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Precise solutions to the Hirota equation and Hirota-Maccari system by using the extended rational methods

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Abstract

This paper adopts the extended rational sinh-cosh as well as sine-cosine procedures to find precise solutions to the Hirota equation and Hirota-Maccari equation. It is illustrated that seeking the precise solutions for these equations plays a foremost and effectual role in solving the numerous kinds of PDEs applied in optics, fluid mechanics, plasma physics and solid physics. Furthermore, we are able to obtain some consequences of dark and cusp wave solutions. Besides, two-dimensional and three-dimensional surfaces have been drawn in order to acknowledge the concept of the acquired equations

Keywords: sine-cosine and sinh-cosh methods, Hirota equation, precise solutions, Hirota-Maccari system
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1 Introduction

In the last decades, research on nonlinear partial differential equations has been one of the most effectual and exciting fields of research in the numerous areas of sciences and engineering [12, 13, 14, 3, 11, 5, 7, 8, 9, 2, 10]. A wide range of powerful approaches are intermittently used and a diversification of perspectives have been studied.

The evolution of mathematical techniques, that give readers more accurate consequences for the extraction of solitons, is a highly salient area of applied physics, fluid mechanics, and optics. A graceful way of seeking the precise soliton solutions of nonlinear sciences and fractional systems is to propose a transformation to obtain a solvable ODEs (ordinary partial differential equations) employing analytical methods such as Hirota bilinear transformation, the (G/G')-expansion technique and the other ones [4, 1, 6, 15]. In this article, we employ these methods for looking for the precise solutions of two fundamental and important of these (Hirota equation and Hirota-Macarri equation) physical models.

Assume that the Hirota equation has the subsequent formation:

$$i\varrho_t + \varrho_{xx} + 2|\varrho|^2\varrho + i\Omega\varrho_{xxx} + 6i\Omega|\varrho|^2\varrho_x = 0, \tag{1}$$

in which x, t and Ω illustrates the spatial and temporal variables, Ω is a small parameter respectively. Secondly, assume that the Hirota-macarri system in the following formation:

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$$i\varrho_t + \varrho_{xy} + i\varrho_{xxx} + \varrho\varphi - i|\varrho|^2\varrho_x = 03\varphi_x + (|\varrho|^2)_y = 0, \tag{2}$$

in which $\varrho(x, y, t)$ and $\varphi(x, y, t)$ demonstrate the filed of complex scalar and the real one. Temporal variable and spatial variables are illustrated by t, x and y. The residual sections of this paper are conformed as follows: in section 2, the algorithms of the extension rational sin-cos as well as sinh-cosh techniques are presented, in section 3 and 4, these techniques are represented with the Hirota equation and Hirota-Macarri system respectively. Furthermore, the representation of some acquired solutions in section 5 will be considered.

2 General solution of the technique

Consider the nonlinear partial differential equation (PDE) be of the form

$$H(\vartheta, \vartheta_t, \vartheta_x, \varrho_{xx}, ...) = 0, \tag{3}$$

in which traveling wave solution is denoted by $\vartheta = \vartheta(x, t)$. By using the subsequent transformation:

$$\vartheta(x, t) = \vartheta(\xi), \quad \xi = mx + gt, \tag{4}$$

where wave speed is illustrated by g. Equation (3) can be turned into the subsequent ordinary differential equation:

$$G(\vartheta, \vartheta', \vartheta'', ...) = 0. \tag{5}$$

2.1 The extension rational sin-cos technique

Pace1. Assume that Equation (5) has the solution in the conformation of

$$\vartheta(\xi) = \frac{\beta_0 \sinh(\zeta\xi)}{\beta_2 + \beta_1 \cosh(\zeta\xi)}, \quad \cosh(\zeta\xi) \neq -\frac{\beta_2}{\beta_1}, \tag{6}$$

or

$$\vartheta(\xi) = \frac{\beta_0 \cosh(\zeta\xi)}{\beta_2 + \beta_1 \sinh(\zeta\xi)}, \quad \sinh(\zeta\xi) \neq -\frac{\beta_2}{\beta_1}, \tag{7}$$

where ζ is the wave number and the parameters of β_0, β_1,and β_2 will be determined.

Pace 2. In this pace, we concatenated one of the equations mentioned above into Eq.(5), then by gathering all words with the identical powers of $cosh(\zeta\xi)^r$ or $sinh(\zeta\xi)^r$ and equalling to zero, all the coefficients of $cosh(\zeta\xi)^r$ or $sinh(\zeta\xi)^r$ leads to a set of algebraic equations. By using mathematics software, algebraic equations' solutions will be found.

Pace 3. In this step, by concatenating the values of β_0, β_1 and β_2 and ζ in Eq.(6) or Eq.(7) the solution to the Eq.(5) will be resolved.

2.2 The extension rational sin-cos technique

Pace 1. Assume that Equation (5) has the solution in the conformation of

$$\vartheta(\xi) = \frac{\beta_0 \sin(\zeta\xi)}{\beta_2 + \beta_1 \cos(\zeta\xi)}, \quad \cos(\zeta\xi) \neq -\frac{\beta_2}{\beta_1}, \tag{8}$$

or

$$\vartheta(\xi) = \frac{\beta_0 \cos(\zeta\xi)}{\beta_2 + \beta_1 \sin(\zeta\xi)}, \quad \sin(\zeta\xi) \neq -\frac{\beta_2}{\beta_1}, \tag{9}$$

where ζ is the wave number and the parameters of β_0, β_1,and β_2 will be determined.

Pace 2. In this pace, we concatenated one of the equations mentioned earlier into Eq.(5), then by gathering all words with the identical powers of $cos(\zeta\xi)^r$ or $sin(\zeta\xi)^r$ and equalling to zero, all the coefficients of $cos(\zeta\xi)^r$ or $sin(\zeta\xi)^r$ leads to a set of algebraic equations. By using mathematics software, algebraic equations' solutions will be found.

Pace 3. In this step, by concatenating the values of β_0,β_1,β_2 and ζ in Eq.(8) or Eq.(9) the solution to the Eq.(5) will be resolved.

2.3 The implementation of the extension rational sinh-cosh technique

Assume that Eq. (12) has solutions in the conformation of

$$\vartheta(\xi)=\frac{\beta_0\,\sinh(\zeta\xi)}{\beta_2+\beta_1\cosh(\zeta\xi)} \tag{10}$$

By concatenating Eq. (13) into Eq. (12) and then gathering all words with the identical powers of $cosh(\zeta\xi)^r$ as well as calculating all the coefficients of $cosh(\zeta\xi)^r$ to zero, then we have:

$cosh(\zeta\xi)^2\,:\,-\beta_0\,\left(\left(-s^3\Omega+s^2+r\right)\beta_1{}^2+6\Omega\beta_0{}^2s-2\beta_0{}^2\right)$

$cosh(\zeta\xi)^1\,:\,-2\beta_0\beta_1\,\left(-s^3\Omega-\frac{3}{2}s\zeta^2\Omega+s^2+\frac{1}{2}\zeta^2+r\right)\beta_2$

$cosh(\zeta\xi)^0\,:\,-\beta_0\,\left(\left(3s\zeta^2\Omega-s^3\Omega-\zeta^2+s^2+r\right)\beta_2{}^2-6\,\left(\Omega s-\frac{1}{3}\right)\left(\beta_1{}^2\zeta^2+\beta_0{}^2\right)\right).$

These algebraic equations will be solved by software and then we have the subsequent solutions:

part 1)

$$\zeta=\pm\sqrt{-\frac{s^3\Omega-s^2-r}{6\Omega s-2}},\quad \beta_0=\pm\sqrt{-\frac{-s^3\Omega+s^2+r}{6\Omega s-2}}\beta_1\quad \beta_1=\beta_1,\quad \beta_2=0. \tag{11}$$

part 2)

$$\zeta=\pm\sqrt{-\frac{2s^3\Omega-2s^2-2r}{3\Omega s-1}},\quad \beta_0=\pm\sqrt{-\frac{-s^3\Omega+s^2+r}{6\Omega s-2}}\beta_2\quad \beta_1=\beta_2,\quad \beta_2=\beta_2. \tag{12}$$

part 3)

$$\zeta=\pm\sqrt{-\frac{2s^3\Omega-2s^2-2r}{3\Omega s-1}},\quad \beta_0=\pm\sqrt{-\frac{-s^3\Omega+s^2+r}{6\Omega s-2}}\beta_2\quad \beta_1=-\beta_2,\quad \beta_2=\beta_2. \tag{13}$$

Case 1: Taking part 1 into consideration, and the solution of (12) can be obtained as

$$\vartheta_1(x,t)=\pm\sqrt{-\frac{-s^3\Omega+s^2+r}{6\Omega s-2}}\tanh\left(\sqrt{-\frac{s^3\Omega-s^2-r}{6\Omega s-2}}\xi\right) \tag{14}$$

by merging the equations (10) and (17), we get

$$\varrho_1(x,t)=\pm\sqrt{-\frac{-s^3\Omega+s^2+r}{6\Omega s-2}}\tanh\left(\sqrt{-\frac{s^3\Omega-s^2-r}{6\Omega s-2}}\,(vt+x)\right)\mathrm{e}^{i(rt+sx)}. \tag{15}$$

Case 2: Likewise, for part 2, the solutions of (12) can be obtained as

$$\vartheta_2(x,t)=\pm\sqrt{-\frac{-s^3\Omega+s^2+r}{6\Omega s-2}}\sinh\left(\sqrt{-\frac{2s^3\Omega-2s^2-2r}{3\Omega s-1}}\xi\right)\left(1+\cosh\left(\sqrt{-\frac{2s^3\Omega-2s^2-2r}{3\Omega s-1}}\xi\right)\right)^{-1} \tag{16}$$

by merging equations (10) and (19), we get

$$\varrho_2(x,t)=\pm\sqrt{-\frac{-s^3\Omega+s^2+r}{6\Omega s-2}}\sinh\left(\sqrt{-\frac{2s^3\Omega-2s^2-2r}{3\Omega s-1}}\,(vt+x)\right)\left(1+\cosh\left(\sqrt{-\frac{2s^3\Omega-2s^2-2\,r}{3\Omega s-1}}\,(vt+x)\right)\right)^{-1}\mathrm{e}^{i(rt+sx)}. \tag{17}$$

Case 3: Likewise for part 3, the solutions of (12) can be obtained as

$$\vartheta_3(x,t)=\pm1\sqrt{-\frac{-s^3\Omega+s^2+r}{6\Omega s-2}}\sinh\left(\sqrt{-\frac{2s^3\Omega-2s^2-2r}{3\Omega s-1}}\xi\right)\left(1-\cosh\left(\sqrt{-\frac{2s^3\Omega-2s^2-2r}{3\Omega s-1}}\xi\right)\right)^{-1} \tag{18}$$

by merging equations (10) and (21), we get

$$\varrho_3(x,t)=\pm\sqrt{-\frac{-s^3\Omega+s^2+r}{6\Omega s-2}}\sinh\left(\sqrt{-\frac{2s^3\Omega-2s^2-2r}{3\Omega s-1}}\,(vt+x)\right)\left(1-\cosh\left(\sqrt{-\frac{2s^3\Omega-2s^2-2\,r}{3\Omega s-1}}\,(vt+x)\right)\right)^{-1}\mathrm{e}^{i(rt+sx)}. \tag{19}$$

or assume that Equation (12) has solutions in the conformation of

$$\vartheta(\xi) = \frac{\beta_0 \cosh(\zeta\xi)}{\beta_2 + \beta_1 \sinh(\zeta\xi)} \tag{20}$$

By concatenating Eq. (23) in Eq. (12) and then gathering all terms with the identical powers of $sinh(\zeta\xi)^r$ and equating to zero. All coefficients of $sinh(\zeta\xi)^r$ will obtain and we have the subsequent set of algebraic equations in the following stage:

$\sinh(\zeta\xi)^2 : -2\left(\left(-\frac{1}{2}s^3\Omega + \frac{1}{2}s^2 + \frac{r}{2}\right){\beta_1}^2 + 3\Omega{\beta_0}^2 s - {\beta_0}^2\right)\beta_0$

$\sinh(\zeta\xi)^1 : -2\beta_2\beta_1\left(-s^3\Omega - \frac{3}{2}\Omega\zeta^2 s + s^2 + \frac{1}{2}\zeta^2 + r\right)\beta_0$

$\sinh(\zeta\xi)^0 : -\left({\beta_2}^2\left(3\Omega\zeta^2 s - s^3\Omega - \zeta^2 + s^2 + r\right) + 6\left({\beta_1}^2\zeta^2 + {\beta_0}^2\right)\left(s\Omega - \frac{1}{3}\right)\right)\beta_0.$

These algebraic equations will be solved by software and then we have the subsequent solutions:

part 4)

$$\zeta = \pm\sqrt{-\frac{s^3\Omega - s^2 - r}{6s\Omega - 2}}, \quad \beta_0 = \pm\sqrt{-\frac{-s^3\Omega + s^2 + r}{6s\Omega - 2}}\beta_1 \quad \beta_1 = \beta_1, \quad \beta_2 = 0. \tag{21}$$

part 5)

$$\zeta = \pm\sqrt{-\frac{2s^3\Omega - 2s^2 - 2r}{3\Omega s - 1}}, \quad \beta_0 = \sqrt{-\frac{s^3\Omega - s^2 - r}{6\Omega s - 2}}\beta_2, \quad \beta_1 = I\beta_2, \quad \beta_2 = \beta_2. \tag{22}$$

Case 4: we take part 4 into consideration, the solution of (12) can be obtained as

$$\vartheta_4(x,t) = \pm\sqrt{-\frac{-s^3\Omega + s^2 + r}{6s\Omega - 2}}\coth\left(\sqrt{-\frac{s^3\Omega - s^2 - r}{6s\Omega - 2}}\xi\right) \tag{23}$$

by merging equations (10) and (26), we get

$$\varrho_4(x,t) = \pm\sqrt{-\frac{-s^3\Omega + s^2 + r}{6s\Omega - 2}}\coth\left(\sqrt{-\frac{s^3\Omega - s^2 - r}{6s\Omega - 2}}\,(vt + x)\right)\mathrm{e}^{i(rt+sx)}. \tag{24}$$

Case 5: Likewise, for part 5, the solutions of (12) can be obtained as

$$\vartheta_5(x,t) = \pm\sqrt{-\frac{s^3\Omega - s^2 - r}{6\Omega s - 2}}\cosh\left(\sqrt{-\frac{2s^3\Omega - 2s^2 - 2r}{3\Omega s - 1}}\xi\right)\left(1 + i\sinh\left(\sqrt{-\frac{2s^3\Omega - 2s^2 - 2r}{3\Omega s - 1}}\xi\right)\right)^{-1} \tag{25}$$

by merging equations (10) and (28), we get,

$$\varrho_5(x,t) = \pm\sqrt{-\frac{s^3\Omega - s^2 - r}{6\Omega s - 2}}\cosh\left(\sqrt{-\frac{2s^3\Omega - 2s^2 - 2r}{3\Omega s - 1}}\,(tv + x)\right)\left(1 + i\sinh\left(\sqrt{-\frac{2s^3\Omega - 2s^2 - 2r}{3\Omega s - 1}}\,(tv + x)\right)\right)^{-1}\mathrm{e}^{i(rt+sx)}. \tag{26}$$

2.4 Extension rational sine-cosine method

Assume that Equation (12) has solutions in the conformation of

$$\vartheta(\xi) = \frac{\beta_0 \sin(\zeta\xi)}{\beta_2 + \beta_1 \cos(\zeta\xi)}. \tag{27}$$

By concatenating Eq. (30) in Eq. (12) and then gathering all terms with the identical powers of $cos(\zeta\xi)^r$ and equalling to zero. All coefficients of $cos(\zeta\xi)^r$ will obtain and we have the subsequent set of algebraic equations in the follow- ing stage:

$\cos(\zeta\xi)^2 : -\beta_0\left(\left(-s^3\Omega + s^2 + r\right){\beta_1}^2 - 6\Omega{\beta_0}^2 s + 2{\beta_0}^2\right)$

$\cos(\zeta\xi)^1 : -2\beta_0\beta_1\left(-s^3\Omega + \frac{3}{2}s\zeta^2\Omega + s^2 - \frac{1}{2}\zeta^2 + r\right)\beta_2$

$cos(\zeta\xi)^0$: $-\beta_0\left(\left(-3s\zeta^2\Omega-s^3\Omega+\zeta^2+s^2+r\right){\beta_2}^2+6\left(\Omega s-\frac{1}{3}\right)\left({\beta_1}^2\zeta^2+{\beta_0}^2\right)\right)$

These algebraic equations will be solved by software and then we have the subsequent solutions:

part 6)

$$\zeta=\pm\sqrt{-\frac{-s^3\Omega+s^2+r}{6\Omega s-2}},\quad \beta_0=\pm\sqrt{-\frac{s^3\Omega-s^2-r}{6\Omega s-2}}\beta_1\quad \beta_1=\beta_1,\quad \beta_2=0. \tag{28}$$

part 7)

$$\zeta=\pm\sqrt{-\frac{-2s^3\Omega+2s^2+2r}{3\Omega s-1}},\quad \beta_0=\pm\sqrt{-\frac{s^3\Omega-s^2-r}{6\Omega s-2}}\beta_2\quad \beta_1=\beta_2,\quad \beta_2=\beta_2. \tag{29}$$

part 8)

$$\zeta=\pm\sqrt{-\frac{-2s^3\Omega+2s^2+2r}{3\Omega s-1}},\quad \beta_0=\pm\sqrt{-\frac{s^3\Omega-s^2-r}{6\Omega s-2}}\beta_2\quad \beta_1=-\beta_2,\quad \beta_2=\beta_2. \tag{30}$$

Case 6: Taking part 6 into consideration, the solutions of (12) can be obtained as

$$\vartheta_6(x,t)=\pm\sqrt{-\frac{s^3\Omega-s^2-r}{6\Omega s-2}}\tan\left(\sqrt{-\frac{-s^3\Omega+s^2+r}{6\Omega s-2}}\xi\right) \tag{31}$$

by merging equations (10) and (34), then we get

$$\varrho_6(x,t)=\pm\sqrt{-\frac{s^3\Omega-s^2-r}{6\Omega s-2}}\tan\left(\sqrt{-\frac{-s^3\Omega+s^2+r}{6\Omega s-2}}\,(x+v\cdot t)\right)\mathrm{e}^{i(rt+sx)}. \tag{32}$$

Case 7: Likewise, for part 7, the solutions of (12) can be obtained as

$$\vartheta_7(x,t)=\pm\sqrt{-\frac{s^3\Omega-s^2-r}{6\Omega s-2}}\sin\left(\sqrt{-\frac{-2s^3\Omega+2s^2+2r}{3\Omega\, s-1}}\xi\right)\left(1+\cos\left(\sqrt{-\frac{-2s^3\Omega+2s^2+2r}{3\Omega s-1}}\xi\right)\right)^{-1} \tag{33}$$

by merging equations (10) and (36), then we get

$$\varrho_7(x,t)=\pm\sqrt{-\frac{s^3\Omega-s^2-r}{6\Omega s-2}}\sin\left(\sqrt{-\frac{-2s^3\Omega+2s^2+2r}{3\Omega s-1}}\xi\right)\left(1+\cos\left(\sqrt{-\frac{-2s^3\Omega+2s^2+2r}{3\Omega s-1}}\xi\right)\right)^{-1}\mathrm{e}^{i(rt+sx)}. \tag{34}$$

Case 8: Likewise, for part 8, the solutions of (12) can be obtained as

$$\vartheta_8(x,t)=\pm\sqrt{-\frac{s^3\Omega-s^2-r}{6\Omega s-2}}\sin\left(\sqrt{-\frac{-2s^3\Omega+2s^2+2r}{3\Omega\, s-1}}\xi\right)\left(1-\cos\left(\sqrt{-\frac{-2s^3\Omega+2s^2+2r}{3\Omega s-1}}\xi\right)\right)^{-1} \tag{35}$$

by merging equations (10) and (38), then we get

$$\varrho_8(x,t)=\pm\sqrt{-\frac{s^3\Omega-s^2-r}{6\Omega s-2}}\sin\left(\sqrt{-\frac{-2s^3\Omega+2s^2+2r}{3\Omega s-1}}\left(x+v\cdot t\right)\right)\left(1-\cos\left(\sqrt{-\frac{-2s^3\Omega+2s^2+2r}{3\Omega s-1}}\left(x+v\cdot t\right)\right)\right)^{-1}\mathrm{e}^{i\left(rt+sx\right)} \tag{36}$$

or assume that Equation (12) has solution in the conformation of

$$\vartheta(\xi)=\frac{\beta_0\,\cos(\zeta\xi)}{\beta_2+\beta_1\sin(\zeta\xi)} \tag{37}$$

By concatenating Eq. (40) into Eq. (12) and then gathering all words with the identical powers of $sin(\zeta\xi)^r$ as well as equalling all the coefficients of $sin(\zeta\xi)^r$ to zero, then we have:

$\sin(\zeta\xi)^2$: $2\left(-\left(\frac{1}{2}s^3\Omega-\frac{1}{2}s^2-\frac{r}{2}\right){\beta_1}^2-3\Omega{\beta_0}^2s+{\beta_0}^2\right)\beta_0$

$\sin(\zeta\xi)^1$: $2\beta_1\beta_2\left(-s^3\Omega+\frac{3}{2}\Omega\zeta^2s+s^2-\frac{1}{2}\zeta^2+r\right)\beta_0$

$\sin(\zeta\xi)^0$: $\left({\beta_2}^2\left(-3\Omega\zeta^2s-s^3\Omega+\zeta^2+s^2+r\right)+6\left(\Omega s-\frac{1}{3}\right)\left({\beta_1}^2\zeta^2+{\beta_0}^2\right)\right)\beta_0.$

These algebraic equations will be solved by mathematics software and then we have the subsequent solutions:
set 9)

$$\zeta=\pm\sqrt{-\frac{-s^3\Omega+s^2+r}{6s\Omega-2}},\quad \beta_0=\pm\sqrt{-\frac{s^3\Omega-s^2-r}{6s\Omega-2}}\beta_1\beta_1=\beta_1,\quad \beta_2=0 \tag{38}$$

part 10)

$$\zeta=\pm\sqrt{-\frac{-2s^3\Omega+2s^2+2r}{3\Omega s-1}},\quad \beta_0=\pm\sqrt{-\frac{s^3\Omega-s^2-r}{6\Omega s-2}}\beta_2\beta_1=\beta_2,\quad \beta_2=\beta_2 \tag{39}$$

part 11)

$$\zeta=\pm\sqrt{-\frac{-2s^3\Omega+2s^2+2r}{3\Omega s-1}},\quad \beta_0=\pm\sqrt{-\frac{s^3\Omega-s^2-r}{6\Omega s-2}}\beta_2\quad \beta_1=-\beta_2,\quad \beta_2=\beta_2. \tag{40}$$

Case 9: we take set 9 into consideration, the solutions of (12) can be obtained as

$$\vartheta_9(x,t)=\pm\sqrt{-\frac{s^3\Omega-s^2-r}{6s\Omega-2}}\cot\left(\sqrt{-\frac{-s^3\Omega+s^2+r}{6s\Omega-2}}\xi\right) \tag{41}$$

by merging equations (10) and (44), then we get

$$\varrho_9(x,t)=\pm\sqrt{-\frac{s^3\Omega-s^2-r}{6s\Omega-2}}\cot\left(\sqrt{-\frac{-s^3\Omega+s^2+r}{6s\Omega-2}}\,(x+v\cdot t)\right)\mathrm{e}^{i(rt+sx)}. \tag{42}$$

Case 10: Likewise, for part 10, the solutions of (12) can be obtained as

$$\vartheta_{10}(x,t)=\pm\sqrt{-\frac{s^3\Omega-s^2-r}{6\Omega s-2}}\cos\left(\sqrt{-\frac{-2s^3\Omega+2s^2+2r}{3\Omega\, s-1}}\xi\right)\left(1+\sin\left(\sqrt{-\frac{-2s^3\Omega+2s^2+2r}{3\Omega s-1}}\xi\right)\right)^{-1} \tag{43}$$

by merging equations (10) and (46), then we get

$$\varrho_{10}(x,t) \tag{44}$$

$$=\pm\sqrt{-\frac{s^3\Omega-s^2-r}{6\Omega s-2}}\cos\left(\sqrt{-\frac{-2s^3\Omega+2s^2+2r}{3\Omega s-1}}\,(x+v\cdot t)\right)\left(1+\sin\left(\sqrt{-\frac{-2s^3\Omega+2s^2+2\,r}{3\Omega s-1}}\,(x+v\cdot t)\right)\right)^{-1}\mathrm{e}^{i(rt+sx)} \tag{45}$$

Case 11: Likewise, for part 11, the solutions of (12) can be obtained as

$$\vartheta_{11}(x,t)=\pm\sqrt{-\frac{s^3\Omega-s^2-r}{6\Omega s-2}}\cos\left(\sqrt{-\frac{-2s^3\Omega+2s^2+2r}{3\Omega\, s-1}}\xi\right)\left(1-\sin\left(\sqrt{-\frac{-2s^3\Omega+2s^2+2r}{3\Omega s-1}}\xi\right)\right)^{-1} \tag{46}$$

by merging equations (10) and (48), then we obtain

$$\varrho_{11}(x,t)$$

$$=\pm\sqrt{-\frac{s^3\Omega-s^2-r}{6\Omega s-2}}\cos\left(\sqrt{-\frac{-2s^3\Omega+2s^2+2r}{3\Omega s-1}}\,(x+v\cdot t)\right)\left(1-\sin\left(\sqrt{-\frac{-2s^3\Omega+2s^2+2\,r}{3\Omega s-1}}\,(x+v\cdot t)\right)\right)^{-1}\mathrm{e}^{i(rt+sx)} \tag{47}$$

3 Implementation of these methods on Hirota-Maccari system

Suppose the Hirota-Maccari equation, then by adopting the following transformations

$$\varrho(x,y,t)=e^{-i\zeta}\vartheta(\xi),\quad \varphi(x,y,t)=\Psi(\xi),\quad \xi=x+y+kt,\quad \zeta=sx+ry+kt. \tag{48}$$

Then, we have the subsequent ordinary differential equations:

$$3(1-3s)\vartheta''+3(s^3-sr-k)\vartheta+(3s-1)\vartheta^3=0\qquad \Psi=-\frac{\vartheta^2}{3}, \tag{49}$$

where $s\neq\frac{1}{3}$.

3.1 The implementation of the extension rational sinh-cosh technique

Assume that Equation (51) has solutions in the conformation of

$$\vartheta(\xi) = \frac{\beta_0 \sinh(\zeta\xi)}{\beta_2 + \beta_1 \cosh(\zeta\xi)} \tag{50}$$

By concatenating Eq. (52) in Eq. (51) and then gathering all terms with the identical powers of $cosh(\zeta\xi)^r$ and equating to zero all coefficients of $cosh(\zeta\xi)^r$, the subsequent set of algebraic equations are obtained in the following stage:
$\cosh(\zeta\xi)^2 : -3\beta_0 \left(\left(-s^3 + sr + k\right) {\beta_1}^2 - {\beta_0}^2 \left(s - \frac{1}{3}\right)\right)$
$\cosh(\zeta\xi)^1 : -6\beta_0\beta_2 \left(-s^3 + \left(-\frac{3}{2}\zeta^2 + r\right) s + \frac{1}{2}\zeta^2 + k\right) \beta_1$
$\cosh(\zeta\xi)^0 : -3\beta_2^2 - 6 \left(s - \frac{1}{3}\right) \left(\beta_1\beta_0 \left(\left(-s^3 + \left(3\zeta^2 + r\right) s - \zeta^2 + k\right) {}^2\zeta^2 - \frac{1}{6}{\beta_0}^2\right)\right)$.

These algebraic equations will be resolved by mathematics software and then we have the subsequent solutions:

set 1)

$$\zeta = \pm\sqrt{-\frac{s^3 - sr - k}{6s - 2}}, \quad \beta_0 = \pm\sqrt{-\frac{3s^3 - 3sr - 3k}{3s - 1}}\beta_1 \quad \beta_1 = \beta_1, \quad \beta_2 = 0. \tag{51}$$

set 2)

$$\zeta = \pm\sqrt{-\frac{2s^3 - 2sr - 2k}{3s - 1}}, \quad \beta_0 = \pm\sqrt{-\frac{3s^3 - 3sr - 3k}{3s - 1}}\beta_2 \quad \beta_1 = \beta_2, \quad \beta_2 = \beta_2. \tag{52}$$

set 3)

$$\zeta = \pm\sqrt{-\frac{2s^3 - 2sr - 2k}{3s - 1}}, \quad \beta_0 = \pm\sqrt{-\frac{3s^3 - 3sr - 3k}{3s - 1}}\beta_2 \quad \beta_1 = -\beta_2, \quad \beta_2 = \beta_2. \tag{53}$$

Case 1: Taking set 1 into consideration, and then the solutions of equation (51) can be obtained as

$$\vartheta_1(x, y, t) = \pm\sqrt{-\frac{3s^3 - 3sr - 3k}{3s - 1}} \tanh\left(\sqrt{-\frac{s^3 - sr - k}{6s - 2}}\xi\right) \tag{54}$$

by merging the equations (50) and (56), we get

$$\varrho_1(x, y, t) = \pm\sqrt{-\frac{3s^3 - 3sr - 3k}{3s - 1}} \tanh\left(\sqrt{-\frac{s^3 - sr - k}{6s - 2}} (kt + x + y)\right) \mathrm{e}^{-i(kt+ry-sx)} \tag{55}$$

the second part of (51) is expressed as follows:

$$\Psi_1(x, y, t) = \pm\frac{1}{3}\frac{3s^3 - 3sr - 3k}{3s - 1} \left(\tanh\left(\sqrt{-\frac{s^3 - sr - k}{6s - 2}} (kt + x + y)\right)\right)^2. \tag{56}$$

Case 2: Likewise, for set 2, the solutions of (51) can be gained as

$$\vartheta_2(x, y, t) = \pm\sqrt{-\frac{3s^3 - 3sr - 3k}{3s - 1}} \sinh\left(\sqrt{-\frac{2s^3 - 2sr - 2k}{3s - 1}}\xi\right) \left(1 + \cosh\left(\sqrt{-\frac{2s^3 - 2sr - 2k}{3s - 1}}\xi\right)\right)^{-1} \tag{57}$$

by merging equations (50) and (59), we get

$$\varrho_2(x, y, t) = \pm \sqrt{-\frac{3s^3 - 3sr - 3k}{3s - 1}} \sinh\left(\sqrt{-\frac{2s^3 - 2sr - 2k}{3s - 1}} (kt + x + y)\right) \left(1 + \cosh\left(\sqrt{-\frac{2s^3 - 2sr - 2k}{3s - 1}} (kt + x + y)\right)\right)^{-1} \mathrm{e}^{-i(kt+ry-sx)}. \tag{58}$$

The second part of (51) is expressed as follows:

$$\Psi_2(x, y, t) = \pm\frac{1}{3}\frac{3s^3 - 3sr - 3k}{3s - 1} \left(\sinh\left(\sqrt{-\frac{2s^3 - 2sr - 2k}{3s - 1}} (kt + x + y)\right)\right)^2 \left(1 + \cosh\left(\sqrt{-\frac{2s^3 - 2sr - 2k}{3s - 1}} (kt + x + y)\right)\right)^{-2} \tag{59}$$

Case 3: Likewise for set 3, the solutions of (51) can be got as

$$\vartheta_3(x,y,t) = \pm\sqrt{-\frac{3s^3-3sr-3k}{3s-1}}\sinh\left(\sqrt{-\frac{2s^3-2sr-2k}{3s-1}}\xi\right)\left(1-\cosh\left(\sqrt{-\frac{2s^3-2sr-2k}{3s-1}}\xi\right)\right)^{-1} \tag{60}$$

by merging equations (50) and (62), we get

$$\begin{aligned}&\varrho_3(x,y,t)\\ &= \pm\sqrt{-\frac{3s^3-3\,sr-3k}{3s-1}}\sinh\left(\sqrt{-\frac{2s^3-2sr-2k}{3s-1}}\,(kt+x+y)\right)\left(1-\cosh\left(\sqrt{-\frac{2s^3-2sr-2k}{3s-1}}\,(kt+x+y)\right)\right)^{-1}\mathrm{e}^{-i(kt+ry-sx)}\end{aligned} \tag{61}$$

the second part of (51) is expressed as follows:

$$\Psi_3(x,y,t) = \pm\frac{1}{3}\frac{3s^3-3sr-3k}{3s-1}\left(\sinh\left(\sqrt{-\frac{2s^3-2sr-2k}{3s-1}}\,(kt+x+y)\right)\right)^2\left(1-\cosh\left(\sqrt{-\frac{2s^3-2sr-2k}{3s-1}}\,(kt+x+y)\right)\right)^{-2} \tag{62}$$

or assume that Eq.(51) has solutions in the conformation of

$$\vartheta(\xi) = \frac{\beta_0\,\cosh(\zeta\xi)}{\beta_2+\beta_1\sinh(\zeta\xi)}. \tag{63}$$

By concatenating Eq. (65) in Eq. (51) and then gathering all terms with the identical powers of $\sinh(\zeta\xi)^r$ and equating to zero all coefficients of $\sinh(\zeta\xi)^r$, the subsequent set of algebraic equations are obtained in the following stage:

$\sinh(\zeta\xi)^2 : -6\beta_0\left(\left(-\frac{1}{2}s^3+\frac{1}{2}sr+\frac{k}{2}\right)\beta_1{}^2-\frac{1}{2}\beta_0{}^2\left(s-\frac{1}{3}\right)\right)$

$\sinh(\zeta\xi)^1 : -6\beta_0\beta_2\left(-s^3+\left(-\frac{3}{2}\zeta^2+r\right)s+\frac{1}{2}\zeta^2+k\right)\beta_1$

$\sinh(\zeta\xi)^0 : -3\beta_0\left(\beta_2{}^2\left(-s^3+\left(3\zeta^2+r\right)s-\zeta^2+k\right)+6\left(\beta_1{}^2\zeta^2-\frac{1}{6}\beta_0{}^2\right)\left(s-\frac{1}{3}\right)\right)$

These algebraic equations will be solved by mathematics software and then we have the subsequent solutions: set 4)

$$\zeta = \pm\sqrt{-\frac{s^3-sr-k}{6s-2}},\quad \beta_0 = \pm\sqrt{-\frac{3s^3-3sr-3k}{3s-1}}\beta_1\quad \beta_1=\beta_1,\quad \beta_2=0 \tag{64}$$

set 5)

$$\zeta = \pm\sqrt{-\frac{2s^3-2sr-2k}{3s-1}},\quad \beta_0 = \sqrt{-\frac{-3s^3+3sr+3k}{3s-1}}\beta_2,\quad \beta_1=I\beta_2,\quad \beta_2=\beta_2 \tag{65}$$

Case 4: we take set 4 into account, the solutions of (51) can be obtained as

$$\vartheta_4(x,y,t) = \pm\sqrt{-\frac{3s^3-3sr-3k}{3s-1}}\coth\left(\sqrt{-\frac{s^3-sr-k}{6s-2}}\xi\right) \tag{66}$$

by merging equations (50) and (68), we get

$$\varrho_4(x,y,t) = \pm\sqrt{-\frac{3s^3-3sr-3k}{3s-1}}\coth\left(\sqrt{-\frac{s^3-sr-k}{6s-2}}\,(kt+x+y)\right)\mathrm{e}^{-i(kt+ry-sx)} \tag{67}$$

the second part of equations (51) is expressed as follows:

$$\Psi_4(x,y,t) = \pm\frac{1}{3}\frac{3s^3-3sr-3k}{3s-1}\left(\coth\left(\sqrt{-\frac{s^3-sr-k}{6s-2}}\,(kt+x+y)\right)\right)^2 \tag{68}$$

Case 5: Likewise, for set 5, the solutions of (51) can be gained as

$$\vartheta_5(x,y,t) = \pm\sqrt{-\frac{-3s^3+3sr+3k}{3s-1}}\cosh\left(\sqrt{-\frac{2s^3-2sr-2k}{3s-1}}\xi\right)\left(1+i\sinh\left(\sqrt{-\frac{2s^3-2sr-2k}{3s-1}}\xi\right)\right)^{-1} \tag{69}$$

by merging equations (50) and (71), we get,

$$\varrho_5(x,y,t) = \pm\sqrt{-\frac{-3s^3+3sr+3k}{3s-1}}\cosh\left(\sqrt{-\frac{2s^3-2sr-2k}{3s-1}}\,(kt+x+y)\right)\left(1+i\sinh\left(\sqrt{-\frac{2s^3-2sr-2k}{3s-1}}\,(kt+x+y)\right)\right)^{-1}\mathrm{e}^{-i(kt+ry-sx)} \tag{70}$$

the second part of equations (51) is expressed as follows:

$$\Psi_5(x,y,t) = \pm\frac{1}{3}\frac{-3s^3+3sr+3k}{3s-1}\left(\cosh\left(\sqrt{-\frac{2s^3-2sr-2k}{3s-1}}\,(kt+x+y)\right)\right)^2\left(1+i\sinh\left(\sqrt{-\frac{2s^3-2sr-2k}{3s-1}}\,(kt+x+y)\right)\right)^{-2} \tag{71}$$

3.2 extension rational sine-cosine method

Assume that Equation (51) has solutions in the conformation of

$$\vartheta(\xi) = \frac{\beta_0\,\sin(\zeta\xi)}{\beta_2+\beta_1 cos(\zeta\xi)} \tag{72}$$

By concatenating Eq. (74) into Eq. (51) and then gathering all words with the identical powers of $cos(\zeta\xi)^r$ and equalling to zero all the coefficients of $cos(\zeta\xi)^r$, then we have:
$\cos(\zeta\xi)^2 : -3\beta_0\left(\left(-s^3+rs+k\right)\beta_1{}^2+\beta_0{}^2\left(s-\frac{1}{3}\right)\right)$
$\cos(\zeta\xi)^1 : -6\beta_0\left(-s^3+\left(\frac{3}{2}\zeta^2+r\right)s-\frac{1}{2}\zeta^2+k\right)\beta_1\beta_2$
$\cos(\zeta\xi)^0 : -3\left(\left(-s^3+\left(-3\zeta^2+r\right)s+\zeta^2+k\right)\beta_2{}^2+6\left(s-\frac{1}{3}\right)\left(\beta_1{}^2\zeta^2-\frac{1}{6}\beta_0{}^2\right)\right)\beta_0.$

These algebraic equations will be solved by mathematics software and then we have the subsequent solutions: set 6)

$$\zeta = \pm\sqrt{-\frac{-s^3+rs+k}{6s-2}},\quad \beta_0 = \pm\sqrt{-\frac{-3s^3+3rs+3k}{3s-1}}\beta_1\quad \beta_1=\beta_1,\quad \beta_2=0 \tag{73}$$

set 7)

$$\zeta = \pm\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}},\quad \beta_0 = \pm\sqrt{-\frac{-3s^3+3sr+3k}{3s-1}}\beta_2\quad \beta_1=\beta_1,\quad \beta_2=\beta_2 \tag{74}$$

set 8)

$$\zeta = \pm\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}},\quad \beta_0 = \pm\sqrt{-\frac{-3s^3+3sr+3k}{3s-1}}\beta_2\quad \beta_1=-\beta_2,\quad \beta_2=\beta_2 \tag{75}$$

Case 6: Taking part 6 into consideration, the solutions of (51) can be got as

$$\vartheta_6(x,y,t) = \pm\sqrt{-\frac{-3s^3+3rs+3k}{3s-1}}\tan\left(\sqrt{-\frac{-s^3+rs+k}{6s-2}}\xi\right) \tag{76}$$

by merging equations (50) and (78), then we get

$$\varrho_6(x,y,t) = \pm\sqrt{-\frac{-3s^3+3rs+3k}{3s-1}}\tan\left(\sqrt{-\frac{-s^3+rs+k}{6s-2}}\,(kt+x+y)\right)\mathrm{e}^{-i(kt+ry-sx)} \tag{77}$$

the second part of (51) is expressed as follows:

$$\Psi_6(x,y,t) = \pm 1/3\frac{-3s^3+3rs+3k}{3s-1}\left(\tan\left(\sqrt{-\frac{-s^3+rs+k}{6s-2}}\,(kt+x+y)\right)\right)^2 \tag{78}$$

Case 7: Likewise, the solutions of (51) for set 7 can be got as

$$\vartheta_7(x,y,t)=\pm\sqrt{-\frac{-3s^3+3sr+3k}{3s-1}}\sin\left(\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}}\xi\right)\left(1+\cos\left(\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}}\xi\right)\right)^{-1} \tag{79}$$

by merging equations (50) and (81), then we get

$$\varrho_7(x,y,t)=\pm\sqrt{-\frac{-3s^3+3sr+3k}{3s-1}}\sin\left(\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}}(kt+x+y)\right)\left(1+\cos\left(\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}}(kt+x+y)\right)\right)^{-1}\mathrm{e}^{-i(kt+ry-sx)} \tag{80}$$

the second part of (51) is expressed as follows:

$$\Psi_7(x,y,t)=\pm 1/3\frac{-3s^3+3sr+3k}{3s-1}\left(\sin\left(\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}}(kt+x+y)\right)\right)^2\left(1+\cos\left(\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}}(kt+x+y)\right)\right)^{-2} \tag{81}$$

Case 8: Likewise, the solutions of (51) for part 8 can be obtained as

$$\vartheta_8(x,y,t)=\pm\sqrt{-\frac{-3s^3+3sr+3k}{3s-1}}\sin\left(\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}}\xi\right)\left(1-\cos\left(\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}}\xi\right)\right)^{-1} \tag{82}$$

by merging equations (50) and (84), then we get

$$\varrho_8(x,y,t)=\pm\sqrt{-\frac{-3s^3+3sr+3k}{3s-1}}\sin\left(\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}}(kt+x+y)\right)\left(1-\cos\left(\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}}(kt+x+y)\right)\right)^{-1}\mathrm{e}^{-i(kt+ry-sx)} \tag{83}$$

the second part of (51) is expressed as follows:

$$\Psi_8(x,y,t)=\pm\frac{1}{3}\frac{-3s^3+3sr+3k}{3s-1}\left(\sin\left(\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}}(kt+x+y)\right)\right)^2\left(1-\cos\left(\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}}(kt+x+y)\right)\right)^{-2} \tag{84}$$

or assume that Eq.(51) has solutions in the conformation of

$$\vartheta(\xi)=\frac{\beta_0\cos(\zeta\xi)}{\beta_2+\beta_1\sin(\zeta\xi)} \tag{85}$$

By concatenating Eq. (87) in Eq. (51) and then gathering all terms with the identical powers of $sin(\zeta\xi)^r$ and equating to zero all coefficients of $sin(\zeta\xi)^r$, the subsequent set of algebraic equations are obtained in the following stage:

$\sin(\zeta\xi)^2:\ 6\left(-\left(\frac{1}{2}s^3-\frac{1}{2}sr-\frac{k}{2}\right)\beta_1{}^2+\frac{1}{2}\left(s-\frac{1}{3}\right)\beta_0{}^2\right)\beta_0$

$\sin(\zeta\xi)^1:\ 6\beta_1\beta_2\left(-s^3+\left(\frac{3}{2}\zeta^2+r\right)s-\frac{1}{2}\zeta^2+k\right)\beta_0$

$\sin(\zeta\xi)^0:\ 3\beta_0\left(\left(-s^3+\left(-3\zeta^2+r\right)s+\zeta^2+k\right)\beta_2{}^2+6\left(\beta_1{}^2\zeta^2-\frac{1}{6}\beta_0{}^2\right)\left(s-\frac{1}{3}\right)\right)$.

These algebraic equations will be solved by mathematics software and then we have the subsequent solutions:

set 9)

$$\zeta=\pm\sqrt{-\frac{-s^3+sr+k}{6s-2}},\quad \beta_0=\pm\sqrt{-\frac{-3s^3+3sr+3k}{3s-1}}\beta_1\quad \beta_1=\beta_1,\quad \beta_2=0. \tag{86}$$

set 10)

$$\zeta=\pm\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}},\quad \beta_0=\pm\sqrt{-\frac{-3s^3+3sr+3k}{3s-1}}\beta_2\quad \beta_1=\beta_2,\quad \beta_2=\beta_2. \tag{87}$$

set 11)

$$\zeta=\pm\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}},\quad \beta_0=\pm\sqrt{-\frac{-3s^3+3sr+3k}{3s-1}}\beta_2\quad \beta_1=-\beta_2,\quad \beta_2=\beta_2. \tag{88}$$

Case 9: we take set 9 into consideration, the solutions of (51) can be obtained as

$$\vartheta_9(x,y,t)=\pm\sqrt{-\frac{-3s^3+3sr+3k}{3s-1}}\cot\left(\sqrt{-\frac{-s^3+sr+k}{6s-2}}\xi\right) \tag{89}$$

by merging equations (50) and (91), then we get

$$\varrho_9(x,y,t)=\pm\sqrt{-\frac{-3s^3+3sr+3k}{3s-1}}\cot\left(\sqrt{-\frac{-s^3+sr+k}{6s-2}}\,(kt+x+y)\right)\mathrm{e}^{-i(kt+ry-sx)} \tag{90}$$

the second part (51) is expressed as follows:

$$\Psi_9(x,y,t)=\pm\frac{1}{3}\frac{-3s^3+3sr+3k}{3s-1}\left(\cot\left(\sqrt{-\frac{-s^3+sr+k}{6s-2}}\,(kt+x+y)\right)\right)^2 \tag{91}$$

Case 10: Likewise, for set 10, the solutions of (51) can be obtained as

$$\vartheta_{10}(x,y,t)=\pm\sqrt{-\frac{-3s^3+3sr+3k}{3s-1}}\cos\left(\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}}\xi\right)\left(1+\sin\left(\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}}\xi\right)\right)^{-1} \tag{92}$$

by merging equations (50) and (94), then we get

$$\varrho_{10}(x,y,t)=\pm\sqrt{-\frac{-3s^3+3sr+3k}{3s-1}}\cos\left(\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}}\,(kt+x+y)\right)\left(1+\sin\left(\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}}\,(kt+x+y)\right)\right)^{-1}\mathrm{e}^{-i(kt+ry-sx)} \tag{93}$$

the second part of (51) is expressed as follows:

$$\Psi_{10}(x,y,t)=\pm\frac{1}{3}\frac{-3s^3+3sr+3k}{3s-1}\left(\cos\left(\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}}\,(kt+x+y)\right)\right)^2\left(1+\sin\left(\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}}\,(kt+x+y)\right)\right)^{-2}. \tag{94}$$

Case 11: Likewise, for set 11, the solutions of (51) can be gained as

$$\vartheta_{11}(x,y,t)=\pm\sqrt{-\frac{-3s^3+3sr+3k}{3s-1}}\cos\left(\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}}\xi\right)\left(1-\sin\left(\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}}\xi\right)\right)^{-1} \tag{95}$$

by merging equations (50) and (97), then we obtain

$$\varrho_{11}(x,y,t)=\pm\sqrt{-\frac{-3s^3+3sr+3k}{3s-1}}\cos\left(\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}}\,(kt+x+y)\right)\left(1-\sin\left(\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}}\,(kt+x+y)\right)\right)^{-1}\mathrm{e}^{-i(kt+ry-sx)} \tag{96}$$

the second part of (51) is expressed as follows:

$$\Psi_{11}(x,y,t)=\pm\frac{1}{3}\frac{-3s^3+3sr+3k}{3s-1}\left(\cos\left(\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}}\,(kt+x+y)\right)\right)^2\left(1-\sin\left(\sqrt{-\frac{-2s^3+2sr+2k}{3s-1}}\,(kt+x+y)\right)\right)^{-2} \tag{97}$$

4 Graphical Representation

In this section, by drawing two-dimensional and three-dimensional shapes based on the appropriate values of the parameters to some of the acquired results as well as by describing their values in their captions we finalizes the last section. By employing the extension rational sin-cos as well as sinh-cosh techniques the wave behaviours of the Hirota equation and Hirota-Maccari system have been checked.

In this section, Figure 1 demonstrates the two-dimensional and three-dimensional surfaces of the dark wave soliton solution of the Eq. $|\varrho_1(x,t)|$ for the values $r = 0.5$, $s = 0.5$, and $\Omega = 1$. In Figure 2, by adopting the appropriate values of $s = 1$, $r = 0.5$ and $\Omega = 1$ the 2D and 3D surfaces of the cusp wave solution of the Eq. $|\varrho_4(x,t)|$ has drawn. Figure 3 illustrates two-dimensional and three-dimensional dark wave surfaces of Eq. $|\varrho_6(x,t)|$ for the proper values of $s = 1$, $r = 0.5$ and $\Omega = 1$ for the Hirota equation. The three-dimensional and two-dimensional of Figure 4 illustrates the behaviours of cusp wave soliton solution of the Eq. $|\varrho_9(x,t)|$ by the proper values of $s = 1$, $r = 0.5$ and $\Omega = 1$. Figure 5 demonstrates the two-dimensional and three-dimensional surfaces of the dark wave soliton solution of the Eq. $|\varrho_1(x,y,t)|$ for the values $r = 0.5$, $s = 0.5$, $k = 0.5$.

The three-dimensional and two-dimensional of Figure 6 illustrates the behaviours of the Eq. $|\Psi_1(x,y,t)|$ by proper values of $s = 0.5$, $r = 0.5$, $k = 0.5$ which has dark wave soliton solution. In Figure 7, by adopting the appropriate values of $r = 0.5$, $s = 0.5$, $k = 0.5$ the 2D and 3D surfaces of the cusp wave solution of the Eq. $|\varphi_4(x,y,t)|$ has drawn. In figure 8, by indicating the 2D and 3D surfaces of the cusp soliton solution of the Eq. $|\Psi_4(x,y,t)|$ by the values of $r = 0.5$, $s = 0.5$, $k = 0.5$ the visualization of the Hirota-maccari equation will be completed. Figure 9 illustrates two-dimensional and three-dimensional dark wave surfaces of Eq. $|\varphi_6(x,y,t)|$ for the proper values of $r = 0.5$, $s = 0.5$, $k = 0.5$ for the Hirota-Maccari system.

Figure 10, by adopting the suitable values of $r = 0.5$, $s = 0.5$, $k = 0.5$ we could draw the 2D and 3D dark wave solution surfaces of the Eq. $|\Psi_6(x,y,t)|$.
The three-dimensional and two-dimensional of Figure 11 illustrates the behaviours of cusp wave soliton solution of the Eq. $|\varphi_9(x,y,t)|$ by the proper values of $r = 0.5$, $s = 0.5$, $k = 0.5$. Figure 12 demonstrates the two-dimensional and three-dimensional cusp wave soliton solutios surfaces of the Eq. $|\Psi_9(x,y,t)|$ for the values $r = 0.5$, $s = 0.5$, $k = 0.5$.

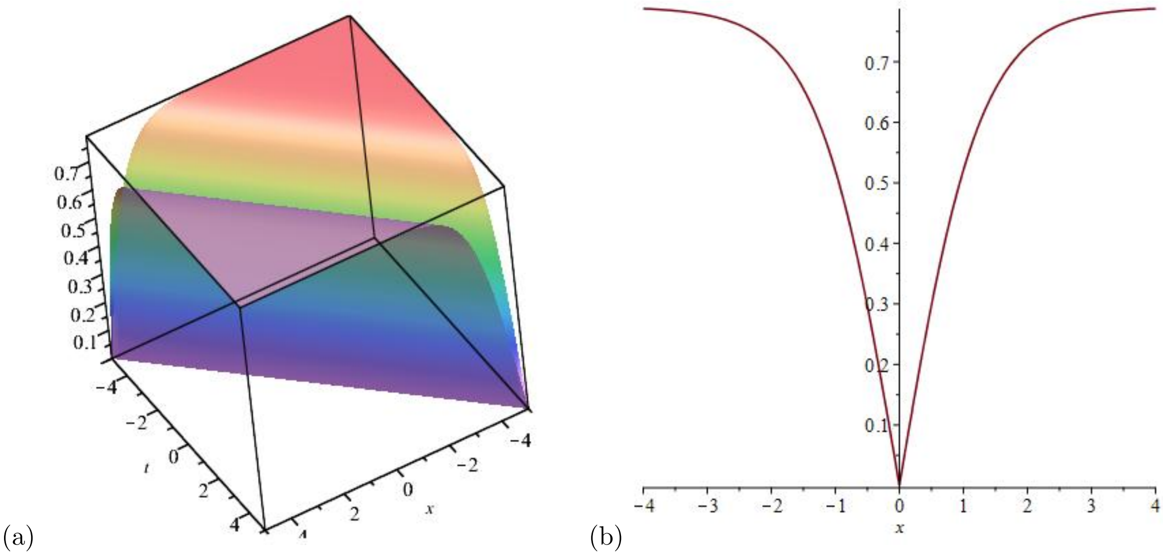


Figure 1: The three-dimensional surfaces of the Equation (18) by regarding the values $r = 0.5$, $s = 0.5$, $\Omega = 1$ in figure. (a) and fig. (b) represents the two-dimensional surface of the Equation (18) by regarding the values of $r = 0.5$, $s = 0.5$, $\Omega = 1$.

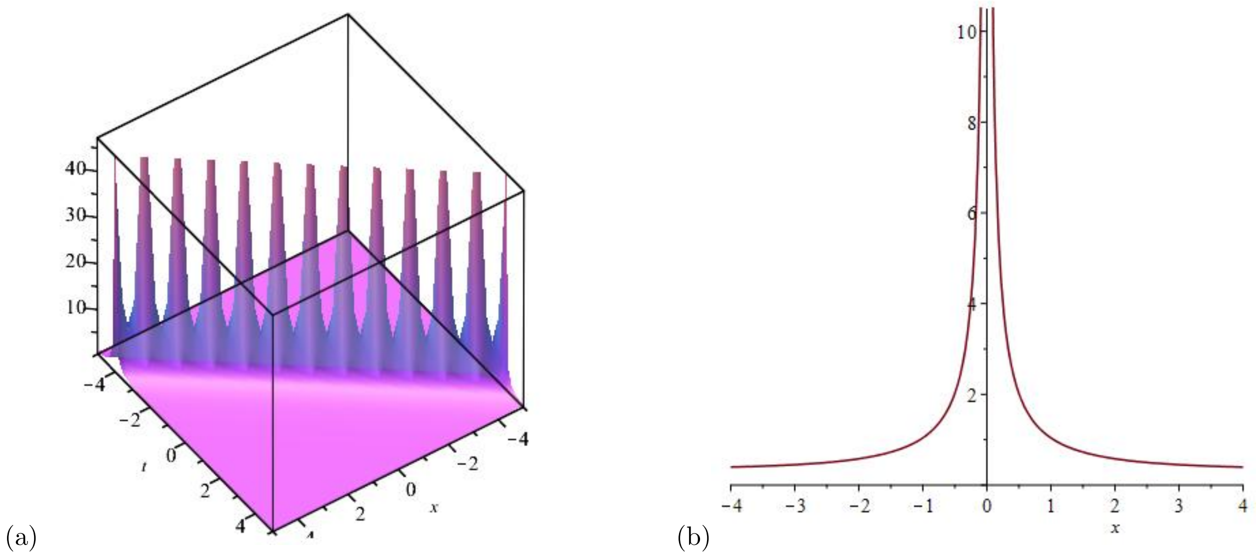


Figure 2: The three-dimensional surfaces of the Equation (27) by regarding the values $r = 0.5$, $s = 1$, $\Omega = 1$ in figure. (a) and fig. (b) represents the two-dimensional surface of the Equation (27) by regarding the values of $r = 0.5$, $s = 1$, $\Omega = 1$.

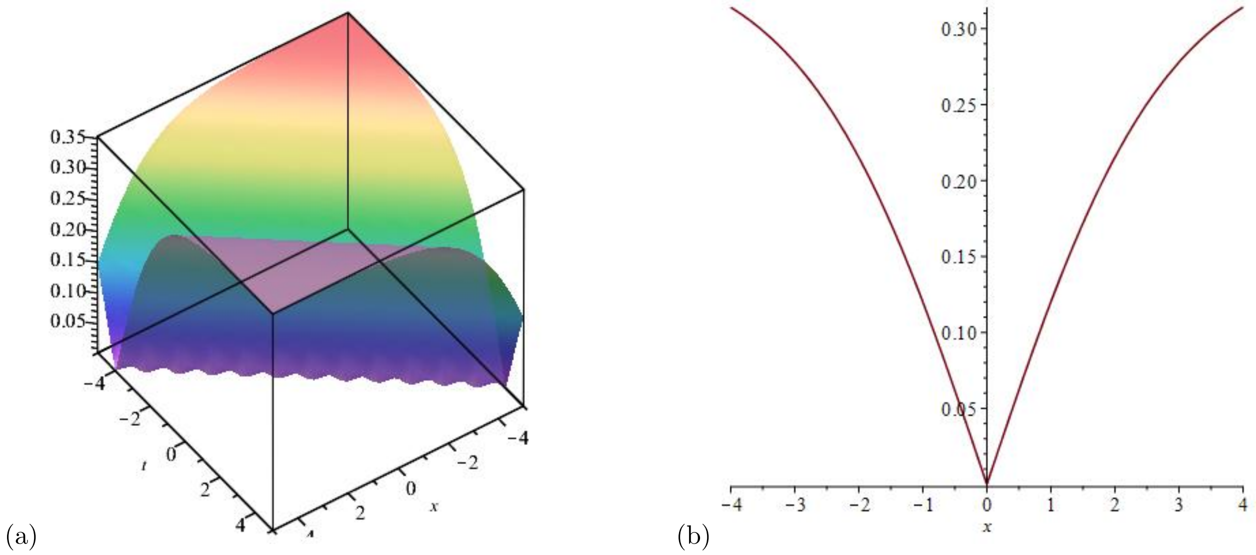
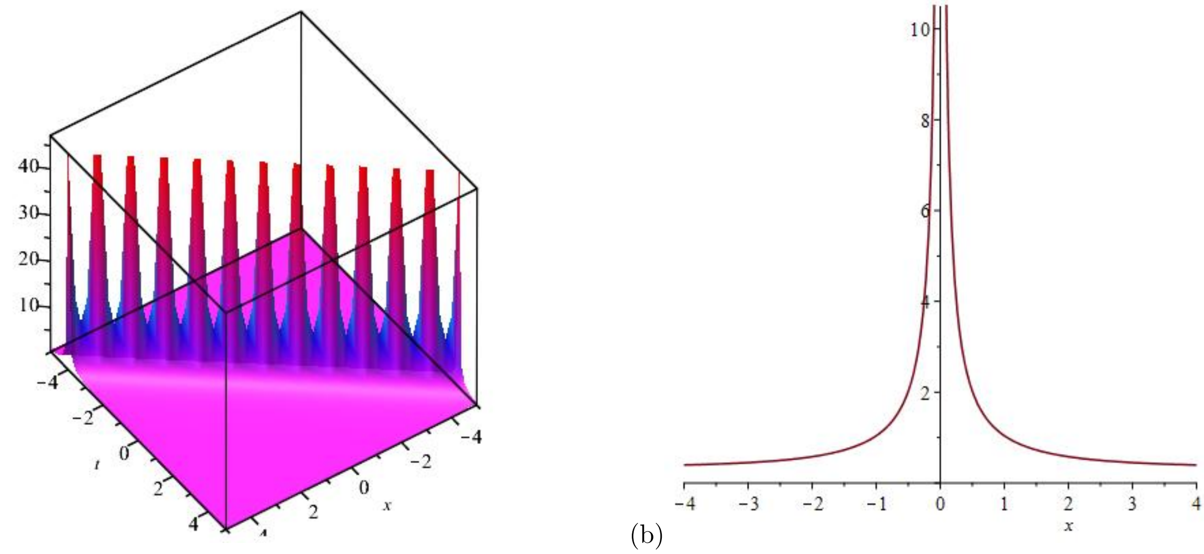
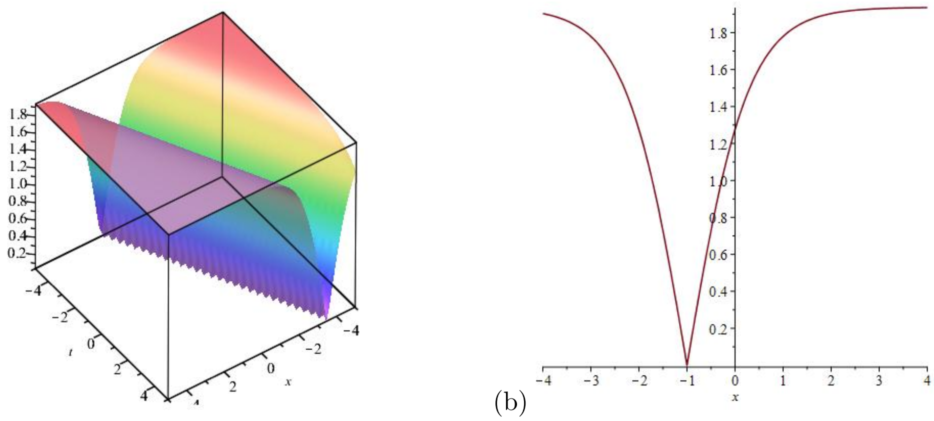


Figure 3: The three-dimensional surfaces of the Equation (35) by considering the values $r = 0.5$, $s = 1$, $\Omega = 1$ in figure. (a) and fig. (b) represents the Equation (35) by considering the values of $r = 0.5$, $s = 1$, $\Omega = 1$.



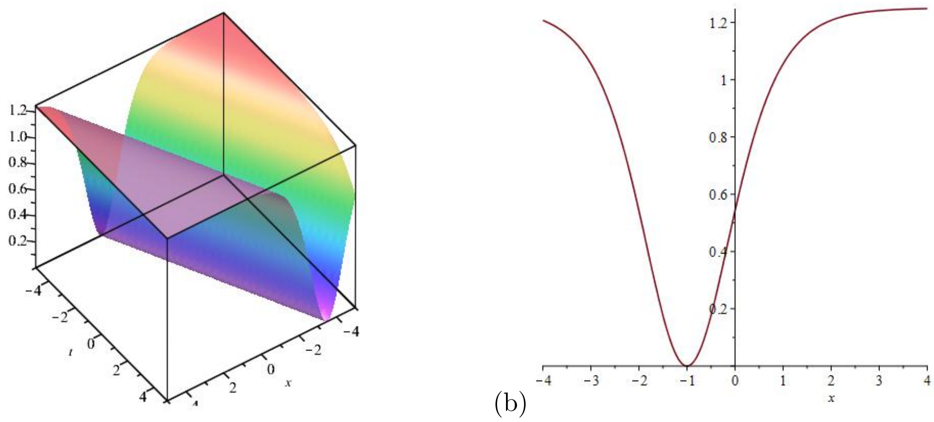
(a) (b)

Figure 4: The three-dimensional forms of the Equation (45) by considering the values $r = 0.5$, $s = 1$, $\Omega = 1$ in figure. (a) and fig. (b) represents the Equation (45) by regarding the values of $r = 0.5$, $s = 1$, $\Omega = 1$.



(a) (b)

Figure 5: The three-dimensional forms of the Equation (57) by considering the values $r = 0.5$, $s = 0.5$, $k = 0.5$ in figure. (a) and fig. (b) represents the Equation (57) by regarding the values of $r = 0.5$, $s = 0.5$, $k = 0.5$.



(a) (b)

Figure 6: The three-dimensional surfaces of the Equation (58) by considering the values $r = 0.5$, $s = 0.5$, $k = 0.5$ in figure. (a) and fig. (b) represents the Equation (58) by regarding the values of $r = 0.5$, $s = 0.5$, $k = 0.5$.

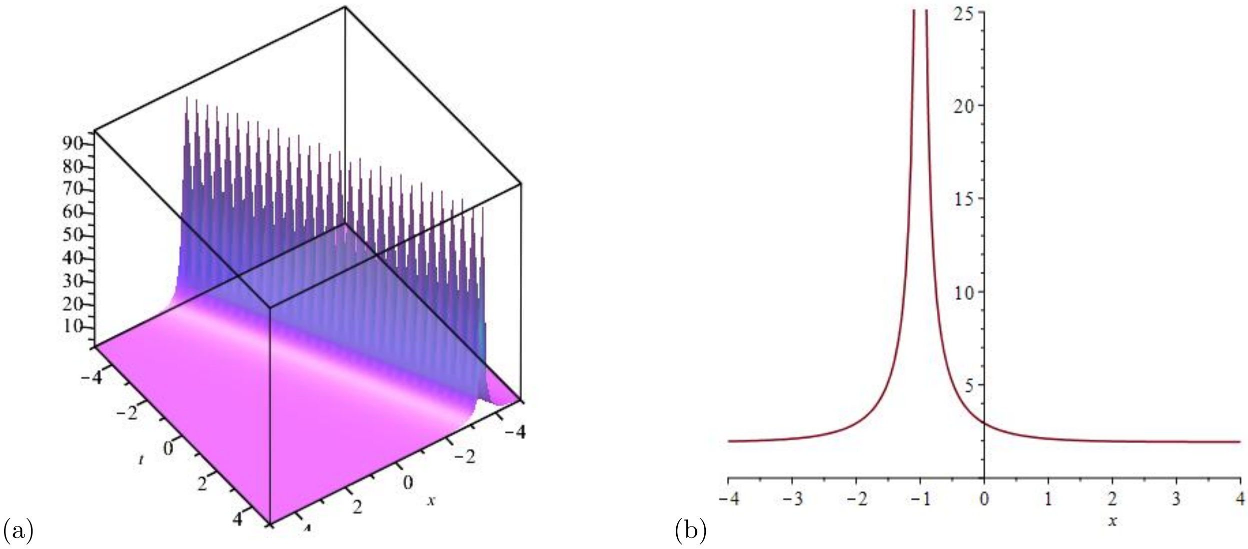


Figure 7: The three-dimensional forms of the Equation (69) by considering the values $r = 0.5$, $s = 0.5$, $k = 0.5$ in figure. (a) and fig. (b) represents the Equation (69) by regarding the values of $r = 0.5$, $s = 0.5$, $k = 0.5$.

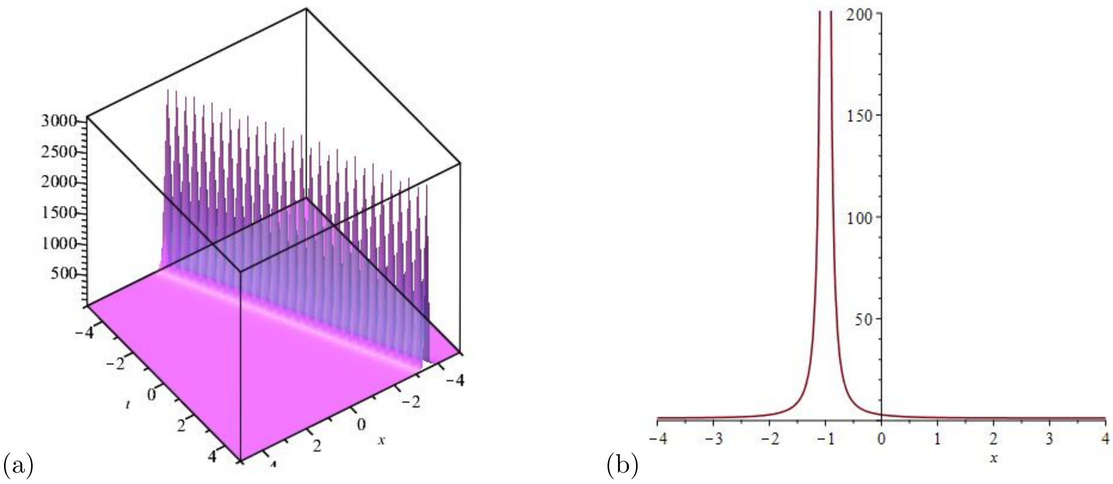


Figure 8: The three-dimensional surfaces of the Equation (70) by considering the values $r = 0.5$, $s = 0.5$, $k = 0.5$ in figure. (a) and fig.(b) represents the Equation (70) by regarding the values of $r = 0.5$, $s = 0.5$, $k = 0.5$.

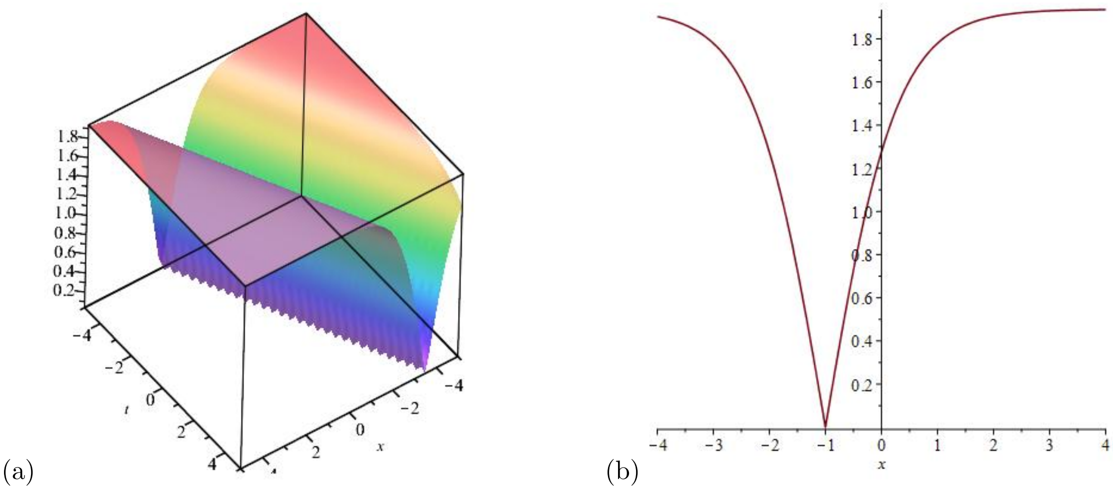
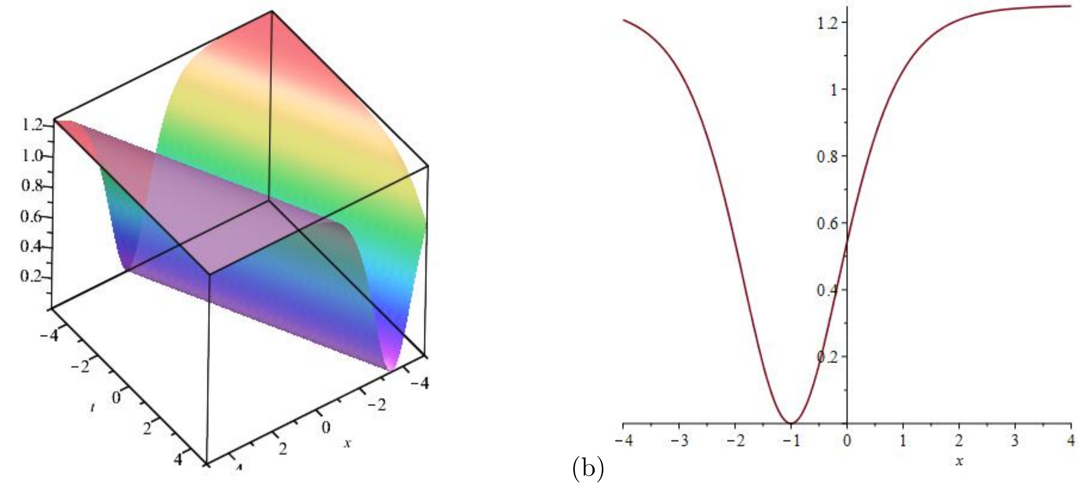
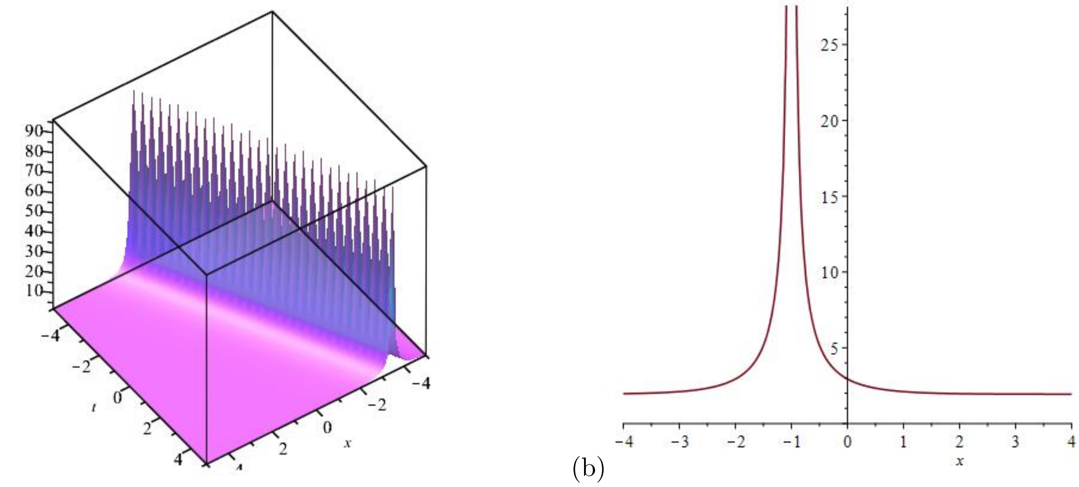


Figure 9: The three-dimensional forms of the Equation (79) by regarding the values $r = 0.5$, $s = 0.5$, $k = 0.5$ in figure. (a) and fig. (b) represents the Equation (79) by regarding the values of $r = 0.5$, $s = 0.5$, $k = 0.5$.



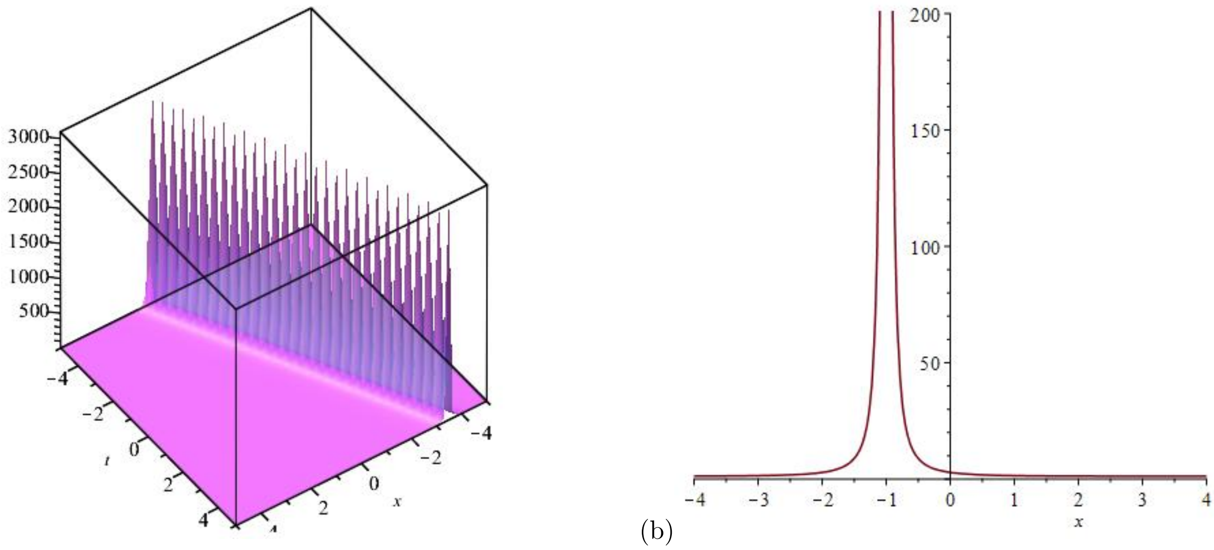
(a) (b)

Figure 10: The three-dimensional surfaces of the Equation (80) by considering the values $r = 0.5$, $s = 0.5$, $k = 0.5$ in figure. (a) and fig. (b) represents the Equation (80) by regarding the values of $r = 0.5$, $s = 0.5$, $k = 0.5$.



(a) (b)

Figure 11: The three-dimensional surfaces of the Equation (92) by considering the values $r = 0.5$, $s = 0.5$, $\omega = 1$ in figure. (a) and fig. (b) represents the two-dimensional surface of the Equation (92) by regarding the values of $r = 0.5$, $s = 0.5$, $\omega = 1$.



(a) (b)

Figure 12: The three-dimensional forms of the Equation (93) by considering the values $r = 0.5$, $s = 0.5$, $k = 0.5$ in figure. (a) and fig. (b) represents the two-dimensional form of the Equation (93) by considering the values of $r = 0.5$, $s = 0.5$, $k = 0.5$.

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