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Steel price volatility forecasting; application of the artificial neural network approach and GARCH family models

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Abstract

GARCH family models are the most widely-used methods for forecasting price volatility. Given that this approach usually has extremely high forecast errors, continuous studies have been conducted to improve forecast models using different techniques. In the present manuscript, we expanded the fields of expert systems, forecast, and modeling using an artificial neural network (ANN) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) method that created an ANN-GARCH model. The hybrid ANN-GARCH model was used to forecast steel price volatility, and its accuracy was evaluated based on mean absolute error (MAE) and mean square error (MSE) evaluation criteria. The results indicated a general improvement in forecasting using ANN-GARCH compared to the GARCH method alone. The results were realized using copper price returns, the dollar index, gold price returns, and oil price returns as inputs. We also discussed the research implications for this field in addition to practical applications. The research results indicated better performance of the hybrid ANN/GARCH/N model than other models. Furthermore, the neural-network-based hybrid models could better forecast prices than other time series models.

Keywords: steel price volatility, forecast, the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model, the Exponential General Autoregressive Conditional Heteroskedasticity (EGARCH) model, Artificial Neural Network (ANN), hybrid model
2020 MSC: 68T07, 93C30

Introduction

Steel is an essential base metal that constitutes the backbone of the industry. The price of steel products has been a critical issue for policymakers in companies or the whole country. Therefore, steel price forecasting is highly demanded by various institutions. Steel price has become more volatile due to globalization, and thus transactions as a financial tool in mercantile exchange; hence, steel price forecasting has significant effects on the decision-making process at all levels to manage the risk of price changes.

In this regard, it is important to forecast steel price volatility more accurately for commodity markets and the global economy. A common method is the use of GARCH family models to forecast volatility. However, this approach often has very high forecast errors that have the potential to cause large economic losses to those who use a faulty

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model. Furthermore, shortcomings in modeling approaches contribute to greater inefficiencies in the market. Therefore, improved modeling approaches are continuously seeking to reduce risk and improve market efficiency.

Classical studies to reduce errors in models are seeking to include variables that seem to be more important in explaining steel price volatility for a target period, and thus they improve the explanatory power of a particular model. However, this classical approach usually lacks predictability outside the sample period. This manuscript creates innovation by changing the focus of models to the ability to forecast future volatility instead of their explanation. Therefore, the results of this study are more useful in predicting steel prices. The present study can be important for government officials in countries where the steel industry constitutes a large portion of their economy, and for investors who want to make investment decisions in the commodity market to be more efficient in asset allocation and have better portfolio diversification.

Tully and Lucey [18] modeled gold price volatility using the asymmetric power GARCH (AP) model and concluded that oil price and Financial Times Stock Exchange 100 Index (FTSE) were the most relevant variables affecting gold price volatility. Kristjanpoller et al. [5] indicated that an expert system, especially the ANN-GARCH model, increased the accuracy of volatility forecasts predicted by GARCH models. The expert system is sensitive to the behavior between the variables so that the results improve the forecast.

The ANN-GARCH approach makes it possible to determine the effects that the variables exert, thereby resulting in gradual improvements in the accuracy of forecasts compared to the classical form of their relationship in model fitting. The ANN-GARCH approach not only considers the GARCH forecasts as an input of the artificial neural network (ANN) model but also allows the combination of other input variables in the ANN. Therefore, financial variables, which are important for forecasting the steel price or volatility in classical models, can be included as inputs in this model. This fact is very important because the focus is not on the in-sample behavior, but rather the variables are measured based on their contribution to the out-of-sample forecast. According to previous studies, the present study used an ANN-GARCH model to forecast volatility beyond the sample period to the out-of-sample population, thereby improving current capabilities and knowledge. The importance of this current measure is based on two findings.

First, we determine the improvement in forecast accuracy using a hybrid model as opposed to classical GARCH models. In particular, we show greater accuracy in hybrid ANN-GARCH forecasts of steel price volatility for GARCH family models with different distributions. Second, the ability to include financial variables as inputs in the ANN allows for determining the effects of variables on the steel price volatility estimation. This is important since the improved accuracy of steel price estimation and volatility leads to decision-making by policymakers and investors and improves market efficiency. The rest of this measure is classified into four more sections. The next section presents a brief literature review of the forecast models, ANNs, and their various applications. The following section explains the research methodology and analyzes data. The findings are then interpreted, and finally, the last section provides a conclusion of the main results.

Literature review

Authors have focused on modeling and forecasting volatility in financial series in recent years because they are very important for describing markets, optimizing portfolios, and valuing assets. The case of steel is not an exception to this rule. There are numerous studies that focus on base metals, either in spot price or volatility analysis. The present study models the steel cash price volatility using the ANN-GARCH model to determine important macroeconomic variables for forecasting.

In studies in Iran, Bakhtiaran and Zolfaghari [1] investigated the forecast of gold daily returns in a research titled "Designing a model to forecast the world price of gold". In this research, they used a combination of GARCH family models with ANNs. Their results indicated the superiority of the hybrid model over the current models in forecasting the time series of gold price returns. Therefore, the use of a convolutional neural network strengthens the predictive power of GARCH family time series models. In a research titled "A market direction prediction model for gold coin trades in Iran Mercantile Exchange using long short-term memory (LSTM) algorithm", Zoghi et al. [21] obtained wavelet noise of input data and then gave data as input to LSTM architecture to predict the market. Their results confirmed the higher accuracy and efficiency of the long short-term memory model.

In a study titled "The evaluation and validation of the optimal architecture of deep learning in stock price forecast with the long short-term memory (LSTM) algorithm approach", Sharif-far et al. [17] investigated the architecture of neural network models, including the number of nodes, layers, group size, optimizer, and activation functions. To this end, they utilized the data of Esfahan Steel Company in 2017-2019. Their results indicated the better performance of the LSTM architecture with the Drop Out layer than its simple model and also RNN model. In a study titled "Bitcoin price prediction using the hybrid ARIMA model and deep learning", Mohammad Sharifi et al. [12] utilized three types of recurrent neural networks, including regression neural network (RNN), long short-term memory (LSTM), and Gated recurrent units (GRUs) to forecast the Bitcoin price. Their results indicated that the ARIMA-GRU model led to better results than other models in the evaluation criteria.

In a study titled "Designing a model to forecast the Stock Exchange total index returns by the hybrid models of ANNs and GARCH family models using the daily data of the Stock Exchange total index during 2018-2019", Zolfaghari and Sahabi [22] compared the forecast accuracy of hybrid models with individual models. Their research results indicated that hybrid models had higher forecast accuracy than individual models.

In another study titled "Designing a model to forecast the stock exchange index return with an emphasis on the hybrid models of ANNs and LSTM", Zolfaghari et al. [14] introduced hybrid models of the GARCH family and ANN. Their findings indicated that the hybrid ANN-FIEGARCH model with a student's t-distribution was more efficient in forecasting the stock exchange total index return and had a lower forecast error than other rival models.

In a study titled "The evaluation of factors affecting the iron ore price using ANNs", Moghadam et al. [11] examined the effects of several economic parameters such as steel price, GDP index, crude oil price, aluminum price, gold price, interest rate, inflation rate, dollar value, stock value, and the rate of iron and steel production on the monthly price of iron ore. After finding the optimal neural model and performing sensitivity analysis, their results indicated that China's GDP, gold price, and oil price had the greatest effects, and the interest rate, inflation rate, and dollar value had the lowest effects among the input parameters on the iron ore price. In a study titled "iron ore price forecasting using a time series model", Mohammadi et al. [13] sought to forecast the iron ore price. The performance of the model was evaluated based on two criteria, Akaike information criterion (AIC), and Root mean square error (RMSE). Their results indicated that the time series model led to acceptable results about the forecast of iron ore prices.

In studies other than in Iran, Yan Hu et al. [20] investigated a hybrid method for predicting copper price volatility from January 1, 2008, to December 31, 2018. This hybrid method was a combination of GARCH family models, ANNs, and recurrent networks. The experimental results also indicated that GARCH forecasts could act as information features to significantly increase the predictive power of neural network models. Furthermore, the integration of LSTM and ANNs was an effective approach to building useful deep neural network structures to enhance forecast performance.

Kim and Won [19] combined a new model with the long short-term memory (LSTM) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models and compared the new hybrid models with traditional models using the KOSPI200 index data from January 1, 2010, to the end of 2016. Their results indicated that the new models had the lowest forecast errors in terms of mean absolute error and mean square error.

Kristjanpoller et al. [7] measured the volatility of Bitcoin price using a hybrid GARCH-ANN model from December 13, 2011, to August 26, 2017, for three periods, 10, 22, and 44 days. In this research, the hybrid model significantly increased the accuracy of price volatility forecast compared to individual and rival models. Similarly, Lahmiri [8] used hybrid models of the GARCH family and ANNs with normal distribution, student's t-distribution, and Generalized Error Distribution (GED) and forecasted volatility for a period of 20 days using daily data of Canadian dollar and Euro from 2010 to 2015, and then compared the results using two the absolute mean error and the mean square error. The results indicated that the hybrid EGARCH-ANN model with GED could better forecast the Canadian dollar and Euro volatility.

Kristjanpoller and Hernández [4] predicted the volatility of three base metals, gold, silver, and copper, with hybrid models, ANN-GARCH and ANN-APGARCH, using explanatory variables of oil prices, financial market indices, and currency pairs. Their results indicated that the use of a hybrid neural network model increased the power of forecasting out-of-sample volatility for these three metals.

Lu et al. [10] compared the prediction performance of the hybrid artificial neural network and GARCH and EGARCH models to forecast the volatility of China's energy share return from December 31, 2013, to March 10, 2016, and concluded that the hybrid models of EGARCH-ANN had lower forecast error and higher forecast power. Lahmiri and Boukadoum [9] investigated the volatility of the S&P 500 index using a hybrid model of an artificial neural network with error Back Propagation (BP) algorithm and GARCH family, including GARCH and EGARCH models with GED and Student's t-distribution from February 28, 2011, to March 11 of the same year, and analyzed the results using RMSE and MAE indices. The research results indicated that the hybrid EGARCH-BP model has a lower prediction error than other individual models. Similarly, in research titled "copper price forecasting by ANNs and ARIMA", Sanchez et al. [16] predicted the price of copper. To this end, they used the monthly copper price from January 2, 2002, to January 16, 2014. The results indicated that the performance of the neural network model was better than the ARIMA model. In another study titled “The gold price volatility forecast using the ANN-GARCH model”, Kristjanpoller & Minutolo [6] investigated hybrid models for forecasting the gold price using economic input variables and predicting the GARCH family models. The results indicated the better performance of the hybrid models than the classical models. Similarly, Kriechbaumer et al. [3] conducted a study titled an improved wavelet–ARIMA approach for forecasting aluminium, copper, lead, and zinc prices. To this end, monthly prices of these metals were used from January 2008 to April 2014. The results indicated that the prediction accuracy of the wavelet–ARIMA approach was higher than the wavelet transform and function. In another study, Kristjanpoller et al. [5] used a hybrid ANN-GARCH model to forecast volatility in three Latin American indices of Brazil, Chile, and Mexico. Their studies indicated that hybrid neural network models could improve the forecasts of GARCH models.

Research questions

1- The first question: How is the forecast of steel price using the hybrid model?

2- The second question: How is the accuracy of the hybrid method compared to conventional models?

Research hypothesis

The hybrid model proposed in the present research had lower error than single and classical models in a short-term forecast of global steel price returns.

Research models and methods of measuring the variables

Since classical models, such as OLS, are not suitable for situations where there is heteroskedasticity and it is true for financial time series, Engle [2] introduced models based on autoregressive conditional heteroskedasticity (ARCH). However, the results of forecasting these GARCH models may not be very satisfactory owing to the existence of a complex nonlinear correlation structure among the variables and the larger size of the data set.

ARMA and ARIMA models

ARIMA (p, d, q) process for variable x can be displayed as follows:

$$\Phi\left(B\right)\Delta^{d}x_{t}{=}\theta_{0+}\theta\left(B\right)\varepsilon_{t} \tag{0.1}$$

Experimental studies by Stocks and Watson on forecasting economic variables indicated that adding explanatory variables improved the capability of this model in forecasting time series. Adding explanatory variables as a matrix to the ARIMA model causes the adjusted ARIMAX (p, d, q) as follows:

$$\Phi\left(L\right)\Delta^{d}Y_{t}{=}\theta\left(L\right)X_{t}{+}\theta\left(L\right)\varepsilon_{t} \tag{0.2}$$

It is an autoregressive integrated moving average process from the orders q, d and p that represent the number of autoregressive terms (p), the order of differentiation (d), and the number of moving average terms (q) respectively, and X_t means the integrated order d which is displayed by $I(d)$. If d is zero, the ARIMA process becomes ARMA. Box-Jenkins method is used to estimate the ARIMA and ARMA models. In this method, the number of terms p and q is calculated using Autocorrelation (AC) and Partial Autocorrelation (PAC) functions, and their accuracy is calculated using the Akaike information criterion (AIC), Schwarz Bayesian Information Criterion (SBC), or Hannan-Quinn Information Criterion (HQIC) criteria, and then the number of terms p and q is measured by targeting the minimum rates of these criteria.

The ARIMA model is used because its AR term in time series indicates that the autoregressive model is used if the values of static time series depend on their previous values. Furthermore, the I term refers to cointegration, and its value is equal to zero when the model is static. In this model, the MA term is used to forecast the static time series that deviates from its stable trend. According to the characteristics of each of the moving average and autoregressive methods, it is possible to combine these two methods within the framework of the ARMA(p, q) model. Furthermore, ARIMA (p, I, q) is used when cointegration is also proven in the model based on the stationary test. In addition to the above-mentioned explanations, the ARIMA model was also used as a basic model in many previous studies. In other words, this model indicates the equation of the average (or conditional average) of the time series so that the error terms should be estimated based on the conditional variance model of the GARCH family by verifying the existence of a correlation between its error terms (based on the ARCH effect test). It is worth noting that if there is a long memory in the conditional average model, that model is considered ARFIMA.

The GARCH family models

The return of time series as r in the existence of Ω_{t-1} information is displayed as follows to provide a proper image of the volatility models of conditional average and conditional variance:

$$r_t = \Phi_0 + \sum_{i=1}^{p} \Phi_i r_{t-i} - \sum_{i=1}^{q} \theta_i a_{t-i} + a_t \tag{0.3}$$

$$E\left[r_t|\Omega_{t-1}\right] = \Phi_0 + \sum_{i=1}^{p} \Phi_i r_{t-i} - \sum_{i=1}^{q} \theta_i a_{t-i} + a_t \tag{0.4}$$

Equations (0.3), (0.4) represents the autoregressive integrated moving average (ARIMA) model. Here, other exogenous variables can be added to the right side of the equation (e.g. seasonal calendar effects and dummy variables). This second-order momentum can be defined using the definition of conditional variance. Autoregressive conditional heteroskedasticity (ARCH) was first presented by Engle [2]. Even though ARCH models are superior due to their simplicity, sometimes a significant number of a_i parameters are needed to describe volatility. To this end, Bollerslev (1988) defines a model which has fewer parameters and is in a better position than the ARCH model in terms of the adequacy of the exponential characteristic of the model that is called the GARCH model as shown below:

$$r_t = \Phi_0 + \sum_{i=1}^{p} \Phi_i r_{t-i} - \sum_{i=1}^{q} \theta_i a_{t-i} + a_t; \quad a_t = \sigma_t \varepsilon_t \tag{0.5}$$

$$\sigma^2 = \omega_0 + \sum_{i=1}^{m} a_i a_{t-i}^2 + \sum_{j=1}^{n} \beta_j \sigma_{t-j}^2 \tag{0.6}$$

The exponential general autoregressive conditional heteroskedasticity (EGARCH) is another form of the GARCH family, which was presented by Nelson [15] by including positive and negative asymmetric effects on returns.

$$\ln \sigma_{j,t}^2 = \omega_j + \beta_j \ln\left(\sigma_{t-1}^2\right) + \gamma \frac{|\varepsilon_{t-1}|}{|\sigma_{t-1}|} + a\left[\frac{|\varepsilon_{t-1}|}{|\sigma_{t-1}|} - \sqrt{\frac{2}{\pi}}\right] \tag{0.7}$$

where σ^2 is the conditional heteroskedasticity and $\gamma, \beta, \alpha, \omega$ are the coefficients of its parameters. This model eliminates the need to impose limitations on the parameters of the GARCH model under which the variance always remains positive by defining the conditional heteroskedasticity in a logarithmic form. Therefore, this model can well explain the fact that negative shocks lead to a larger conditional heteroskedasticity than positive shocks.

Along with the standard GARCH model, this model can add explanatory variables to its body to increase the modeling and forecasting ability just like the ARIMA models. b represents the explanatory variable in the model (0.3).

$$\sigma^2 = \omega_0 + \sum_{i=1}^{m} a_i a_{t-1}^2 + \sum_{j=1}^{n} \beta_j \sigma_{t-j}^2 + \sum_{j=1}^{n} b_j \tag{0.8}$$

Normal, Student's t, and Generalized Error Distributions (GED)

In the calculation of volatility models, it is empirically assumed that the error terms follow three distributions: normal, student's t, and generalized error distributions. The Probability Density Function (PDF) of normal distribution for random variable z is as follows:

$$F(z) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(z-\mu)^2}{2\sigma^2}\right) \tag{0.9}$$

where μ and σ represent the mean and variance respectively. Likewise, the probability cumulative function of the student's t-distribution is as follows:

$$F\left(z,\mu,\sigma,\upsilon\right)=\frac{C}{\sigma^2}\left(1+\frac{\left(Z-\mu\right)^2}{\sigma^2(\upsilon-2)}\right)^{\frac{-(\upsilon+1)}{2}} \tag{0.10}$$

In Equation (0.3), μ is the mean, σ is the variance, and ν is the degree of freedom. In this equation, parameter C is defined as follows:

$$C=\frac{\Gamma\left(\frac{\upsilon+1}{2}\right)}{\sqrt{\mu(\upsilon-2)}\Gamma\left(\frac{\nu+1}{2}\right)} \tag{0.11}$$

where $\Gamma(0)$ represents the gamma function, and the parameter υ controls the thickness and kurtosis of the density function. The GED cumulative probability function is shown below.

$$F\left(z_t,\mu,\sigma,\beta\right)=\frac{\beta}{2\sigma\Gamma\left(\frac{1}{\beta}\right)}\exp\left(-\left(\frac{|z_t-\mu|^\beta}{\sigma}\right)\right) \tag{0.12}$$

where, μ and σ represent mean and variance respectively, and β controls the unevenness and thickness or thinness of the distribution domain behavior. $+$, zero, and -2∞ for β indicate positive, normal, and negative kurtosis respectively.

Artificial Neural Network (ANN)

In addition to the above-mentioned cases, some researchers have used these methods to predict stock prices in recent years due to the development of artificial intelligence models. The use of artificial intelligence neural networks, which can adapt to the volatility of market variables without being limited to certain models, is an appropriate method for modeling financial variables. ANN is the basis of Artificial Intelligence (AI) that seeks to simulate human brain functioning and solves problems that would be impossible or difficult according to human or statistical standards. ANNs have a self-learning capability that allows them to obtain better results by providing more information. In the present research, we use feedforward artificial neural networks such as ANN.

A multilayer perceptron (MLP) has one or more hidden layers of neurons in addition to the input and output layers. A feedforward neural network with a hidden layer, an activation function in the hidden layer, an output layer, and a sufficient number of neurons in the hidden layer is able to accurately approximate any function.Therefore, this type of neural network is called the Universal Approximator. There are different types of neural network models. Multilayer perceptron is the most common neural network structure that is used in studies. A neural network with a hidden layer is usually a function as $f{:}R^D \to R^L$, in which D is the size of the input layers of the vector x, and L is the size of the output of vector $f\left(x\right)$ as shown in Figure 1.

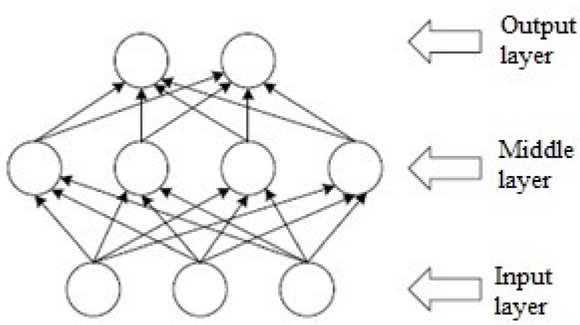

Figure 1: Neural network

In a matrix form:

$$f\left(x\right)=G(b_2+W_2\left(s\left(b_1+W_1x\right)\right) \tag{0.13}$$

where, b_1 and b_2 are deviation vectors and W_1 and W_2 are weight matrices. G and s are activation functions. The mathematical form of the hidden layer is as follows:

$$h\left(x\right)=\Phi\left(x\right)=s\left(b_1+W_1x\right) \tag{0.14}$$

where, $W_1 \in R_{D\times D_k}$ is the weight matrix that is the interface between the input and hidden layers. Each column W_1 assigns a weight from the input data to the i^{th} hidden layer. The usual choices for S include the Rectified Linear Unit (Relu), the hyperbolic tangent function (Tanh), and the sigmoid function. Each element on the vector can produce a unique vector. Finally, the form of the output vector is as follows:

$$o\left(x\right)=G(b_2+W_2h\left(x\right)). \tag{0.15}$$

The learning process in the neural network refers to the estimation of weights and parameters of the model. The most famous learning is with the supervision of the Back Propagation (BP) algorithm that is used in the present research.

Neural network activation functions

Hyperbolic tangent (Tanh) function

Tanh function is S-shaped like a sigmoid function, but it has more strengths than the sigmoid function. This function is zero-axis; hence, it helps the model to have negative, neutral, and positive input values. In other words, negative values, strongly negative, and zero values are mapped close to zero in the hyperbolic tangent graph. Its function is uniform but its derivative is not uniform.

$$f\left(x\right)=\frac{e^x-e^{-x}}{e^x+e^{-x}}. \tag{0.16}$$

Rectified Linear Unit (Relu)

The Rectified Linear Unit (Relu) activation function is often used and very famous in the field of deep learning. This function works in a way that it considers negative values (below zero) as zero and positive values (greater than zero) and values equal to zero as their own values. It is computationally very efficient and allows the network to converge quickly because its relationship is linear, and thus it works faster than the Tanh function.

$$f\left(x\right)=\max(0,x) \tag{0.17}$$

Prediction evaluation criteria

The error rates of real values compared to the predicted values are calculated using the mean square error (MSE) and the mean absolute error (MAE) to examine the performance of steel price forecast models after estimating and predicting these models so that the optimal model can be chosen based on the mean error of the lowest forecast error. The desired models are displayed as follows:

$$\text{MSE}=\frac{1}{m}\sum_{i=1}^{m}\left(y_i-\hat{y}_i\right)^2 \tag{0.18}$$

$$\text{MAE}=\frac{1}{m}\sum_{i=1}^{m}\left|y_i-\hat{y}_i\right| \tag{0.19}$$

where, y_i represents the real values, $\hat{y}_i$ refers to the forecasted values, and m represents the number of observations.

Hybrid models

According to the above-mentioned general structure, d is examined before the first step of the stationary test and inclusion of cointegration, and then the conditional average of the steel price time series is estimated using ARIMA models according to three steps of the Box-Jenkins approach in which the number of autoregressive terms (p) and the number of moving average terms (q) are calculated and then reviewed based on AIC, SBC, and HQIC. The second step is classified into two sections. In the first section, the data is given to the individual models of the ANN once to forecast the time series of the price data using these networks. Different combinations of parameters affecting neural networks such as activation functions are also investigated. The data is divided into 80% of the training section for building the model, and 20% of the data for testing the model to implement model. The training stage of each model is repeated 250 times to improve the learning process. In another section, the ARCH effect of this model is investigated, and if the existence of the ARCH effect is confirmed, the conditional variance of the GARCH family models is estimated based on normal and student's t distribution and GED to improve the learning process. Given that gold, oil, dollar, and copper return variables are used as control variables in this study, the returns of these variables are used for conditional average and thus a total of 6 models are estimated. In the third step, the outputs of GARCH family models are given as new input control variables to ANNs to predict prices with new hybrid models. In the fourth step, the results of the GARCH and EGARCH family models are compared with ANN and new hybrid models using MAE and MSE indices, and in the fifth step, the optimal model with the least error is predicted and selected.

Data analysis

Descriptive statistics

Data of this research include prices of steel ingots in the CIS region, which are collected weekly from 22.03.2016 to 30.08.2022 with 338 observations. Furthermore, the control variables, namely the West Texas Intermediate Crude Oil (WTI) price, world gold ounce price, world copper price, and dollar index are used. First, the return time series of these 5 variables is calculated by Equation (0.20):

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{0.20}$$

where r_t is the return, and P_t and P_{t-1} are the closing price of the current week and the closing price of the last week of the variables respectively. Table 1 presents descriptive statistics of the series along with their returns.

Table 1: Descriptive statistics of data

Statistic	**Steel price**	**Steel return**	**Copper price**	**Copper return**	**Oil price**	**Oil return**	**Dollar index**	**Dollar index return**	**Gold price**	**Gold return**
Mean	469.34	0.00	3.15	0.00	59.32	0.01	95.92	0.00	1508.38	0.00
Median	438.00	0.00	2.96	0.00	56.09	0.01	95.91	0.00	1356.90	0.00
Maximum	820.00	0.29	4.79	0.09	123.70	0.99	108.75	0.04	2043.30	0.09
Minimum	272.50	-0.23	2.09	-0.10	10.01	-0.50	88.98	-0.03	1133.60	-0.08
Sd	109.85	0.04	0.70	0.03	18.28	0.08	3.78	0.01	263.74	0.02
Skewness	0.76	1.36	0.74	-0.23	0.98	4.17	0.67	0.25	0.36	-0.10
Kurtosis	0.19	17.64	-0.49	0.75	4.54	64.10	3.75	4.11	-1.50	4.76
Jarque-Bera statistic	32.94	4347.00	34.54	10.15	87.52	53558.30	33.40	20.00	1.50	44.44
Probability	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00

According to Table 1, the data of steel return, dollar index, and oil return have positive kurtosis coefficients. Furthermore, the steel and oil return, and the dollar index are skew to the right, and the gold and copper returns are skew to the left. The difference between the mean and the standard deviation indicates the existence of volatility during the study. Given that the variable statistics are not zero or close to zero, the Jarque-Bera statistic and its probability rate indicate the non-normality of the indices and their returns.

According to the chart of the CIS steel ingot price, the price volatility is sharp but intermittent in 2016 and 2017 as shown in the price standard deviation chart. The price is growing from 2020 onwards. As shown in Figure 2, the standard deviation range of the price is also increasing.

A lower correlation coefficient between the explanatory variables indicates that more information can be extracted by the models for a better fit. Therefore, the study is conducted on explanatory variables with correlation analysis and principal component analysis (PCA). Both analyses are evaluated in terms of returns. Figure 3 shows the heat map of the variable return correlation matrix. According to the dispersion of correlation coefficients in the heat map, these control variables can be good sources for analyzing the main variable. The data stationary is examined using the Augmented Dickey-Fuller Unit Root Test, Phillips-Perron test (PP), and Kwiatkowski-Phillips-Schmidt-Shin (KPSS).

As presented in Table 2, the statistics are significant in all three tests and all three levels. In the Augmented Dickey-Fuller Unit Root and Phillips-Perron tests, the null hypothesis of statistical non-stationary is rejected at all levels. Furthermore, the null hypothesis of the stationary statistic is confirmed at all levels in the KPSS test.

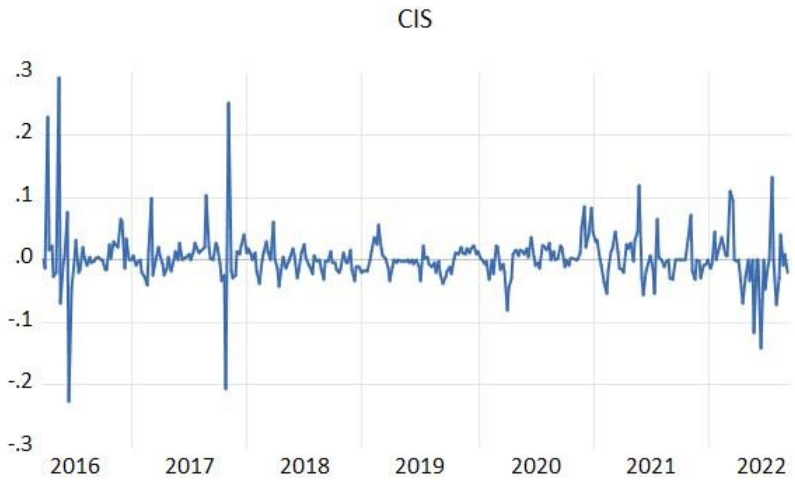

Figure 2: Chart of price and standard deviation of CIS steel ingot price

Oil	Dollar index	Gold	Copper	Steel	
				1.0000	Steel
			1.0000	0.0221	Copper
		1.0000	0.2297	-0.0502	Gold
	1.0000	-0.3820	-0.3800	0.0272	Dollar index
1.0000	-0.0838	0.1134	0.2113	0.0440	Oil

Figure 3: Heat map of the correlation coefficient matrix of returns of the variables

Examining the assistance of long-term memory

It is essential to examine the effect of long-term memory on the return of the conditional average of data before forecasting the data. Two tests, R/S and GPH, are used to examine the presence of long-term memory. Table 3 presents the results.

Table 2: Unit root test (stationary)

Statistic	**Augmented Dickey-Fuller Unit Root Test**			**Phillips-Perron test**			**KPSS**		
P-value	1%	5%	10%	1%	5%	10%	1%	5%	10%
Statistic return	-3.45	-2.87	-2.571	-3.45	-2.87	-2.571	0.7390	0.4630	0.3470
Steel return	-18.28232			-18.32216			0.136854		
Copper return	-15.66026			-15.51923			0.10303		
Oil return	-13.44695			-16.43994			0.046813		
Dollar index return	-14.29384			-18.94742			0.309774		
Gold return	-10.44354			-21.21554			0.092059		

Table 3: Long-term memory test

Test	R/S	GPH
Statistic	2.13	0.189

The test results indicate the absence of long-term memory in the return component of this data series. Therefore, the ARMA model is a suitable model for forecasting the average.

ARMA model estimation

According to the results of Tables 2 and 3, the average model is suitable for the steel return of the ARMA model. Given that four variables, namely copper price return, West Texas Oil price, US dollar index, and gold price, are used as control variables in this study, four relevant variables are used in the conditional average component to investigate the effect of these variables on the average component. Therefore, the ARMA model is estimated for each index in the conditional average component. The Box-Jenkins method is used to estimate ARMA. In this method, the number of autocorrelation terms (p) and moving average (q) terms are measured using autocorrelation (AC) and Partial Autocorrelation (PAC) functions, and their accuracy is calculated using Akaike information criterion (AIC), Schwarz Bayesian Information Criterion (SBC), and Hannan–Quinn Information Criterion (HQIC). The number of terms p and q is obtained by targeting the minimum rates of the criteria. Due to the use of normal and student's t distributions, and GED in the estimation of models in this research, the optimal model in each of these three distributions is selected according to the significance of p and q terms. Tables 4 and 5 separately examine AIC, SBC, and HQIC for GARCH family models. These tables indicate the number of p and q terms for GARCH models and their different distributions. As shown, the numbers of autocorrelation (p) and moving average (q) terms are different in each distribution.

Lagrange Multiplier (LM) test

After estimating the conditional average model and examining the presence of the ARCH effect, the Lagrange Multiplier (LM) test on the residuals of the steel price return time series and the F-value and χ^2 statistics indicate that this return has ARCH effects. In other words, the variances of the error term have autocorrelation.

GARCH family models

After estimating the ARMA model and performing the LM test of the ARCH effect, we estimate two models from the GARCH family (GARCH and EGARCH) based on three distributions, normal, student's t, and GED, that result in six models. Table 7 presents the results of the 6 models.

Table 4: Examination of the number of ARMA (p, q) terms in GARCH models

	AR	**MA**	**AIC**	**SBC**	**HQIC**
GARCH/N	1	1	-4.02752	-3.95951	-4.00041
	1	2	-4.02576	-3.94642	-3.99414
	2	1	-4.04348	-3.96396	-4.01178
	2	2	-4.04179	-3.95090	-4.00556
GARCH/T	1	1	-4.46961	-4.39026	-4.43798
	1	2	-4.46369	-4.37300	-4.42754
	2	1	-4.47950	-4.38862	-4.44327
	2	2	-4.50133	-4.39909	-4.46057
	3	3	-4.51845	-4.39321	-4.46852
	3	2	-4.52225	-4.40840	-4.47686
GARCH/GED	1	1	-4.24921	-4.18120	-4.22210
	1	2	-4.46513	-4.37444	-4.42898
	2	1	-4.47188	-4.38100	-4.43565
	2	2	-4.46878	-4.36654	-4.42803
	3	3	-4.47471	-4.34947	-4.42478
	3	2	-4.48456	-4.37070	-4.43917

Table 5: Examining the number of ARMA(p, q) terms in EGARCH models

	AR	**MA**	**AIC**	**SBC**	**HQIC**
EGARCH/N	1	1	-4.04580	-3.96645	-4.01417
	1	2	-4.04931	-3.95863	-4.01317
	2	1	-3.93830	-3.84742	-3.90207
	2	2	-3.47523	-3.37299	-3.43448
EGARCH/T	1	1	-4.47734	-4.38666	-4.44120
	1	2	-4.47152	-4.36950	-4.43085
	2	1	-4.48751	-4.38527	-4.44675
	2	2	-4.48545	-4.37184	-4.44016
	3	3	-4.51330	-4.37667	-4.45883
	3	2	-4.52057	-4.39533	-4.47064
EGARCH/GED	1	1	-4.46566	-4.37498	-4.42951
	1	2	-4.46001	-4.35799	-4.41935
	2	1	-4.47956	-4.37732	-4.43880
	2	2	-3.46928	-3.35568	-3.42400
	3	3	-4.47549	-4.33887	-4.42102
	3	2	-4.47022	-4.36298	-4.43829
	2	3	-3.46356	-3.33860	-3.41375

Table 6: Lagrange Multiplier test (ARCH effect)

Test	F**-statistic**	**Chi-Squared**
Test statistic	8.669680	8.501425
Probability	0.0035	0.0035

Table 7: The average model of estimate variance (ARMA- GARCH) for the steel price return

GARCH	Normal	Conditional average	$y_t = 0.001 + 0.389y_{t-1} + 0.176\nu_{copper} + 0.010\nu_{wti} + 0.417\nu_{usdx} - 0.069\nu_{\text{gold}}$
		Conditional average	$\sigma_t^2 = 0.0001 + 0.433\varepsilon_{t-1}^2 + 0.608\sigma_{t-1}^2$
	Student's t	Conditional average	$y_t = 0.001 + 0.390y_{t-1} + 0.177\nu_{\text{copper}} + 0.010\nu_{wti} + 0.417\nu_{usdx} - 0.069\nu_{\text{gold}}$
		Conditional average	$\sigma_t^2 = 0.0001 + 0.434\varepsilon_{t-1}^2 + 0.609\sigma_{t-1}^2$
	GED	Conditional average	$y_t = 0.001 + 0.165y_{t-1} + 0.073\nu_{\text{copper}} + 0.0141\nu_{wti} + 0.129\nu_{usdx} - 0.026\nu_{\text{gold}}$
		Conditional average	$\sigma_t^2 = 0.0001 + 0.302\varepsilon_{t-1}^2 + 0.580\sigma_{t-1}^2$
EGARCH	Normal	Conditional average	$y_t = 0.0007 + 0.347y_{t-1} + 0.169\nu_{\text{copper}} + 0.0111\nu_{wti} + 0.477\nu_{usdx} - 0.036\nu_{\text{gold}}$
		Conditional average	$\log(\sigma_t^2) = 0.753 + 0.514\left\lvert\frac{\varepsilon_{t-1}}{\sigma_{t-1}^2}\right\rvert + 0.014\frac{\varepsilon_{t-1}}{\sigma_{t-1}^2} + 0.940\ \log(\sigma_{t-1}^2)$
	Student's t	Conditional average	$y_t = -1.35e^{-6} + 0.318y_{t-1} + 0.122\nu_{\text{copper}} + 0.010\nu_{wti} + 0.039\nu_{usdx} - 0.062\nu_{\text{gold}}$
		Conditional average	$\log(\sigma_t^2) = -2.207 + 0.995\left\lvert\frac{\varepsilon_{t-1}}{\sigma_{t-1}^2}\right\rvert + 0.067\frac{\varepsilon_{t-1}}{\sigma_{t-1}^2} + 0.710\log(\sigma_{t-1}^2)$
	GED	Conditional average	$y_t = 0.0006 + 0.0685y_{t-1} + 0.072\nu_{\text{copper}} + 0.015\nu_{wti} + 0.093\nu_{usdx} - 0.0004\nu_{\text{gold}}$
		Conditional average	$\log(\sigma_t^2) = -1.408 + 0.498\left\lvert\frac{\varepsilon_{t-1}}{\sigma_{t-1}^2}\right\rvert + 0.001\frac{\varepsilon_{t-1}}{\sigma_{t-1}^2} + 0.841\log(\sigma_{t-1}^2)$

In Table 7, the significance of the variables is confirmed in all six models. Copper has a positive effect on the steel price volatility return trend in all the models. Similarly, oil has a positive minor effect on the steel return. The dollar return index has a positive effect on steel price return. Furthermore, the effect of gold return on steel price return is negative in the above six models.

In terms of steel price performance, copper price return is significant in all models. Given the positive signs of its coefficients, the copper price return has a direct effect on the conditional average return of steel price. In terms of oil return, it has a positive significant effect on steel price return. Furthermore, the dollar index return has a positive and direct effect on steel price return in all models. The coefficients are significant in terms of gold return, but due to the negative sign, it can be concluded that the gold return has an inverse effect on the conditional average of the steel price return. This issue can be explained as follows: Gold is always a safe haven for investors in economic turmoils and crises. Assuming that the world economy is in crisis or has no prospects for prosperity, on the one hand, investors buy gold, and the gold price increases, and on the other hand, buyers are less willing to buy basic metals and commodities.

In the EGARCH model, there is no significant difference between the coefficients of the parameters of the average model and the conditional variance in steel price return based on normal and student's t distributions and GED, and the significance of the coefficient $\left\lvert\frac{\varepsilon_{t-1}}{\sigma_{t-1}^2}\right\rvert$ shows the asymmetric reaction of steel price returns to external shocks and the leverage effect (effects of positive and negative volatility). Therefore, asymmetry is accepted in returns.

The target models are re-estimated after entering the variables, namely copper, West Texas oil, the dollar index, and gold returns into the conditional average equations.

In the GARCH model, there is no significant difference between the coefficients of the parameters of the average model and the conditional variance in steel price based on normal and student's t distributions, and GED. The coefficients in two models are not significant in terms of the control variable, copper. Similarly, the coefficients are not significant in 2 cases in terms of oil. In this model, the non-significance of these two variables indicates the lack of effect on the ARCH and GARCH coefficients and no change in the conditional variance of the model. The coefficients in the return variable of the dollar index are significant in all models, indicating the effects on the ARCH and GARCH coefficients. The significance of this variable can indicate the effect of the dollar index on the conditional variance of steel price returns in the short term.

In the EGARCH model, there is no significant difference between the average and conditional variance, and all coefficients are significant based on normal and student's t distributions, and GED. The significance of the coefficient $\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}^2}\right|$ indicates the asymmetric reaction of steel price return to external shocks and leverage effect. In this model, the significance of the returns of the variables indicates the contagion effect of the price volatility of copper, West Texas oil, the dollar index, and gold variables on the volatility of the steel price returns.

ANN model

Like the previous models, in which the steel price return variable is used as the dependent variable, and volatility of copper price, West Texas oil, the dollar index, and gold are used as control variables, the order of using the data is also similar in this model, but the artificial neural network model is used in this section. The mathematical equation of the ANN model is as follows:

$$f(x) = f(f(x_i w_{ij} + b_j) w_{jk} + b_k). \tag{0.21}$$

In Equation (0.21), f is the activation function, x is the input, b is the bias vector, and w is the output weight. The output of this layer is the input of the next layer to finally result in Y which is the prediction value. In this model, we use the Rectified Linear Unit (Relu) and Hyperbolic tangent (Tanh) function. To this end, different choices of the activation function are tested and compared in each layer. The comparative tables present each configuration of the ANN model and the types of activation functions in the model.

Choosing the optimal model

GARCH family models with the input variables, namely copper, oil, the dollar index, and gold, in normal and Student's t distributions and GED are first forecasted for a period of 14 days, and then the first ANN-GARCH model keeps the GARCH forecast constant, which is consistent with research by Kristjanpoller et al. [20]. Thereafter, they add other control variables using the initial parameters to the ANN model. After modeling and forecasting steel price volatility, we use GARCH family models in different distributions and hybrid ANN/GARCH models and use MSE and MAE criteria to examine the forecasting accuracy of the relevant models during a 14-day period. Table 8 shows the forecast results of the above-mentioned models.

Table 8: Forecast errors based on MSE and MAE criteria for different ANN hybrid neural network configurations

Row	**Model configuration**	**Activation function**	**A 14-day period**	
			MSE	**MAE**
1	GARCH/N	—	0.00138	0.02586
2	GARCH/T	—	0.00133	0.02501
3	GARCH/GED	—	0.00123	0.02432
4	ANN	RELU	0.00111	0.02471
5	ANN	TANH	0.00122	0.02508
6	ANN/GARCH/N	RELU	0.00105	0.02468
7	ANN/GARCH/N	TANH	0.00097	0.03017
8	ANN/GARCH/T	RELU	0.00105	0.02554
9	ANN/GARCH/T	TANH	0.00100	0.02373
10	ANN/GARCH/GED	RELU	0.00108	0.02577
11	ANN/GARCH/GED	TANH	0.00100	0.02422

Table 8 presents the forecast error of GARCH models, ANNs, and hybrid ANN models with different GARCH distributions for steel price volatility. The table also presents key observations. First, GED performs best in forecasting in individual GARCH models. Individual neural network models have generally better forecasts of price volatility than individual GARCH models. In the individual models of the ANN, the network with the Relu function has a better forecast of price volatility. Second, even though the neural network models can perform better than GARCH models, combining the GARCH forecasts as input into ANN network models can increase the predictive power of network models.

For example, the MSE error in the ANN/GARCH/N model with the Tanh function is equal to $9.73e^{-4}$, which has the lowest error in neural network and GARCH models and is about 21% lower than the best forecast error of individual GARCH models and is a significant improvement for forecast in terms of accuracy. This issue is confirmed in the MAE error analysis for hybrid models. The hybrid ANN/GARCH/T model with Tanh activator is equivalent to $2.373e^{-2}$ which is smaller than the forecast errors in individual GARCH models. Similar results can be found for MAE and MSE with a few exceptions.

Thereafter, modeling and forecasting of the steel price volatility are performed using EGARCH family models and hybrid ANN/EGARCH models. The forecast accuracy of the relevant models is then examined using the MSE and MAE criteria. Table 9 presents the forecast results of the models.

Table 9: Forecast error based on MSE and MAE criteria for different configurations of hybrid ANN and EGARCH models

Row	Model configuration	Activation function	A 14-day period	
			MSE	MAE
1	EGARCH/N	—	0.00116	0.02185
2	EGARCH/T	—	0.00123	0.02401
3	EGARCH/GED	—	0.00119	0.02312
4	ANN	RELU	0.00111	0.02471
5	ANN	TANH	0.00122	0.02508
6	ANN/EGARCH/N	RELU	0.00107	0.02510
7	ANN/EGARCH/N	TANH	0.00096	0.02090
8	ANN/EGARCH/T	RELU	0.00109	0.02153
9	ANN/EGARCH/T	TANH	0.00111	0.02407
10	ANN/EGARCH/GED	RELU	0.00107	0.02551
11	ANN/EGARCH/GED	TANH	0.00102	0.02085

Table 9 presents the forecast error rates of EGARCH, ANN, and hybrid ANN and LSTM-ANN models with different EGARCH distributions for steel price volatility. The table also presents the following key observations. In EGARCH models, N distribution has the best performance in forecasting. The neural network models, except for the artificial neural network with Tanh activation function, have better forecasts of price volatility than single EGARCH models. Furthermore, among the individual distributions of the ANN, the network with the Relu function has a better forecast of price volatility. Even though neural network models can perform better than the EGARCH model, combining EGARCH forecasts as input in ANNs can increase the predictive power of network models. For example, the MSE error in the ANN/EGARCH/N model with the Tanh activator is equal to $9.64e^{-4}$, which has the lowest error in the hybrid neural network and EGARCH models, is almost 18% less than the minimum MSE prediction error in the single EGARCH models. The reduction of forecast error is also confirmed in the examination of the MAE error for the hybrid models. The hybrid ANN/EGARCH/GED model with the Relu function is equivalent to $2.085e^{-2}$ that is lower than the forecast errors in the individual EGARCH models. Similar results can be achieved in the comparison of hybrid ANN/EGARCH models and EGARCH family models in MAE and MSE errors, with some exceptions.

According to the comparison of the results of the MSE and MAE error criteria in Tables 8 and 9, among the 12 hybrid models and 8 individual models, the ANN /EGARCH/N model with the Tanh function equal to $9.60e^{-4}$ has the lowest MSE error value that is also confirmed in the MAE criterion. Among the reviewed models, the best estimation of upcoming steel volatility is provided by the hybrid ANN/EGARCH/N model with the Tanh function.

The real and forecasted values are displayed in a diagram to understand the ability of this model to forecast the volatility of the main stock market index in a more intuitive way:

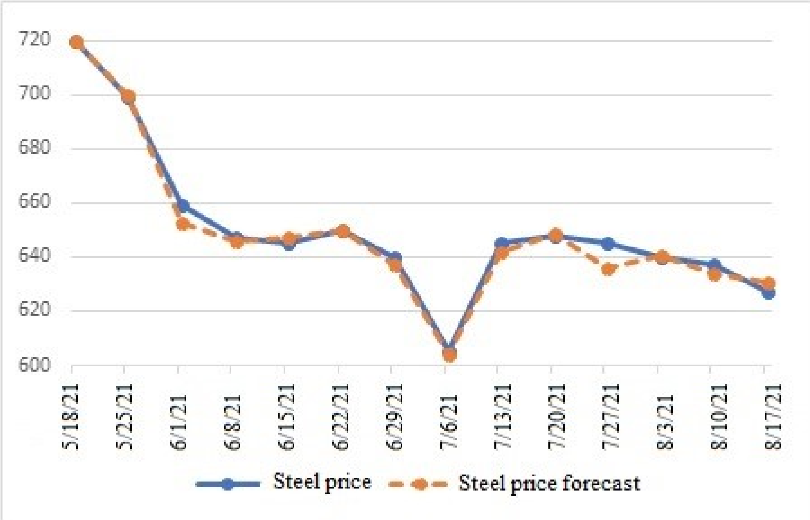

Figure 4: Comparison of forecasted values of the hybrid model with real values

Conclusion

Due to the development of information and data processing technology in the field of artificial intelligence in recent years, a group of financial researchers has utilized this approach to take the advantage of benefits of this tool in the commodity and financial markets through forecasting asset prices. To this end, the present research introduced new hybrid models based on artificial intelligence neural network and GARCH family models and examined the potential of these hybrid models against classical forecast models to predict price volatility in CIS steel ingots. According to the research findings and based on the MSE and MAE criteria, the forecast power of the hybrid ANN/EGARCH/N model with the Tanh function was more accurate than other models. Furthermore, hybrid neural network models and GARCH family models had lower errors in forecasting than classical GARCH family models. As a final result, like other previous studies (e.g. Kristjanpoller [6, 7] and Zolfaghari [22]), hybrid models based on neural networks had a better potential to forecast prices than other time series models.

Since volatility forecasting has many applications in risk management, portfolio valuation, and derivatives pricing, and it is significantly important to understand volatility, accurate forecast, and protecting the portfolio against the costs imposed by this variable on the total value, the use of such a structure, which almost considers all effective factors (political news, internal, and external shocks) in the price, can be practical and useful.

Previous studies used the majority of econometric and artificial neural network models separately, but, on the contrary, hybrid modeling was conducted and ANN with different activation functions was investigated in the present research; however, the results also showed improvement compared to previous studies. The following suggestions are offered about research and future studies based on the results and content of this study, as well as the review of previous studies:

- Using the hybrid econometric family models and meta-heuristic algorithms and hybrid recurrent ANNs such as LSTM and RNN
- Using the hybrid model introduced in the present research to forecast other economic, financial, commodity, and derivative market data such as exchange rates, basic commodity prices, future contracts of stock, coins, and other standard products in commodity exchanges.

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