy quantities and that of their images is also proved by using their proposed ranking method. The problem of fully fuzzy games is solved by Hussein and Abood in which the data is uncertain [14]. In fact, they applied fuzzy payoffs problem using three different ranking functions. In [15], a novel ranking function method is proposed by Ingle and Ghadle in order to solvie fuzzy programing problems (FFPP) with hexagonal (HFN) numbers. A simple general method is proposed by Dutta [7] that is based on the concept of exponential area of the input fuzzy numbers. However, some shortcomings are involved in this method. For example, crisp-valued fuzzy numbers and fuzzy numbers are not differentiable by this method. In 2020, Saneifard and Saneifard [30] used radius of gyration function (ROG) to prove that solving fully fuzzy linear programming (FFLP) can provide accurate fuzzy optimal solutions. This fuzzy relation is considered as a probability based preference intensity index by Dombi and Jónás [5]. In fact, a closed formula is proposed by them to calculate the integral required for determination of this index for fuzzy sets with trapezoidal membership functions. In [22], in order to compare defuzzification algorithms; a cross-domain statistical method is used gaining the concepts of fuzzy sets pairwise similarities and distances operators. The correlation between the changes in defuzzified value of two fuzzy sets is measured by this method and is compared according to their similarity or distance. Moreover, by this method, two support based defuzzification methods mostly used for fuzzy controllers together with two level-based methods mostly used for ranking fuzzy numbers are compared.

In [25], a method is proposed by Rezvani to rank exponential trapezoidal fuzzy numbers based on variance. The index is calculated according to mathematical expectation of probability density function correspondent with exponential trapezoidal fuzzy quantity membership function. In the present paper, the goal is to investigate and analyze the method proposed by Rezvani [25]. Therefore, necessary properties and characteristics of the analysis are presented, then; Rezvani's method is examined by these properties. As a result, the motivation of this paper is:

a) To investigate the correctness of the definition of exponential trapezoidal fuzzy numbers.
b) To investigate and analyze the method proposed by Rezvani [25].

This paper is organized in 6 sections. In section 2, primary definitions of fuzzy concepts are presented. In section 3, existence of exponential trapezoidal fuzzy numbers is investigated. In section 4, necessary properties and characteristics related to the analysis of ranking fuzzy numbers are defined. Thereafter, Rezvani's ranking method is introduced and then evaluated based on properties defined. Finally, results and suggestions are presented in section 5.

2 Primary Definitions

In this section, some primary definitions required for fuzzy theory are defined [11, 37].

Different definitions have been proposed for fuzzy numbers due to their different properties resulted in various applications. Fuzzy numbers play an important role in modeling, fuzzy control analysis, fuzzy decision-making problems, approximate argument, optimization, approximate statistic and probability and etc. In the following, a useful definition of fuzzy numbers is presented that was also inferred by Rezani in [24, 25].

Definition 2.1. Assume $u : \mathbb{R} \to [0, 1]$ is a fuzzy set that satisfy the following conditions:

1) u is a continuous map from $\mathbb{R}$ to the close interval $[0, \omega_A]$.
2) $u(x) = 0$ for each $x \in (-\infty, \alpha) \cup (d, \infty)$.
3) $u(x)$ is strictly ascending in $[a, b]$.
4) $u(x) = \omega_A$ for each $x \in [b, c]$.
5) $u(x)$ is strictly descending in $[c, d]$ where a, b, c and d are real numbers.

The above conditions can be represented in the following form:

$$f_A(x) = \begin{cases} f_A^L(x), & a \leqslant x \leqslant b, \\ \omega_A, & b \leqslant x \leqslant c, \\ f_A^R(x), & c \leqslant x \leqslant d, \\ 0, & otherwise, \end{cases} \tag{2.1}$$

where $0 \leqslant \omega_A \leqslant 1$ is a constant number and $-\infty < a \leqslant b \leqslant c \leqslant d < +\infty$ are real numbers. $f_A^R(x)$ is indicated a right spread which is an ascending function and $f_A^L(x)$ is indicated a left spread which is a descending function. If $\omega_A = 1$, A is a normal fuzzy number.

Definition 2.2. (Zero of fuzzy number set) The set of all fuzzy numbers is shown by. $\tilde{0} = (0, 0, 0, 0)$ is considered as zero fuzzy number or the neutral member of summation in E.

Definition 2.3. (Linear ranking) Generally, ranking function R is called linear if for each $k \in \mathbb{R}$ and each two fuzzy numbers A and B, the following condition is satisfied:

$$R(kA + B) = kR(A) + R(B) \tag{2.2}$$

3 Existence of exponential trapezoidal fuzzy numbers

In this section, the definition of exponential trapezoidal fuzzy numbers is investigated. Also, its conformity with general definition of fuzzy numbers is examined. Therefore, in the following; the definition of exponential trapezoidal fuzzy numbers is presented.

Definition 3.1. (Exponential trapezoidal fuzzy quantity) [24]. An exponential trapezoidal fuzzy quantity is shown as $A = (a, b, \alpha, \beta; \omega)_E$ and its membership function is represented as follow:

$$\mu_A(x) = \begin{cases} \omega e^{-(\frac{b-x}{a})}, & x \leqslant a \\ \omega, & a \leqslant x \leqslant b \\ \omega e^{-(\frac{x-c}{\beta})}, & b \leqslant x \end{cases} \tag{3.1}$$

where $0 < \omega \leqslant 1$. a, b are real numbers and α, β are real positive numbers. If $\omega = 1$, this quantity is shown as $A = (a, b, \alpha, \beta)_E$. In the definition of exponential trapezoidal fuzzy quantity, the membership function for each $x \leqslant a$ is defined as $\omega e^{-(\frac{b-x}{\alpha})}$. Since ω is not zero and the function e^x cannot be zero for each $x \in$, therefore, using the term 'fuzzy number' is in contradiction with the second part of fuzzy number definition 2.1 and this fuzzy quantity cannot be a fuzzy number. To overcome this shortcoming, in 2015; definition 3.1 was modified by Rezvani as follow [25]:

Definition 3.2. (Exponential trapezoidal fuzzy number) Exponential trapezoidal fuzzy number is shown as $A = (a, b, c, d; \omega)_E$ and its membership function is defined as follow:

$$\mu_A(x) = \begin{cases} \omega e^{-(\frac{b-x}{b-a})}, & a \leqslant x \leqslant b \\ \omega, & b \leqslant x \leqslant c \\ \omega e^{-(\frac{x-c}{d-c})}, & c \leqslant x \leqslant d \\ 0, & otherwise \end{cases} \tag{3.2}$$

where $0 < \omega \leqslant 1$. a, b are real numbers and d, c are real positive numbers. If $\omega = 1$, this quantity is shown as $A = (a, b, \alpha, \beta)_E$.

Theorem 3.3. The exponential trapezoidal fuzzy number defined by Rezvani is not a fuzzy number in any sense and can only be considered as fuzzy quantity.

Proof . In fact, Rezvani overcame the problem involved in the second part of definition 2.1 while the new definition is in contradiction with the first part of definition 2.1. Namely, the membership function is not continuous in a and d because the right limit of the function is:

$$\lim_{x\to a^+} \mu_A(x) = \lim_{x\to a^+} \omega e^{-(\frac{b-x}{b-a})} = \omega e^{-1} \neq 0,$$

and the left limit of the function is:

$$\lim_{x\to a^-} \mu_A(x) = 0,$$

that is the function is noncontiguous in a. Moreover, the new definition is in contradiction with the fifth part of the definition 2.1, since d and c are considered as positive real numbers that limits the scope of fuzzy numbers. In fact, negative fuzzy numbers are ignored. □

4 Necessary properties of analysis

Rezvani achieved an index for ranking exponential trapezoidal fuzzy numbers based on variance [25]. This index is calculated according to the mathematical expectation of probability density function corresponds with membership function of exponential trapezoidal fuzzy numbers. In the following, the goal is to investigate and analyze the method proposed by Rezvani. Therefore, some properties and characteristics required for such analysis are defined in the following. Then Rezvani's method is evaluated by these properties. In 2015, Taghi-Nezhad proposed some properties (which are presented in the following) by examining, compiling and investigating advantages and disadvantages of more than hundred common full citation ranking indexes [31]. It is worthwhile to note that investigation of these properties can be considered as pre-conditions for accepting each new ranking index.

1) Ability to rank unnormal fuzzy numbers
Methods must be able to recognize two fuzzy numbers that are different only in terms of normality and non-normality.
2) Ranking symmetric fuzzy numbers with equal core and different spreads
3) Sensitivity to all points of the support of fuzzy numbers
Ranking index must be sensitive to all points of the supports of fuzzy numbers or at least to the points that have high membership degree. Otherwise, it may rank two different fuzzy numbers equally.
4) Simplicity and computation time of the method
In many practical problems with large number of fuzzy numbers, it is very important that a ranking method has short computational time and is simple to implement. For example, assume that method "A " can arrange fuzzy numbers with high accuracy, but requires complex and time consuming calculations and is also very difficult to implement. In contrast, method "B " is less accurate than method "A ", but its computational time is less and it is easier to implement. So, in a large problem consisting of millions of variables and fuzzy parameters, it would be cost-effective to use method "B ".
5) Linearity of the ranking index.
6) Satisfying common properties of ranking fuzzy quantities.
One of the best studies performed in the field of ranking and proposing some logical properties for fuzzy ranking methods is Wang and Kerre's paper (Wang & Kerre, 2001a, 2001b). In the following, common properties of fuzzy ranking methods are investigated briefly:
$\mathbf{A_1}$: For each finite subset Γ of the set E and $A \in \Gamma$ there is $A \gtrsim A$.
$\mathbf{A_2}$: For each finite subset Γ of the set E and $(A, B) \in \Gamma^2$. If $A \gtrsim B$ and $\gtrsim A$, then $A \approx B$.
$\mathbf{A_3}$: For each finite subset Γ of the set E and $(A, B, C) \in \Gamma^3$. If $A \gtrsim B$ and $B \gtrsim C$, then $A \gtrsim C$.
$\mathbf{A_4}$: For each finite subset Γ of the set E and $(A, B) \in \Gamma^2$. If $\inf \operatorname{supp}(A) > \sup \operatorname{supp}(B)$, then $A \gtrsim B$.
$\mathbf{A'_4}$: For each finite subset Γ of the set E and $(A, B) \in \Gamma^2$. If $\inf \operatorname{supp}(A) > \sup \operatorname{supp}(B)$, then $A > B$.
$\mathbf{A_5}$: Assume Γ and Γ' are two arbitrary finite subsets of E, and A and B are in $\Gamma \cap \Gamma'$. It can be said $A > B$ using d_p on Γ' if and only if $A > B$ using d_p on Γ.
$\mathbf{A_6}$: Assume $A, B, A + C$ and $B + C$ are some members of Γ. If $A \gtrsim B$ then $B + C \gtrsim A + C$.
$\mathbf{A'_6}$: Assume $A, B, A + C$ and $B + C$ are some members of Γ. If $A > B$ then $B + C > A + C$.
$\mathbf{A_7}$: Assume $A, B, A.C$ and $B.C$ are some members of Γ and $C > 0$. If $A > B$ then $A.C \gtrsim B.C$.

7) Ranking fuzzy numbers and their symmetries
   A ranking method must satisfy the symmetry property. In the other words, if a ranking method R ranks two fuzzy number A and B as $A < B$, it must rank their symmetry as $-B < -A$.
8) Ranking real numbers
   Each real number can be shown by a fuzzy number. In fact, fuzzy numbers are the augmented form of real numbers. Therefore, it is expected that ranking methods could be able to compare real and fuzzy number simultaneously.

5 Rezvani's ranking method

Rezvani proposed the following index for an exponential trapezoidal fuzzy quantity $A = (a, b, c, d; \omega)$:

$$M_A(t) = \frac{\omega}{a(1-e) - b + c + (e-1)d} \left[\frac{(e-1)b^t + (1-e)c^t - a^t + d^t}{t} \right], \tag{5.1}$$

where e is the Napier number. For a Fuzzy quantity A, the new index was proposed as follow:

$$\sigma_A^2 = M_A(3) - M_A(2)^2. \tag{5.2}$$

According to the index σ, both exponential trapezoidal fuzzy number $A, B \in E$ can be compared as followings:

i. If $\sigma_A^2 < \sigma_B^2$, then $A \prec B$,
ii. If $\sigma_A^2 > \sigma_B^2$, then $A \succ B$,
iii. If $\sigma_A^2 = \sigma_B^2$, then $A \approx B$,

In the following, the method proposed by Rezvani is analyzed and the examples in Rezvani's Paper are investigated.

Theorem 5.1. Rezvani's ranking method is not able to rank certain numbers.

Proof . Assume real number a $A = (a, a, a, a)$ that is shown as its general form namely exponential trapezoidal quantity. Then:

$$M_A(t) = \frac{1}{a(1-e) - a + a + (e-1)a} \left[\frac{(e-1)a^t + (1-e)a^t - a^t + a^t}{t} \right] = \frac{1}{0}[\frac{0}{t}] = \frac{0}{0}.$$

Therefore, for each certain number A, the index $M_A(t)$ is undefinable. So, Rezvani's ranking method is not able to rank certain numbers. □

Note 5.2. Application of Rezvani's method on dominant numbers which are the simplest and the most obvious fuzzy numbers is not investigated.

Note 5.3. Gupta et. al. [12] investigated Rezvani's ranking method [25] and claimed that the modified index must be defined as follow:

$$M_A(t) = \frac{1}{a(1-e) - b + c + (e-1)d} \left[\frac{(e-1)b^t + (1-e)c^t - a^t + d^t}{t} \right],$$

However, this modified index is also unable to rank certain numbers and has the aforementioned shortcomings.

Note 5.4. Rezvani's method was compared with eight different methods in the field of ranking fuzzy numbers. However, only one of them was used for ranking exponential trapezoidal fuzzy quantity. In the other words, the efficiency of Rezvani's algorithm is compared with general methods not with the methods in the same category. In the present paper, Rezvani's method is evaluated based on properties suggested in the previous section that are essential for ranking purposes. The results are shown in the following Table.

Guide for Table 1

In this table, the results achieved by examining each method based on the eight required properties are presented in column 1 to 8, respectively.

Table 1: Results of evaluating Rezvani's method in terms of proposed essential properties

Essential properties	1	2	3	4	5	6	7	8
Rezvani	No	Yes	Not investigated	No	Time consuming	Yes	Yes	Yes

In the first column, if a method is able to recognize normal and unnormal fuzzy number the answer is 'Yes', otherwise is 'No'.

In the second column, if the method under consideration is able to recognize symmetric fuzzy numbers with equal core and different spreads, the answer is 'Yes', otherwise is 'No'. However, some methods involve both 'Yes' and 'No' answers, therefore; the term 'dependent to the decision-maker' is written in the correspondent column to make the table more general.

In the third column, if a ranking method is sensitive to all points of the supports of fuzzy numbers, the answer is 'Yes', otherwise is 'No'.

In the fourth column, one of the linguistic (fuzzy) terms 'Low', 'Medium', 'High' or 'very high' is used for each method. In fact, the methods are ranked in terms of time consumption and implementation complexity based on linguistic terms.

As can be seen from (2-4), this method cannot be deduced as a simple methods due to the use of the Nipper number 'e' and also the use of other complex mathematical operations.

In the fifth column, the methods are ranked based on linearity and the terms 'Yes' or 'No' are used. It can be implied from (2-4) that this method is not qualified to be linear due to the use of multiplication and division operators.

In the sixth column, seven properties proposed by Wang and Kerre are examined for each method. Some methods do not satisfy some of the properties and are investigated in detail. If a method has all these properties the term 'Complete' is used.

In the seventh column, if a method has the symmetric property, the term 'Yes' is used, otherwise 'No' is used.

Finally, in the eighth column, each method is classified according to possibility of ranking real numbers by using the terms 'Yes' or 'No'.

6 Conclusion and results

In 2000, exponential trapezoidal fuzzy number quantity was defined and was called fuzzy number without investigating necessary conditions. In this paper, necessary conditions of fuzzy numbers for exponential fuzzy numbers are presented. The results show that these numbers are not fuzzy numbers using the basic definition in 2000 or the definition modified by Rezvani. In fact, the definition only represented a fuzzy quantity.

Then, one of the most authenticated methods of ranking exponential trapezoidal fuzzy numbers is investigated in this paper. The results reveal serious shortcomings of the method. In fact, the aforementioned method is disable to rank certain numbers and is not linear, also. On the other hand, time consuming computations and not investigating Wang and Kerre's indexes are other shortcomings of Rezvani's method. Also, in this paper; it is shown that the modified index proposed by Gourav Gupta et. al. [12] after investigating Rezvani's method [25] has some shortcomings too.

For future work, the correctness and existence of other fuzzy numbers can be investigated and their proposed features can be analyzed. Moreover, the proposed method can be augmented to the set of bipolar numbers, z-numbers, etc.

7 Results

It is concluded that exponential trapezoidal fuzzy numbers do not satisfy general conditions of fuzzy numbers such as bounded amplitude and continuity. In fact, these are fuzzy quantities not fuzzy numbers. As a result, a ranking method which is proposed for such quantities are examined using common tools and test platforms exist for ranking fuzzy quantities. Moreover, the strength and weakness of the method are reported [2].

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